"Weak" Cosmic Censorship[1]

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It is known from the singularity theorems of general relativity (see Hawking and Ellis, 1973) that, under a variety of circumstances, solutions to Einstein's equation with physically reasonable matter must develop singularities. In particular, for a sufficiently compacted body, trapped surfaces must be present (Schoen and Yau, 1983), and collapse to a singularity must occur. Of crucial importance for the theory of gravitational collapse is the issue of the nature of the final state resulting from such a collapse. The idea that physically realistic gravitational collapse always results in a black hole—so that no "naked singularities", visible to a distant observer, can occur—was first conjectured by Penrose (1969), although it had been implicitly assumed in many discussions and analyses prior to that time.

A related conjecture was later made by Penrose (1979). It states that all singularities occurring in physically reasonable spacetimes (i.e., not merely those resulting from the gravitational collapse of an isolated body) must have a "spacelike or null" character, so that one can never "see" a singularity except by "running into it". Thus, this conjecture also states that, in some sense, singularities are never "visible" to a distant observer.

These two conjectures are commonly referred to as the "weak" and "strong" versions of the cosmic censorship conjecture, respectively. This terminology is somewhat misleading in that the two conjectures are logically independent: "Weak" cosmic censorship does not imply "strong" cosmic censorship, since "weak" cosmic censorship makes no assertions about singularities other than those resulting from the gravitational collapse of an isolated body, nor does it say anything about the character of singularities within a black hole (i.e., such singularities could be "visible" to observers who fall into a black hole). On the other hand, "strong" cosmic censorship does not imply "weak" cosmic censorship: If gravitational collapse resulted in a singularity which propagated to infinity in a spacelike or null fashion, "strong" cosmic censorship would hold for this process, but "weak" cosmic censorship would be violated. Thus, "strong" cosmic censorship is a stronger conjecture only in the sense that processes which violate "weak" but not "strong" cosmic censorship generally are viewed as being highly implausible. Since the terminology of "weak" and "strong" has become standard in referring to these two—related but independent—conjectures concerning "cosmic cen-

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sorship", I will use these terms in the discussion below, and will drop the "quotes" around them hereafter, despite the misleading aspects of this terminology.

The fundamental issue addressed by strong cosmic censorship is determinism. Since the time of Newton, the character of the laws of classical (i.e., non-quantum) physics has been such that the fundamental quantities appearing in these laws obey differential equations, and the solutions to these differential equations are uniquely determined by specifying appropriate "data" at an initial time. Thus, the laws have the character that one can predict everything—for all time in the past and future—in the universe from a complete knowledge of the state of the universe at a single instant of time. (The laws of quantum physics also have this basic character, but in quantum physics, a complete knowledge of the state of the system allows one to make only probabalistic predictions for the results of physical observations.) The occurrence of singularities in general relativity poses a threat to this classical determinism: If a singularity with a timelike character formed, the differential equations of classical physics would not uniquely determine the solutions to these equations in the regions where this singularity is "visible". Thus, either these differential equations would have to be supplemented by new laws of physics governing what can emerge from a singularity, or a new element of indeterminism would appear in classical physics.

Strong cosmic censorship asserts that neither of these possiblities need be confronted. More precisely, the absence of timelike singularities in a (time oriented) spacetime can be shown to be equivalent to global hyperbolicity (Penrose, 1979). In a globally hyperbolic spacetime, there exist achronal hypersurfaces, known as a *Cauchy surfaces*, having the property that every inextendible timelike curve intersects each Cauchy surface at precisely one point. For suitable (i.e., hyperbolic) differential equations, knowledge of appropriate data on a Cauchy surface uniquely determines a solution throughout the spacetime. Thus, a Cauchy surface in a globally hyperbolic spacetime provides a suitable notion of an "instant of time" from which all past and future occurrences can be predicted. Thus, if strong cosmic censorship holds, then despite the presence of singularities, the nature of the physical laws and determinism in general relativity will be of the same fundamental character as in prior theories of classical physics. Since the issues related to strong cosmic censorship are treated in the contribution by Geroch, I will not discuss strong cosmic censorship further here.

The fundamental issue addressed by weak cosmic censorship can be expressed in graphic terms by posing the following question: Could a mad scientist—with arbitrarily large, but finite, resources—destroy the universe? The singularity theorems of general relativity indicate that this might be possible, since, in particular, by gathering a sufficiently large amount of mass into a sufficiently small region, we know that it would be possible for such a mad scientist to create a spacetime singularity. In essence, weak cosmic censorship asserts that he could not destroy the universe in this way: Neither the singularity he could produce nor any of its effects can propagate in such a way as to reach a distant observer. In the remainder of this discussion, I shall give a somewhat more mathematically precise formulation of weak cosmic censorship, and I then shall discuss the evidence concerning the validity of this conjecture within the theory of general relativity.

First, by "finite resources" in the above question, we mean, in essence, that only a finite amount of energy in a finite region of space is availble to our mad scientist, so that whatever he creates can be viewed (initially, at least) as an "isolated system". The notion of an initially isolated system is well modeled in general relativity by the concept of a spacetime which is asymptotically flat on an initial spacelike hypersurface. Here, by "asymptotically flat", we mean that the spacetime metric approaches that of flat (Minkowski) spacetime at large distances from some compact ("central") region on the hypersurface; the precise fall-off conditions on the spacetime curvature need not concern us here. If the spacetime remains asymptotically flat for all time, we can consider the family of observers who remain in the (nearly flat) asymptotic region for all time. The events which are "visible", in principle, to these observers are those which lie to their past. Any events in the spacetime which do not lie to the past of these observers are said to be contained within a *black hole*.

Weak cosmic censorship asserts that all singularities produced in nature (or by mad scientists) from an initially isolated system are confined to the interior of black holes. Thus, the conjecture states that observers in the spacetime can live out their lives in their entirety, free from any catastrophic or non-deterministic effects of the singularities which may occur—provided, of course, that these observers have the prudence to steer clear of any black holes that may form. This idea can be stated more precisely as follows: Consider asymptotically flat initial data on a spacelike hypersurface (with compact "interior region") for a solution of Einstein's equation with *suitable* matter. Then the maximal Cauchy evolution of this data (i.e., the "largest" spacetime uniquely determined by this data and Einstein's equation) *almost always* is asymptotically flat for all time, so that distant observers can "live forever" and never "see" any singularities.

In the above formulation of the weak cosmic censor hypothesis, the notion of asymptotic flatness at null infinity (with complete "scri") provides a suitable, mathematically precise notion of the spacetime being "asymptotically flat for all time". (See, e.g., Wald (1984a) for the definition of asymptotic flatness at null infinity and, more generally, for the definitions of any technical terms occurring in this discussion.) However, the conjecture remains mathematically imprecise on account of the two terms which I italicized above: "suitable" and "almost always". Two obvious necessary conditions on matter for it to be "suitable" are that it be governed by deterministic (i.e., hyperbolic) differential equations and that it have locally positive energy density (more precisely, that its stress-energy tensor obey the dominant energy condition). However, perfect fluids with certain equations of state obey both of these conditions, yet are known to yield counterexamples to the above conjecture. For reasons which will be discussed further below, these counterexamples are generally viewed as resulting from the fluid matter not being a "suitable" model for the fundamental matter fields occurring in nature, rather than as an indication of the failure of weak cosmic censorship. A provisional, additional requirement on matter fields for them to be "suitable" is that when their differential equations are evolved on a fixed, nonsingular, globally hyperbolic spacetime (such as Minkowski spacetime), one always obtains globally nonsingular solutions. (Consequently, any singularities occurring in the Einstein-matter system necessarily would be attributable to gravitational effects.) The perfect fluid matter which is known to yield counterexamples does not satisfy this additional requirement.

The "almost always" condition was inserted in the above statement because it would not be fatal to the physical content of the conjecture if some counterexamples exist, provided that the initial data required for these counterexamples is so special (i.e., a "set of measure zero") that it would be physically impossible to achieve. Indeed, the known fluid counterexamples mentioned above are of this character, so even if one wished to include fluids as "suitable" matter, it is possible that no physically achieveable counterexamples could be constructed with fluids. Whether one needs to include the word "almost" in the conjecture may well depend on the precise requirements imposed upon the matter via the term "suitable". In my view, until we have a deeper understanding of the dynamics implied by Einstein's equation with matter, there is not much point in attempting to refine further the notions of "suitable" and "almost always" appearing in the above formulation of the conjecture.

Does weak cosmic censorship hold, i.e., is it a property of classical general relativity? To answer to this question, one would need to know a great deal about the global properties of solutions to Einstein's equation. However, Einstein's equation comprises a system of nonlinear partial differential equations, and—apart from some local existence and uniqueness theorems—very little is known about the general properties of solutions to such equations. Some important progress has been made in recent years (Christodoulou and Klainerman 1993) toward establishing global existence properties of solutions to Einstein's equation with nearly flat initial data. However, mathematical techniques have not progressed to the stage where a direct attempt at a general proof (or disproof) of weak (or strong) cosmic censorship would be feasible. Thus, the evidence both for and against the validity of weak cosmic censorship is largely circumstantial in nature.

The main evidence in favor of weak cosmic censorship is as follows: First, exactly spherically symmetric gravitational collapse of a fluid body (with no matter outside of its surface) always produces a black hole as the final state. (This is basically an immediate consequence of Birkhoff's theorem, which states that the Schwarzschild solution—which describes a black hole—is the only spherically symmetric solution of Einstein's equation in vacuum.) However, since the dynamical degrees of freedom of the gravitational field are suppressed by the assumption of spherical symmetry, it is far from clear that spherically symmetric collapse is a representative case.

Nevertheless, it is tractable to study small perturbations of spherical collapse, wherein all the dynamical degrees of freedom are restored. Theorem 2 of Kay and Wald (1987) provides a rigorous statement and proof of the result (obtained previously in a less rigorous manner by Price (1972)) that, in the linear approximation, spherical collapse to a Schwarzschild black hole is stable. (Only perturbations produced by a scalar field are treated in this theorem, but there should be no difficulty extending the theorem to Einstein-fluid perturbations.) This indicates—but does not prove—that gravitational collapse with sufficiently small departures from spherical symmetry also will result in a black hole. The fact that the Kerr black hole also has been shown to be stable in linear perturbation theory (Whiting 1989) lends further important plausibiltiy to the idea that black holes describe final states of gravitational collapse. However, the linear perturbation analyses cannot be used to draw any conclusions about what may happen in a highly nonspherical gravitational collapse.

A third, and final, important body of evidence in favor of weak cosmic censorship comes from the remarkable internal consistency of the theory of black holes. Much of the theory of black holes developed in the past 25 years is founded upon the assumption of weak cosmic censorship, and many key results—such as the "area theorem", which states that the surface area of a black hole cannot decrease with time—explicitly assume the validity of this conjecture. The fact that many nontrivial results have been derived assuming weak cosmic censorship, but no inconsistencies have ever been uncovered, thus provides some evidence in favor of its validity.

An illustration of this internal consistency of the theory of black holes is provided by the following simple example: It is known that a stationary (i.e., time independent) black hole solution in general relativity cannot have its electric charge, Q, be greater than its mass, M, (in units where the gravitational constant, G, and speed of light, c, are set equal to 1). Hence, a contradiction with weak cosmic censorship would be obtained if, starting with a black hole with $Q = M$, we could drop in a particle with $q >$

m, where q and m denote, respectively, the charge and mass of the particle. However, it turns out that such a particle will not fall into the black hole, since the Coulomb repulsion on the particle will exceed its gravitational attraction. (The particle could be "thrown" into the black hole, but the additional energy thereby given to the particle will increase the mass of the black hole in such a way as to keep the black hole mass at least as great as its electric charge.) Other, similar, "gedanken experiments" to destroy a black hole fail in a similar manner (Wald,1974), as have other attempts to uncover contradictions with weak cosmic censorship (see, e.g., Jang and Wald 1977).

The evidence in favor of weak cosmic derived from the failure of attempts to find inconsistencies within the theory of black holes is, of course, is quite indirect and hard to quantify. However, many researchers (including myself) find this evidence quite compelling.

We turn, now, to the evidence against the validity of weak cosmic censorship in general relativity. An important body of evidence against this conjecture comes from the study of the collapse of spherically symmetric fluid bodies. Although the collapse of such bodies ultimately produces a Schwarzchild black hole, for suitable equations of state of the fluid (such as "dust", i.e., a fluid without pressure), solutions exist in which singularities develop at "early times" (prior to the formation of the black hole) which are "visible" from infinity. In particular, for dust, one can produce "shell-crossing singularities", wherein a "shell" of dust initially at one radius overtakes a similar "shell" of dust at an initially smaller radius, resulting in infinite density of the dust at the moment when these shells cross. This can occur long before any collapse to a black hole takes place. "Shell focusing" singularities in which the density of the dust becomes infinite at the origin in a manner visible from infinity also can occur. In addition, for other equations of state, solutions with shock waves and other fluid singularites visible from infinity also exist.

The main reason why I (and, I believe, most other researchers) do not view these solutions as valid counterexamples to weak cosmic censorship is that perfect fluids are only an effective, macroscopic approximation to a fundamental description of matter, and they possess properties that are not shared by what are believed to be fundamental descriptions of matter at the classical level. A fluid is described by assigning an energy density and four-velocity at each event within the fluid. (The pressure is then determined from the energy density via the equation of state of the fluid.) In particular, for dust in a fixed, nonsingular, background spacetime (such as Minkowski spacetime), starting with a smooth, bounded density and velocity of the dust, one easily can arrange to concentrate a finite amount of mass-energy into zero volume by "aiming" the dust appropriately. It is precisely the ability to do this that allows the above mentioned "shell-crossing" and "shell-focusing" singularities to occur in gravitational collapse. However, this kind of concentration of energy into zero volume cannot be done for fields which are believed to be realistic, fundamental descriptions of matter at the classical level. In particular, for the electromagnetic field (which is the only fundamental, classically describable field—apart from gravity itself—known to exist in nature), any initially nonsingular solution of Maxwell's equations in an arbitrary, nonsingular, globally hyperbolic spacetime remains nonsingular for all time; one cannot focus a finite amount of energy in electromagnetic waves into zero volume. It seems plausible that all other fundamental (as opposed to effective, macroscopic) classical matter will share this property. As already indicated in our above discussion of the formulation of weak cosmic censorship, in order to obtain a convincing counterexample, it will be necessary for the matter to be such that no singularities in the matter can develop when it is evolved in a fixed, nonsingular, globally hyperbolic background spacetime.

Another reason why the naked singularities which can be produced in the collapse of fluids are generally viewed as not providing valid counterexamples to weak cosmic censorship is that the known examples are "non-generic". In particular, on account of the careful aiming that is needed to produce "shell-crossing" or "shell-focusing" singularities, the initial data which gives rise to such singularities comprises only a "set of measure zero". However, I would not be surprised if generic examples could be constructed of fluid collapse with nonzero pressure in which there occur shocks or other similar singularities which are visible from infinity. Thus, in my view, it is the apparent failure of fluids to provide an adequate model of fundamental matter—rather than the non-generic character of the known solutions with naked singularities—which invalidates the fluid solutions as counterexamples to weak cosmic censorship.

In some recent work, Christodoulou (1993) has shown that naked singularities do occur in some spherically symmetric solutions to Einstein's equation with a scalar field source. The scalar field meets all reasonable mathematical criteria for "fundamental matter", and hence certainly should qualify as "suitable" matter in the weak cosmic censor conjecture. However, Christodoulou (unpublished) also has proven that these solutions comprise a "set of measure zero" within the class of all spherically symmetric Einstein-scalar-field solutions, so that, in fact, weak cosmic censorship (with a suitable definition of "almost always") does hold in this case.

A second body of evidence sometimes cited as suggesting the failure of weak cosmic censorship comes from the study of collapse of matter with cylindrical symmetry. (By cylindrical symmetry, we mean rotational symmetry about one axis, and translational symmetry in the direction along that axis; more precisely, we have two commuting spacelike Killing fields, one of which has closed orbits and vanishes on an "axis".) Cylindrically symmetric fluids can collapse to singularities, but it is known that trapped surfaces never form in this case (Thorne 1972; Chrusciel 1990), and that the singularities are "visible" from infinity. These cylindrically symmetric solutions do not provide direct counterexamples to weak cosmic censorship because the spacetimes involved are not asymptotically flat in the usual sense: The matter distribution extends out to infinity along the symmetry axis, and the gravitational field falls off too slowly in the direction perpendicular to the symmetry axis. However, these examples suggest that weak cosmic censorship could be violated in the collapse of very long, but finite, cylinders. Indeed, the singularity produced in the collapse of a sufficiently long but finite cylinder must be of the same local character as that occurring in the collapse of an infinite cylinder, so it should be possible for a light ray emitted from the singularity to propagate "far away" from it. It is less clear that this light ray must actually reach infinity, but, for sufficiently long cylinders, it would appear quite plausible that this would be the case.

The main reason why I do not feel that cylindrical collapse provides strong evidence against weak cosmic censorship is the same reason as for spherical collapse: The known examples involve fluids and appear to be based upon the ability to concentrate (in a nonsingular, background spacetime) a finite amount of energy of matter into zero volume. Indeed, it seems plausible that the singularities produced in the collapse of dust cylinders are analogous to the "shell focusing" singularities of spherical dust collapse mentioned above. This view is supported by the fact that the collapse of a cylindrical dust shell to a singularity is "non-generic"; an arbitrarily small amount of rotation causes a "bounce" (Apostolatos and Thorne 1992). No examples are known of cylindrical collapse to a singularity with matter described by fundamental fields. Indeed, it appears very likely that one could prove that no singularities can develop from suitable, nonsingular, data for cylindrically symmetric gravitational waves or Einstein-Maxwell solutions. (The vacuum case where the two Killing fields are orthogonal should be straightforward to treat, whereas it should be possible to treat the general case using recent results of Christodoulou and Tahvildar-Zadeh (1993).) The issue of absence of singularities for cylindrically symmetric solutions with fundamental fields would appear to be worthy of further investigation.

The marked difference in behavior between spherical collapse (where a Schwarzschild black hole is—generically, at least—the final state) and cylindrical collapse (where no analog of a black hole ever occurs) has spawned an idea, known as the *hoop conjecture*, which has played a role in some discussions of cosmic censorship. The hoop conjecture states that "black holes with horizons form when and only when a mass, M, gets compacted into a region whose circumference in *every* direction is less than or equal to $4\pi M$" (Thorne 1972; see also "box 32.3" of Misner, Thorne, and Wheeler 1973). (Here, one envisions "passing a hoop" of circumference $4\pi M$ around the matter in every direction to test this criterion.) This conjecture is quite vague for at least the following two reasons. First, there is no local definition of (total) mass density (including gravitation) in general relativity, so it is far from clear what one means by "compacting a mass, M, into a region". (The difficulties involved in making sense of this idea become particularly striking if one restricts the hoop conjecture to the pure vacuum case and attempts to get criteria for determining when gravitational waves might collapse to a black hole.) Second, it is far from clear that any sensible notion of the "circumference" of a region (as determined by "hoops" exterior to the region) can be given. In particular, arbitrarily nearby (in spacetime) to any given two-dimensional spacelike surface (such as one bounding a given region), one can find surfaces (well approximated by suitably chosen broken null surfaces) which have arbitrarily small "size" in every direction. Thus, one always can "pass a hoop" of arbitrarily small circumference around any world tube in any spacetime.

Nevertheless, a result closely related to the "when" half of the hoop conjecture has been made precise and proven to hold by Schoen and Yau (1983). As already mentioned above, they proved that for a sufficiently compacted body, trapped surfaces must be present. (They avoided the above difficulties by using as the notion of mass in their theorem an integral of the local energy density of matter on a maximal hypersurface, and using as the notion of "size" of the region a measure of the "internal geometry" of the region rather than a "hoop" criterion.) Note, however, that while the presence of a trapped surface implies the occurrence of a singularity, it implies the formation of a black hole only under the assumption of weak cosmic censorship, so the theorem of Schoen and Yau does not prove the "when" half of the hoop conjecture.

For the reasons just indicated above, I feel that it is unlikely that a sensible formulation (no less proof) of the "only when" half of the hoop conjecture can be given. However, even if we suppose that a sensible formulation of the "only when" half of the hoop conjecture can be given, its validity does not seem to me to be at all incompatible with weak cosmic censorship: It could well be that (fundamental) matter which fails to be sufficiently compacted in every direction simply fails to undergo collapse to a singularity. Indeed, this view is highly compatible with the views expressed above on the nature of the singularities occurring in cylindrically symmetric collapse. Nevertheless, there appears to be a close correspondence between researchers who believe in the validity of the "only when" half of the hoop conjecture and researchers who believe that weak cosmic censorship will fail for highly nonspherical collapse.

A third piece of evidence cited as indicating a failure of weak cosmic censorship comes from recent numerical simulations of gravitational collapse by Shapiro and Teukolsky (1991). They considered the collapse of prolate spheroids of dust matter, and numerically evolved the Einstein equations (with the constant time hypersurfaces chosen to be maximal slices) until a singularity was reached. They then searched for trapped surfaces on their constant time surfaces. For spheroids which initially were not excessively prolate, they found that trapped surfaces were present. However, for highly prolate spheroids, no trapped surfaces were found. Furthermore, the singularity encountered in the highly prolate case occurred outside of the collapsing matter, thus indicating that its presence was not an artifact of an unrealistic choice of matter distribution. These results were interpreted by them, the New York Times, and others as evidence for the failure of weak cosmic censorship for highly nonspherical collapse, and as support for the "only when" half of the hoop conjecture discussed above.

The absence of trapped surfaces in a spacetime does not imply the absence of a black hole. Indeed, the "extreme" charged Kerr black holes do not possess trapped surfaces. However, the event horizon of a stationary black hole always is a marginally outer trapped surface. Furthermore, the intuitive picture held by most researchers on the nature of gravitational collapse (which, however, is based mainly on the analysis of spherical collapse) suggests that collapse to a black hole should be accompanied by the presence of trapped surfaces. Thus, if no trapped (or marginally outer trapped) surfaces were present in the entire spacetime describing the collapse of the dust spheroid, it would be natural to interpret this as providing strong evidence against weak cosmic censorship. However, in fact, only a portion of this spacetime was numerically constructed by Shapiro and Teukolsky, since their computer code "crashes" as soon as a singularity is reached at any gridpoint on the spacelike slices used as constant time hypersurfaces. Thus, it is quite possible that trapped surfaces do form around the collapsing body, but that the portion of the spacetime that they construct covers only an "early time" region around the collapsing body (at some angles), so that no trapped surfaces have yet formed on their time slices. This possibility is made more plausible by the fact that, for a standard, spherically symmetric collapse to a Schwarzschild black hole, it is possible to choose a time slice which "touches" the singularity within the black hole (outside of the collapsing matter) at one angle, such that no trapped surfaces lie on this slice or anywhere within its past (Wald and Iyer 1991). Thus, until more of the spacetime considered by Shapiro and Teukolsky can be explored numerically, it will not be known whether their failure to find trapped surfaces is a property of the spacetime or merely an artifact of the time slicing they chose. Unfortunately, it will be a major project to revise their computer code to enable much more general choices of time slicing, so this issue is not likely to be fully resolved in the very near future.

A final argument against weak cosmic censorship arises from outside the domain of classical general relativity. Quantum particle creation occurring near a black hole will cause it to radiate energy to infinity (Hawking,1975). As a result of this process, an isolated black hole should radiate away all of its energy—and thus "evaporate" completely—within a finite amount of time. However, one can show quite generally (Kodama 1979; Wald 1984b) that any classical spacetime model in which a black hole forms and evaporates must possess a naked singularity. Of course, such a naked singularity would not count as a true counterexample to weak cosmic censorship, since it lies outside the context of that conjecture. However, if a gross violation of cosmic censorship by a quantum process occurred, this would suggest that classical violations might also be possible. In fact, however, as I have argued elsewhere (Wald 1984b), the violation of weak cosmic censorship associated with the black hole evaporation process appears to be quite mild, and quite compatible with the spirit of classical cosmic censorship. Thus, I do not view the quantum black hole evaporation process as providing evidence against weak cosmic censorship.

In summary, given that the validity of the weak cosmic censor conjecture has been widely recognized as one of the most important issues in classical general relativity for more than 20 years, there is remarkably little evidence either for or against this conjecture.

What are the prospects for this situation to change in the near future? As already indicated above, in the absence of some truly dramatic, new breakthroughs in mathematics, it seems unlikely that a proof (or disproof) of weak cosmic censorship will be forthcoming in the foreseeable future. However, it is very likely that the validity of weak cosmic censorship will be probed and tested quite stringently within the next few years by numerical simulations. "Numerical relativity" has reached the stage where it is possible to reliably study the evolution of nonspherical spacetimes in which gravitational collapse to a singularity occurs, as exemplified by the work of Shapiro and Teukolsky cited above. A number of groups are well advanced in such projects, and it should not be too long before a considerable body of evidence concerning the conjecture will be obtained. Thus, I am optimistic that in the near future, much more will be known about the validity of weak cosmic censorship than is known today.

Note

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