Part II

CHAOS THEORY

https://doi.org/10.1017/S0270864700009176 Published online by Cambridge University Press

A Philosophical Evaluation of the Chaos Theory "Revolution"[1]

Stephen H. Kellert

Indiana University

1. Introduction

I would like to begin by addressing two pressing questions that may be raised regarding chaos theory: Is it actually about "chaos" at all? And is it actually a "theory?" The short answers to these questions are, respectively, "no" and "it depends." With respect to the first question, scientists have appropriated the ordinary term that means utterly unintelligible disorder. But chaotic behavior in the contemporary scientific sense is manifestly intelligible, and the word "chaos" may lead to misunderstandings as well as legitimate interest. So, in the first part of my presentation, I will offer a characterization of chaos theory and briefly address the use of this term in scientific enterprises.

That discussion will enable me to address the second and more pressing question of the status of these enterprises. Is chaos theory a theory at all? Is it a *science* at all? My intention in posing these questions is not to be belligerent or merely clever. Rather, I mean for these questions to lead to a philosophical evaluation of chaos theory. I will use various philosophical conceptions of the nature and structure of scientific theories to help understand this field of research, and I hope in the process to reflect on the adequacy of these philosophical conceptions as well. So in the second part of my presentation I will consider chaos theory in terms of Kuhnian notions, including that of disciplinary matrices. I will consider chaos theory in terms of the semantic conception of theories as models, and of Hacking's notion of a "style" of scientific reasoning. These considerations will lead me to suggest that chaos theory, while not qualifying as a Kuhnian scientific revolution, nonetheless represents an important new development in the sciences: the emergence of a new constellation of models and techniques and a new practice of scientific reasoning.

2. A Description of Chaos Theory

The central insight of chaos theory is that complex and unpredictable behavior can occur in systems governed by mathematically simple equations. This field of research has enjoyed tremendous growth in recent years. In 1980, no full-length books had been published on chaos, but by 1990 there were 125 books or conference reports, as well as over 4000 research papers (Dresden 1992, p. 10). But what exactly is

PSA 1992, Volume 2, pp. 33-49

chaos theory? I suggest the following definition: chaos theory is *the qualitative study of unstable aperiodic behavior in deterministic nonlinear dynamical systems*.

Beginning at the very end of this definition, we should note that the word "system" commonly means a situation investigated by scientists. By specifying the numerical value of all quantitative features of a system, one obtains a compact description of the way the system is at a certain time: the "state" of the system. A *dynamical system* includes both a recipe for producing such a mathematical description of the instantaneous state of a system and a set of rules, or "evolution equations," for transforming the current state description into a description for some future, or perhaps past, time.

Given the state of a dynamical system and the evolution equations, it is possible to calculate the state at other times by computing the changes in a system's variables in small increments. This procedure has the disadvantage of often becoming computationally unwieldy—such an "open-form" solution can require burdensome calculations. Some dynamical systems can be manipulated to yield a closed-form solution: a simple formula that allows one simply to plug the final time into the formula and find the final state of the system. Although very few problems allow a closed-form solution, this approach was long presumed to be the norm for most physical sciences: if a system did not allow such a solution, one sought an approximation.

One distinguishing mark of the dynamical systems of interest for chaos theory is the presence of *nonlinear* terms in the equations, terms such as x^2 or sin(x) that may stem from the inclusion of such factors as frictional forces or limits to biological populations. The nonlinearity of the equations usually renders a closed-form solution impossible. So researchers into chaotic phenomena seek a *qualitative* account of the behavior using mathematical techniques to "provide some idea about the long-term behavior of the solutions" (Devaney 1986, p. 4).

A closed-form solution may allow one to predict, for example, when three planets orbiting a star will line up, whereas a qualitative study would be more interested in discovering what circumstances will lead to elliptical orbits as opposed to, say, circular or parabolic ones. Mathematical research in this field goes by the name "dynamical systems theory." It typically asks such questions as, what characteristics will solutions of this system ultimately exhibit? And how does this system change from exhibiting one kind of behavior to another kind? Chaos theory is a specialized application of dynamical systems theory, or "dynamics."

While qualitative questions can be asked about almost any dynamical system, chaos theory focuses on certain forms of behavior—behavior which is *unstable* and *aperiodic*. Instability means that the system never settles into a pattern of behavior that resists small disturbances. A system marked by stability, on the other hand, will shrug off a small jostle and continue about its business like a marble which, when jarred, will come again to rest at the bottom of a bowl.

Aperiodic behavior occurs when the state of the system never exactly repeats itself. Unstable aperiodic behavior is thus highly complex: it never repeats and it continues to manifest the effects of any small perturbation. Consider how much of physics is concerned with periodic behavior—the harmonic oscillator and the two-body problem. There was a methodological presumption that if you could not approximate to such behavior, you had noise. But sometimes that noise is chaos. A distinguishing feature of the systems studied by chaos theory, and a large part of what makes the field so exciting to researchers, is that unstable aperiodic behavior can be found in mathematically simple systems. These systems bear the label *deterministic* because the equations make no explicit reference to chance mechanisms: no averaging over subsystems, no probabilistic branching, no noise terms.

Descriptions of chaotic behavior celebrate their 100th anniversary this year. Chaos theory, on the other hand, is only three decades old at most. As an aside, it should be noted chaos is an appropriate label only when such behavior occurs in a bounded system. An explosion does not qualify as chaotic behavior in this sense. Note also that the systems studied by chaos theory are strictly classical—medium sized objects travelling at moderate speeds. To illustrate the new and distinctive image of complexity that operates in chaos theory, I turn now to a discussion of a long-standing problem in classical physics: the problem of the onset of turbulence.

Turbulence remains an unsolved problem for classical physics. One of the few theoretical approaches to this problem before chaos theory was the account suggested by Lev Landau (1944). The Landau model seeks to understand turbulence by describing how smooth fluid flow becomes disrupted as the speed (for example) of the fluid past an object is increased. By understanding how turbulence begins, it is hoped that some clues can be found to the nature of full-blown turbulent behavior (see the discussion in Kellert, Stone and Fine 1990).

Imagine a creek in which water flows past a large rock, and a device downstream from the rock measuring the velocity of the water at one point. For steady flow, the device will register a constant value, but as the water increases in speed, the smooth flow lines around the rock begin to bend, causing undulations that detach into small eddies that move downstream in time. As one of these eddies passes our measuring device, the velocity will register an increase, then a decrease, and then return to the undisturbed value until another eddy passes by. The sequence of velocity measurements—the time series—changes from constant to periodic behavior.

Perhaps the most important arena for understanding dynamical systems is state space (sometimes called "phase space"), a mathematically constructed space where each dimension corresponds to one variable of the system. Thus, every point in state space represents a full state description of the system, and the evolution of the system manifests itself as the tracing out of a path, or trajectory, in state space. This method is extremely useful, because it is often possible to study the geometric features of these trajectories without explicit knowledge of the solutions they represent. The characterization of possible trajectories in state space according to their "shape"—a kind of topological taxonomy—constitutes a major element of the mathematical apparatus of chaos theory.

Consider a mathematical representation of all possible states of the creek—every point in this state space corresponds to a different configuration of the fluid flow. In the case of steady flow, the behavior of the system is characterized by a single attracting point. No matter where you begin in the state space, no matter how you stir up the creek, the system will eventually wind up moving along smoothly with a constant velocity everywhere in the creek bed. The transition to the "small eddies" behavior has a mathematical counterpart in the change from an attractive point to an attractive periodic cycle in state space. Such a change in the nature of a system's behavior as a parameter is varied is termed a bifurcation, and dynamics often involves the investigation of different types of bifurcations.

As the speed of the creek increases, the behavior of the flow becomes more complicated. But we can follow one scenario for the transition to turbulence, where the faster flow makes smaller eddies appear within the original ones. The time series of velocity measurements will now vary with two frequencies. The system has undergone another bifurcation and the point representing the state of the system in abstract state space now spirals around a two-dimensional torus, or doughnut shape. If the two frequencies are not rationally commensurate, the trajectory will wind around the torus forever and never return to its starting point. This situation is labeled quasiperiodic (or multiply periodic) behavior and can best be appreciated by imagining a clock with two hands, one of which circles the clock face every hour and one of which takes π hours to describe a circle. If we start the clock with both hands at twelve, they will never again meet at twelve; such a system is deterministic and easily predictable, but not periodic.

The essence of the Landau model's explanation of the onset of turbulence is this: as the flow rate is increased, the quasiperiodic motion on the two-dimensional torus becomes unstable; some small disturbance will lead to three-dimensional quasiperiodic motion, then four-dimensional, and so on to infinity. The onset of turbulence represents the piling up of huge numbers of incommensurable frequencies, representing the excitation of more and more degrees of freedom—more and more eddies within eddies. Quasiperiodic motion on a very high dimensional torus (and, in the limit, a torus of infinite dimensions) will never repeat itself and will be utterly unpredictable. So the Landau model suggests that complex, apparently random turbulent behavior is best understood as akin to a clockmaker's shop with a huge number of clocks each ticking at a different, irrational rhythm.

One of the birthplaces of chaos theory was in an alternative account for the onset of turbulence, an account that challenged this picture of complexity. Known as the Ruelle-Takens-Newhouse (RTN) model, this account rejects the idea that complex behavior must be modeled by the agglomeration of incommensurable frequencies. The transition to turbulence is explained instead by the appearance in state space of an attractor that represents very complicated dynamical behavior, yet is described by a very simple set of mathematical equations. Such a novel mathematical object is called a "strange attractor."

In the RTN model, the behavior of fluid flow past an obstacle follows the path laid down by Landau only up to the appearance of a two-dimensional torus. After that point, a further increase in the flow rate can render this attractor unstable, and the behavior will change to weak turbulence characterized by motion on a strange attractor (Ruelle and Takens 1971; Newhouse, Ruelle, and Takens 1978). The strange attractor has several important characteristics: (1) it is an attractor, that is, an object with no volume in state space toward which all nearby trajectories will converge; (2) it typically has the appearance of a fractal, a stack of two-dimensional sheets displaying a self-similar packing structure; (3) motion on it exhibits a form of instability known as sensitive dependence on initial conditions, which I will discuss below; and (4) it can be generated from a very simple set of dynamical equations.

The idea that complex and unpredictable behavior such as turbulence can be understood by investigating simple dynamical systems is at the heart of chaos theory. The RTN model made this idea mathematically plausible, and the work of other mathematicians, theoreticians, and experimenters in the seventies has added evidence of the fruitfulness of chaos theory. The meteorologist Edward Lorenz laid the groundwork for this approach with his discovery of a strange attractor in a highly simplified set of equations derived from a model for fluid convection (Lorenz 1963). Lorenz took the Navier Stokes equations for viscous fluid flow, equations which have no general solution, and truncated them and reduced them to a system of three ordinary differential equations. By using a computer to plot the trajectory of his system, Lorenz created the first picture of a surprising new geometrical object: a strange attractor.

The behavior of this system, known as the Lorenz system, exhibits the form of instability known as *sensitive dependence on initial conditions*, a distinguishing characteristic of chaotic behavior. A dynamical system that exhibits sensitive dependence on initial conditions will produce markedly different solutions for two specifications of initial states that are initially very close together. In fact, given any specification of initial conditions, there is another set of initial conditions close to it that will diverge from it by some required distance, given enough time.

Lorenz spelled out the consequences of his discovery as follows: "It implies that two states differing by imperceptible amounts may eventually evolve into two considerably different states. If, then, there is any error whatever in observing the present state—and in any real system such errors seem inevitable—an acceptable prediction of an instantaneous state in the distant future may well be impossible" (1963, 133).

In the case of a strange attractor, the action of chaotic dynamical systems often produces objects with the appearance of infinite puff pastry—stacks of sheets that are themselves two-dimensional, but stacked in a never-ending self-similar structure that seems to intrude into the three-dimensional space. The fractal dimension of such an object, conceived as a measure of its "intrusiveness," is more than 2 but less than 3. The fractal dimension of an attractor thus provides a quantitative means for characterizing its topological features. Note that not all strange, or fractal, attractors are the result of chaotic dynamics. Nonetheless, much of the work in chaos theory studies chaotic strange attractors, and I am concentrating on the work of this so-called "strange attractor" school of nonlinear dynamics.

The qualitative approach used in chaos theory emphatically does not mean that no precise numerical results are available. Another quantitative characterization of chaotic systems is given by the Lyapunov exponents, which measure the degree of sensitivity to initial conditions and thus the degree of unpredictability. The measurement of Lyapunov exponents can be used even when the trajectories of a dynamical system do not lie on an attractor: one chooses a trajectory as a standard and measures the growth or shrinking of small displacements from it. It should be noted that strange attractors appear only in chaotic dissipative systems. For Hamiltonian systems, where energy is conserved, there is no convergence onto an attractor. Instead, trajectories are confined to a surface of constant energy. Chaotic behavior can occur in such systems, but instead of strange attractors with fractal dimension, the trajectories will fill the allowed energy surface which may itself display an extremely complicated structure (Walker and Ford 1969). In this paper I concentrate on dissipative systems, but permit me to note that chaos theory investigates energy-conserving systems such as planetary orbits and particle accelerators as well.

In conservative systems, as in dissipative ones, much attention has been focused on the way a system changes from ordinary behavior to chaos. One simple dynamical system, the logistic map, has served as an exemplary model for the transition to chaos. In this dynamical system, initially used to study fluctuations in animal populations, x_n represents the population in year n and the evolution equation is $x_{n+1} = \alpha x_n(1 - x_n)$. As the parameter α is varied, this extremely simple system displays an extraordinary range of behavior. For instance, the long-term behavior of the system will change from stable equilibrium to periodic fluctuations. With higher values of α, the period of the fluctuations doubles repeatedly, until full-blown chaotic behavior is reached.

Besides this "period-doubling" route to chaos, there are two other transitional scenarios that have received some theoretical attention: the transitions via quasiperiodicity and intermittency. Quasiperiodicity is the route described in the RTN model, where a torus in state space changes into a strange attractor. Intermittency occurs when a pe-

https://doi.org/10.1017/S0270864700009176 Published online by Cambridge University Press

riodic signal is interrupted by random bursts that arrive unpredictably but increasingly often as a parameter is increased. It should be noted that while chaos theory permits qualitative understanding and even some quantitative prediction with regard to these three routes, no one has yet established necessary and sufficient conditions for determining which type of transition will occur in a given system. But when a system manifests aspects of a certain type of transition, the mathematical theory pertaining to that generic type can be applied.

The qualitative study of chaotic dynamical systems is mathematically interesting, but how can it be of use in experimental situations in which we do not know the equations governing the physical system's behavior? One of the most important methods for discovering and analyzing chaos in dissipative systems is the reconstruction of attractors, a procedure that extracts the geometric features of a system's behavior from the time-series record. This method, developed by the physicists N. Packard, J. Crutchfield, J. Farmer, and R. Shaw together with the mathematical work of Floris Takens (1981), allows researchers to study qualitative features without solving (or even knowing) the equations governing a system. The basic idea is to reconstruct a multidimensional attractor from the time series by plotting, say, $x(t)$ versus $x(t+\tau)$ and $x(t + 2\tau)$, where τ is a suitable time-lag (Packard et al. 1980, p. 713). Thus, for the three-dimensional case, three measurements of the same variable serve as three independent variables in order to specify the state of the system. As they write, "the evolution of any single component of a system is determined by the other components with which it interacts. Information about the relevant components is thus implicitly contained in the history of any single component" (Crutchfield et al. 1986, p. 54).

The reconstruction of attractors creates a simulated state space. Because the most important properties of strange attractors are topological, almost any set of coordinates can be used to discern these properties, so this method is usually a reliable guide to the dynamical behavior of the system under study (Shaw 1981, 222). Numerical simulations have confirmed that reconstructing an attractor in this way yields a representation in the constructed state space that is "faithful" to "the dynamics in the original x, y, z space" (Packard et al. 1980, 714).[2] Faithful, that is, to the important geometric features responsible for the qualitative behavior of the system: features such as the stretching and folding that produces fractal layering, and the number of attracting, repelling, and saddle points in the state space.

Once a reconstructed picture of the dynamics is available, researchers may wish to determine the dimension of the attractor. The fractal dimension may be computed by various techniques that take off from the Hausdorff-Besicovitch definition of topological dimension, including an analysis of the density of points on the attractor within spheres of increasing size (Moon 1987, p. 220). The development of more efficient ways to calculate the dimension of attractors, and the invention of newer, even more informative quantitative measures for their topological features, attract a tremendous amount of interest among those currently working in chaos theory.

Another useful analytical tool for studying reconstructed attractors with few degrees of freedom is the Poincaré surface-of-section. This method involves examining the reconstructed trajectories of a system as they pass through a plane in state space. Imagine a very thin phosphorescent screen that slices the attractor. Instead of trying to visualize the attractor itself, the surface-of-section allows us to pay attention only to the pattern of glowing spots where the trajectories intersect the screen. Since a two-dimensional display is easier to examine, these surfaces-of-section are often used to look for the characteristic doubling of paths on the period-doubling route to chaos or for the intricate folded structure that often signals a chaotic system (Moon 1987, p. 53).

If the system is highly dissipative, the surface-of-section will appear to be very thin—practically a line segment. In this case, another analytic technique is used to discover whether the system is chaotic or "merely stochastic." If such a system were stochastic—meaning, not governed by a few deterministic equations—we would expect the trajectory to be randomized each time it passed through the thin segment on the plane. So we plot the position along the segment versus the position on the next pass through, and we do this repeatedly. If the points so plotted fit along a "well-defined curve" instead of a random scatter, this shows that the irregular behavior is not mere randomness but in fact deterministic chaos (Shaw 1981, p. 224).

This method, which produces what is known as a "first-return map," effectively reduces the study of the system to an analysis of an iterated one-dimensional discrete mapping. If the first-return map has a quadratic extremum, for instance, the entire analysis of period-doubling can be applied (Bergé, Pomeau, and Vidal 1984, p. 219). Moreover, the first-return map can provide a measure of the Lyapunov exponent. By fitting a curve to the points on the map and then averaging the slope over the curve, an approximate measure of the degree of sensitive dependence on initial conditions can be obtained (Shaw 1981, p. 224).

The routes to chaos, sensitive dependence on initial conditions, strange attractors with elaborately folded fractal structure, and other elements of chaos theory have been reported in experimental systems. These systems vary from measurements of brain wave activity to yearly patterns of measles outbreaks to instabilities in the electrical conductivity of crystals to the wobbling of certain coffee-table toys (see Holden 1986; Hao 1984). Allow me to also mention the pulsations of variable stars, the transmission of soundwaves over the ocean bottom, the fibrillations of hearts during cardiac arrest, and the orbit of Pluto. Some of these examples of chaotic behavior have been convincingly documented in laboratory settings, while some of the examples of low-dimensional chaos outside the laboratory are still the subject of lively debate. But the interplay of theory and practice, aided by computer simulations, continues to expand the repertoire of models and techniques. To conclude this section, if we asked, Is it really "chaos," the answer is No, of course not, not in the ordinary sense of the word. But it is certainly not "order" in the ordinary sense, either.

3. An Evaluation of the Study of Chaotic Dynamics

Several writers, working in the physical sciences, philosophy of science, and other fields, have characterized chaos theory as "revolutionary" in the sense of Thomas Kuhn's 1962 book (Devaney 1990, p. 1; Kellert, Stone and Fine 1990, p. 103; Hayles 1990, p. 169; Ruelle 1991, p. 66). James Gleick, in his bestselling journalistic history of chaos theory, explicitly characterizes the development of this field as a scientific revolution in the Kuhnian sense. In the chapter of his book entitled "Revolution" he writes, "A few freethinkers working alone, unable to explain where they are heading, afraid even to tell their colleagues what they are doing—that romantic image lies at the heart of Kuhn's scheme, and it has occurred in real life, time and time again, in the exploration of chaos" (1987, p. 37). But even granting Gleick's documentation of the hostility, resentment, discouragement and resistance encountered by scientists working on chaos, such resistance alone does not constitute sufficient evidence that a scientific revolution has occurred.

Such a romantic characterization of scientific revolutionaries may indeed be present in Kuhn's account, but Gleick overstates the case in suggesting that it forms the heart of an adequate conceptualization of Kuhnian scientific revolutions. Kuhn describes three "defining characteristics" of scientific revolutions: first, the "rejection of one time-honored scientific theory in favor of another incompatible with it." Second, "a shift in the problems available for scientific scrutiny and in the standards" by which to judge acceptable problems or solutions; and third, a transformation of "the scientific imagination," best described as a "transformation of the world within which scientific work [is] done."

Utilizing this understanding of what Kuhn means by a scientific revolution, Katherine Hayles has made perhaps the most convincing case for the revolutionary character of chaos theory. Relying on Gleick's interviews with practitioners in the field such as physicist Bernardo Huberman, Hayles does point out distinct changes wrought by chaos theory in the criteria for interesting problems and successful solutions—the second of Kuhn's defining characteristics. When Huberman presented his chaotic model of eye movements in schizophrenics at a conference of biological systems specialists, it was rejected as irrelevant mathematics. How could an abstract dynamical system say something interesting about the eye, when it ignored the details of neuromuscular function? (Hayles 1990, pp. 169-70). The techniques for reconstructing attractors and establishing useful similarities between mathematical models and experimental systems which I have described in the previous section were seen as unsatisfactory answers addressing uninteresting questions. Hayles suggests that the techniques of chaos theory represent a new paradigm for the investigation of complicated behavior—a new paradigm whose practitioners appeal to incommensurably different standards of successful explanation.

The mention of incommensurability recalls the well-known question of whether scientists live in "different worlds" before and after a revolution, an element of Kuhn's third defining characteristic in the list above. Without touching the general question, one might be willing to accept that chaos theory has brought with it a change in the "scientific imagination," and that it does therefore satisfies Kuhn's third criterion. The only picture of complicated and unpredictable behavior used to be the picture of a welter of competing and interacting systems. But chaos theory reconceptualizes complexity as one possible outgrowth of simple and deterministic systems. The guiding images of order and noise have been reoriented. While Hayles sees the general outline of a paradigm shift in a move away from reductionism (p. 170), others have concentrated on just this new image of intelligibility (Kellert, Stone, and Fine 1990, pp. 103-4).

When we finally consider Kuhn's first criterion, however, it becomes impossible to fit chaos theory into the role of a scientific revolution. This criterion states that a scientific revolution must involve "rejection of one time-honored scientific theory in favor of another incompatible with it." Yet no scientific theory was rejected in order to make way for chaos. As the astronomer John Barrow has written, "there has been no build-up of inconsistences that, suddenly, could be overlooked no longer. Neither has there been an inner crisis within some existing paradigm which undermined the everyday practice of normal puzzle-solving activities" (1988, p. 4).

Recall that chaotic behavior occurs in scrupulously Newtonian systems. Nothing in nonlinear dynamics corresponds to the postulation of the relativity of simultaneity, or the limitations on measurement of noncommuting observables, associated with the revolutions of special relativity and quantum theory, respectively. Chaos theory involves no fundamental theoretical change, while a Kuhnian revolution involves "a reconstruction of the field from new fundamentals, a reconstruction that changes some of the fields' most elementary theoretical generalizations..." (Kuhn 1970, p. 85) .

Neither is there fundamental theoretical change in the specific fields affected by chaos theory. Several writers have contended that the Landau model for the onset of turbulence provided a paradigm in fluid dynamics which was overthrown by the success of the Ruelle-Takens (RTN) theory of strange attractors (see Gleick 1987, p. 130; Kellert, Stone and Fine 1990, p. 104). But the Landau conjecture was long known to turbulence researchers not to correspond to the actual transition in many situations. One fluid dynamicist, in pointing out this fact, notes that it "appears to have been overlooked by many contemporary authors," who were at pains to stress a clear contrast between the two scenarios (Van Atta 1990, p. 66). Indeed, it might even be said that the Landau conjecture was rediscovered in the literature on turbulence, where it had been languishing, in order to be used as an historical foil for the successful RTN approach (Jerry Gollub, private communication).

So, contrary to Hayles, chaos theory cannot be understood as emerging from a full fledged Kuhnian revolution. It fails to satisfy the first of Kuhn's defining characteristics listed above. And in fact the case for the third characteristic seems shaky as well. Chaos theory does satisfy Kuhn's second criterion, a change in what counts as an important problem and an interesting solution, and I will return to this point later. But where Gleick sees the overthrow of an entire scientific worldview, it is more accurate to see the profound continuities between chaos theory and the science that came before it. Those who study nonlinear dynamical systems still strive to apply simplified mathematical models, still seek quantitative results, and still seek to generalize and unify their understanding. The discovery of low-dimensional attractors in hitherto incomprehensible noise, to quote David Porush, "actually reasserts one of the fundamental axioms of science—that the universe can be described by deterministic mathematics" (Porush 1990, p. 438). Indeed, this is precisely the reason chaos theory is so exciting to many researchers.

It is notable that the subtitle of James Gleick's book in fact announces not a chronicle of revolution, but of "making a new science." For if the emergence of chaos theory does not fit Kuhn's picture of a scientific revolution, it seems very likely to fit much better into his picture of the emergence of a newly mature science. Such a new science would be marked by the presence of the three elements of a Kuhnian disciplinary matrix: symbolic generalizations—the formal expressions that meet with general acceptance, models—the analogies and metaphors used in applying these formulae, and exemplars—the paradigmatic cases of solved problems that are used to initiate newcomers into the science (Kuhn 1970, 1977).

It would make sense for an episode marking the birth of a new science to give the appearance of a revolution; as Kuhn remarks, "research during crisis very much resembles research during the pre-paradigm period" (1970, p. 84). Before the emergence of a successful disciplinary matrix, a number of schools compete over a field of scientific inquiry. Each may have exemplars of successful research, but at the transition to maturity there emerges a dominant approach, which allows the practitioners to cease quarreling over fundamentals and commence normal "puzzle-solving" work (Kuhn 1970, pp. 178-9). Certainly some aspects of the development of chaos theory, particularly with regard to the study of fluid turbulence, fit this depiction of the transition to maturity.

In addition to Landau's theoretical account of the onset of turbulence, there were at least two other distinct movements in the modern scientific study of turbulence before the 1980's. The *statistical* approach, which began with Reynolds, focussed on the behavior of various averages associated with variables describing the flow. This viewpoint sought to characterize the turbulent velocity field in terms of a mean flow plus a perturbation, where the perturbations were "unpredictable or incomprehensible in detail" (Chapman 1985, pp. 21-22). The *structural* approach, on the other hand, focussed on coherent structures that appear within turbulent flows, although still conceiving the occurrence of such structures as governed by random events (Chapman 1985, p. 27). Each of these schools produced valuable work in the attempt to understand turbulence, but in a 1978 historical review of fluid dynamics in *Physics Today* we read that "as of now, it cannot be said that any turbulence theory has a greater likelihood of representing real phenomena than any other" (Emrich 1978, p. 39).

The presence of competing schools in fluid dynamics highlights the absence of any successful theoretical treatment of turbulence. In fact, a distinct note of despair can be found in many of the writings on turbulence before 1980. Without any general solution for the Navier-Stokes equations, there was simply no general analysis of fluid motion. The theories of turbulence available were described as "crude," "frustrating," and faced controversy on every point. Practicing engineers seeking to use one of these theories would be "repelled by its complexity and by its inability to tell them anything that they actually wanted to know" (see White 1979, p. 1, p. 311; Bradshaw 1976; Leslie 1973, vii).

One researcher said in 1968 that "the situation of turbulence theory is somewhat depressing, because... our understanding of turbulence is practically nil, despite the last 30 years of effort. But undue pessimism is out of order, as the position could change overnight" (Saffman 1968, p. 611). Chaos theory did not change the situation overnight, but the success of the RTN theory of the transition to turbulence, and especially the news of the detection of a strange attractor in closed-cell convection, generated a great deal of excitement in the world of fluid dynamics. Deterministic chaos raised the hope that a successful exemplary solution to at least one outstanding problem was available.

But the question of whether there *is* a successful new exemplar in the study of turbulence is as yet undecided. The pursuit of chaos in other fields is even more controversial. So the successes of chaos theory are by no means so convincing as to have established entirely new, successfully maturing science. But what about the other elements of a new Kuhnian disciplinary matrix—symbolic generalizations and models? (see Kuhn 1977, p. 463)

With no new theoretical postulates, chaos theory does not license the use of any new symbolic generalizations. Instead, we find drastically different ways to apply old generalizations — f = ma will now systematically be fleshed out with a nonlinear force law, and the mathematical manipulations exercised on Newtonian symbolizations will diverge from the traditions of perturbation analysis and the approximation of exact solutions. But if new symbolic generalizations are a necessary condition for a new science, then chaos theory cannot fill the bill. Kuhn insists that the interpretation or implementation of these formulae are not practiced identically within a scientific community: members of that community will never challenge an utterance of one of these symbolic generalizations, but they may well disagree with how they are "correlated with the results of experiment and observation" (1977, p. 464).

Such an openness to variations in mathematical technique as Kuhn countenances would render all of nonlinear dynamical systems theory as a mere variation on what came before. For no one working on chaotic dynamics denies the validity of the Navier-Stokes equations, for example. They do, however, do a variety of non-classical things with them, of the sort I have discussed above—interpreting, implementing, and applying classical equations in very new ways. Each of these new mathematical techniques has bound up with it conceptual, metaphysical, and experimental commitments. I will have more to say about the role of these new mathematical techniques in the evaluation of chaos theory. Before doing so, I would like to turn to another topic, the role of models.

Models play the role of the final element of a disciplinary matrix. And new models of dynamical behavior form the heart of chaos theory. As such, they provide the best support for conceiving of this research as carried out within a new disciplinary matrix. I will now turn my attention away from Kuhn to a conception of scientific theories which highlights the role of models. This view, known as the semantic conception of theories, may be useful for making sense of the emerging study of chaos (see Kellert 1993, chapter 4).

For Kuhn, models can be heuristically useful analogies or deeply held metaphors that constitute an ontology (1977, p. 463). But they are only part of a disciplinary matrix. The semantic conception of theories seeks to free the essential features of a theory from the metaphoric trappings of its formulation in a particular language, and so conceives of scientific models as abstract mathematical structures that can be articulated in several presentations in language, all equivalent (Suppes 1967). In this way, the notion of a model is both narrower and broader than Kuhn's: in the semantic conception, a model does not include the natural-language formulation and attendant metaphorical connotations that are used to present the mathematical structure for consideration. But the *role* of models is much broader: they do not merely make up part of a disciplinary matrix, they are absolutely central to the constitution of a scientific theory.

In the semantic conception, a theory is just a set of models together with application rules. A model is a mathematical structure, and the application rules detail the construction of structures of empirical data. The theory is applicable to the data if there exists an isomorphism between these structures. This view contrasts with a more traditional view of theories, which holds that a theory is a body of theorems expressed in language, together with a set of coordinating definitions that supply empirical content to the terms in the language.

Bas Van Fraassen, a proponent of the semantic conception, has claimed support for this view from the "actual form of presentation" of scientific theories (1980, p. 65). Chaos theory provides some additional support. This support does *not* come from the fact that both nonlinear dynamics and some formulations of the semantic view make use of state space. I regard this as a coincidence.[3] The deeper affinity is due to the fact that the actual presentation of research in nonlinear dynamics, whether in textbooks or published reports, normally treats the theoretical aspects of the results in terms of mathematical *models*. Models such as the logistic map, the RTN model, and the Roessler attractor provide a toolbox of mathematical structures that are then applied to a wide variety of sources of empirical data.

To my knowledge, no one has ever attempted a formulation of nonlinear dynamics in terms of a set of axioms and deductive consequences; no one is likely to do so in the foreseeable future. Chaos theory remains a cluster of models, and the applicability of one of those models to any given experimental situations cannot be determined in advance. Patrick Suppes describes just this kind of situation when he writes that sometimes what we are faced with is "not a theory with a genuine logical structure but a collection of heuristic ideas" (1962, p. 260). Some may see this lack of a unifying theoretical framework as an indication of the immaturity of chaos theory. I hold that the unifying element of chaos theory lies instead in the dynamical, geometric approach to the analysis of data.

These techniques for analyzing data provide a second realm where the semantic view of theories makes contact with chaos theory. One of the strengths of the semantic view is the opportunity it opens for an in-depth analysis of the procedures used to transform experimental data into a mathematical structure that can be put into an isomorphism with the structures of the theoretical models. Suppes describes a "hierarchy of theories" that constitute a complex "conceptual grinder": an elaborate methodology for the transformation of observational and experimental data into a form that can be compared with theory (1967, p. 62-3). In the case of chaos theory, the techniques of reconstructing attractors, surfaces-of-section, and first-return maps perform just this function of transformation. The appeal to simple coordinating definitions does violence to the rich layers of methodology—the application rules—which should in fact be considered part of the theory.

The role of these techniques of application is absolutely crucial for an accurate characterization of chaos theory. If we focus only on the mathematical structures at the highest level of generality, we run into trouble because these structures remain the same as before; chaos theory does not introduce any new fundamental laws of nature. For example, as I have said with regard to fluid dynamics, the Navier-Stokes equations remain unchallenged, and thus they are already a correct "model" for chaotic behavior, in the strict sense of "model" employed by the semantic view. Every structure revealed by the conceptual techniques of dynamical transformation and reconstruction must be isomorphic to some substructure of the fundamental equations for fluid flow. But while the Navier-Stokes equations are the correct mathematical structure for fluid behavior, these equations fail to provide a useful mathematical technique for understanding most actual fluid behavior.

I think there is a tension here between the technical and the heuristic senses of the word "model." In brief, these equations are a correct model, but worthless for modelling: a good mathematical structure, an inadequate mathematical technique. Chaotic dynamics changes the hierarchy of models, the application rules, but not the highest level models of our theories. This is the reason I contend that the interesting and important aspects of chaos theory, including any possible revisions to our metaphysical or epistemological positions, derive from a change in methodology.

To conclude this discussion of the semantic view of theories, let me emphasize just this point: chaos theory changes our methods for applying models to actual systems. To the extent that the semantic view considers Newtonian mechanics and Hamiltonian mechanics the same theory, merely in different formulations, it obscures the importance of mathematical techniques. But to the extent that the semantic view encourages us to look at the hierarchy of models, the mathematical techniques, used to build bridges between high-level models and actual systems, it brings out just what is important in chaos theory.

Up to this point, I have suggested that chaos theory does not qualify as a scientific revolution in the Kuhnian sense, and that there are serious difficulties in conceiving it as a new science with a new and complete disciplinary matrix. I have made use of the semantic view of theories to point out the importance of mathematical models and methodologies of application in the presentation of nonlinear dynamics. I would now like to apply this emphasis on methodology, technique, and practice to the Kuhnian picture. In doing this, I will follow Joseph Rouse's characterization of the profound change in our conception of a scientific theory that comes from taking very seriously the role of scientific practice in Kuhn's work.

Rouse takes issue with the standard depiction of the Kuhnian disciplinary matrix as a set of assumptions believed by all the members of a scientific community. Instead, he describes it as a set of "exemplary ways of conceptualizing and intervening in particular empirical contexts." Learning to engage in a scientific discipline is thus "more like acquiring and applying a skill than like understanding and believing a statement." (1987, p. 30) Indeed, Rouse highlights the fact that Kuhn's original work included a significant emphasis on "commitments to preferred types of instrumentation" and the way they were to be employed (1970, p. 40), "methods and applications" (p. 85), and norms of practice (p. 103) as elements of a scientific paradigm in the broad sense.

On this understanding, the fact that chaos theory does not challenge any fundamental physical theory becomes considerably less important for our assessment of its status as a new science. Mathematician Morris Hirsch has ridiculed what he calls "the nonexistent science of chaos," and from the perspective of a mathematically rigorous theory, he may be justified (Hirsch 1989, p. 8). But this theory may provide a new disciplinary matrix, as Rouse understands it. If its great promise pans out, as now seems likely, it will constitute a revolutionary development without qualifying as a Kuhnian scientific revolution.

This somewhat paradoxical state of affairs has a precedent in the development and widespread application of statistics in the nineteenth century. Ian Hacking has proposed a useful way to conceptualize this development in terms of "styles of scientific reasoning." According to this account, the successive waves of interest in statistical techniques that transformed European science and society in the nineteenth century should be understood as the result of the success of a new style of scientific reasoning—a pattern of successes which proved revolutionary without being reducible to any set of Kuhnian scientific revolutions (Hacking 1987).

Hacking's notion of a style of scientific reasoning, based on the work of the historian A. J. Crombie, does not deal with personal, idiosyncratic style, but with something considerably more expansive in scale. It is a significantly larger unit of analysis than a Kuhnian paradigm, and a change in style may in fact include changes in the elements of several different disciplinary matrices (Hacking, 1992, pp. 2-3; 1983, pp. 458-9). Examples of such large-scale styles are mathematical postulation and proof, taxonomic ordering, statistical analysis of regularities, and historical analysis of genetic development. Note that working within a style of scientific reasoning does not entail the belief in certain propositions, but rather the practicing of certain ways of reasoning toward propositions (1883, p. 454). Such a unit of analysis accords with Rouse's reading of Kuhn, for "style" is meant to invoke practice as well as thinking and sentence-making: in Hacking's words, "the manipulative hand and the attentive eye" (1992, p. 3).

But perhaps chaos theory, and nonlinear dynamics more generally, should be considered merely a refinement of the long-established style known as the "hypothetical construction of analogical models." After all, dynamical systems theory works by constructing simplified mathematical models and then matching them to experimental situations that display analogous features. I contend that this attempt to assimilate nonlinear dynamics to a preexisting style is no more valid than an attempt to similarly co-opt statistical reasoning. For statistical analysis also constructs hypothesized models and seeks to test how well they serve as analogies for experimental and observational data. The crucial distinction for statistics is that the objects of study are populations, not individual systems. For nonlinear dynamics, the crucial distinction lies in the character of the attempted analogy: it seeks to match the long-term, qualitative features of a system by analyzing the topological features of the dynamics in state-space. Traditional hypothetical models sought to match the detailed, quantitative features of the behavior of single trajectories.

Does chaos theory qualify as a new style of scientific reasoning? Hacking proposes as a necessary condition that a new style should introduce novel types of objects, evidence, sentences, laws or modalities, possibilities, and "ways of being a candidate for truth or falsehood" (1992, pp. 8-9). Chaos theory has introduced such objects as strange attractors and lyapunov exponents, which qualify as "new" in the same sense as the correlation coefficients and unemployment rates introduced by statistical reasoning. And if we consider the entire range of studies in dynamical systems, rather than just the study of chaotic behavior, we find scientists describing their work as utilizing new forms of analysis that complement older techniques. One brain scientist, for example, sees the common element in the new dynamical analysis as the attempt to reconstruct the dynamics generating the signal, to understand its geometric or topological character. This he opposes to classical signal analysis (such as examination of Fourier power spectrum) that only looked at the "signal itself" (Rapp 1990, p. 10). New techniques for transforming data, new methodologies for modelling, new norms for what counts as an interesting question or a possible answer, create new sets of sentences about which it is possible to ask whether they are true or false. Chaos theory introduces a new realm of intelligibility.

Hacking uses the term "autonomy" to refer to two different features of a new style of reasoning. In a narrow sense, a new technique for generating and testing sentences qualifies as autonomous if "it can be used to explain something else, without itself having to be reduced" (Hacking 1987, p. 53). The phenomenon of regression to the mean provides an example of the autonomy of statistical law, for the fact that exceptionally tall parents, for example, have shorter offspring can be explained as a consequence of the fact that height follows a normal distribution. "Reduction of this normal distribution to an underlying causal structure," writes Hacking, "is simply irrelevant to this explanation" (p. 53). Dynamical reasoning asserts its own autonomy by similarly seeking to understand behavior in terms other than underlying causal mechanisms. Fluid dynamicists, for example, write that "the complicated behaviors of a thermal-fluid system [at least for small systems] are attributable to the interaction of a rather small number of degrees of freedom" (Keefe, et al., 1990, p. 56). The RTN model for the onset of turbulence thus explains the emergence of certain type of behavior by elucidating what I call "geometric mechanisms."

In a broader sense, Hacking requires that a style of reasoning prove itself autonomous from the micro-history of its birth. It must persist and grow and take root in an institutional context beyond the circumstances of its origin, and not die out when the fashion passes. A style achieves this form of autonomy by harnessing "its own techniques for self-stabilization. That is what constitutes something as a style of reasoning" (Hacking 1992, p. 8, 12). Has dynamics succeeded in becoming a self-authenticating realm of the true-or-false? Has it ceased to require authorization from other techniques for creating and justifying observations and assertions? Perhaps not yet. But it is a candidate for a new style. The study of dynamical chaos provides an opportunity to observe the tentative emergence of a new style of scientific reasoning, in its attempts to emerge from the contingencies of its birth and prove itself applicable across a broad range of disciplines. Chaos theory, as a theory of turbulence, is still less than a full-fledged theory. Chaos theory, as a representative of dynamics, may become more than a theory. It may be a new style of scientific reasoning.

Notes

[1]I am grateful to Roger Jones, Lisa Heldke, Eric Winsberg, and John Winnie for their comments on earlier drafts of this paper.

[2]This method should not be considered universally applicable, however. For example, one would not expect to get a low-dimensional attractor from data collected at one point of a very large, spatially extended system with many degrees of freedom (like the

Earth's atmosphere). In questionable situations, one may need to collect additional data, or to achieve greater accuracy, or to take data from spatially separated points. Eventually, the experimentalist may decide that there is no low-dimensional attractor at all. I have benefitted from discussions with Jerry Gollub and James Crutchfield on this point.

[3]Van Fraassen and others have suggested that we characterize the mathematical structures of a theory as introducing restrictions on a "state space" that represents all logically possible configurations of a physical system (Van Fraassen 1972, p. 312). But the dynamical systems approach remains at the so-called qualitative level of analysis, rather than making "elementary" statements about individual states.

References

Barrow, J. (1988), Review of Gleick 1987. *New Scientist*, 118 no. 1614 (May 26): 73-74.

Bergé, P., Pomeau, Y., and Vidal, C. (1984), *Order within Chaos*. L. Tuckerman, trans. Paris: J. Wiley & Sons.

Bradshaw, P. (1976), "Introduction" in *Turbulence*, P. Bradshaw (ed.). Berlin: Springer-Verlag, pp. 1-44.

Chapman, G. T. and Tobak, M. (1985), "Observations, theoretical ideas, and modeling of turbulent flows—past, present, and future", in *Theoretical Approaches to Turbulence*, D.L. Dwoyer, M.Y. Hussaini, and R.G. Voight (eds.). New York: Springer-Verlag, pp. 19-49.

Crutchfield, J., Farmer, J.D., Packard, N., and Shaw, R. (1986), "Chaos", *Scientific American* 255:46-57.

Devaney, R.L. (1986), *An Introduction to Chaotic Dynamical Systems*. Menlo Park: Benjamin Cummings.

Devaney, R. L. (1990), "Chaotic Explosions in Simple Dynamical Systems", in *The Ubiquity of Chaos*, S. Krasner (ed.). Washington: American Association for the Advancement of Science, pp. 1-9.

Dresden, M. (1992), "Chaos: a new scientific paradigm—or science by public relations?—part 1", *The Physics Teacher* 30:10-14.

Emrich, R.J., et al. (1978), "Thirty years of fluid dynamics", *Physics Today* 31:38-46.

Gleick, J. (1987), *Chaos: Making a New Science*. New York: Viking Penguin

Hacking, I. (1992), "'Style' for Historians and Philosophers", *Studies in the History and Philosophy of Science* 23:1-17.

Hacking, I. (1983), "The Accumulation of Styles of Scientific Reasoning", in *Kant oder Hegel?*, D. Henrich (ed.). Stuttgart: Klett-Cotta, pp. 453-465.

Hacking, I. (1987), "Was There a Probabilistic Revolution 1800-1930?", in *The Probabilistic Revolution* vol. 1, L. Krüger, L. J. Daston, and M. Heidelberger (eds.). Cambridge: MIT Press, pp. 45-55.

Hao, B.-L., (ed.) (1984), *Chaos*. Singapore: World Scientific.

Hayles, K. (1990), *Chaos Bound*. Ithaca: Cornell University Press.

Hirsch, M.W. (1989), "Chaos, Rigor, and Hype", Review of Gleick 1987. *The Mathematical Intelligencer* 11:6-8.

Holden, A.V., (ed.) (1986), *Chaos*. Princeton: Princeton University Press.

Keefe, L., Moin, P., and Kim, J. (1990), "Applications of Chaos Theory to Shear Turbulence", in *The Ubiquity of Chaos*, S. Krasner (ed.). Washington: American Association for the Advancement of Science, pp. 56-63.

Kellert, S.H. (1993), *In the Wake of Chaos: Unpredictable Order in Dynamical Systems*. Chicago: University of Chicago Press.

Kellert, S.H., Stone, M.A., and Fine, A. (1990), "Models, Chaos and Goodness of Fit" *Philosophical Topics* 18:85-105.

Kuhn, T.S. (1970), [1962]. *The Structure of Scientific Revolutions*. Chicago: University of Chicago Press.

_ _ _ _ _ _. (1977), "Second Thoughts on Paradigms", in *The Structure of Scientific Theories*, F. Suppe (ed.). Urbana, Ill.: University of Illinois Press, 459-82.

Landau, L. (1944), "On the problem of turbulence", *C.R. Academy of Science, URSS*. 44:311-316.

Leslie, D.C. (1973), *Developments in the Theory of Turbulence*. Oxford: Oxford University Press.

Lorenz, E. (1963), "Deterministic Nonperiodic Flow", *Journal of the Atmospheric Sciences*. 20:130-141.

Moon, F. (1987), *Chaotic Vibrations*. New York: John Wiley & Sons.

Neményi, P.F. (1962), "The Main Concepts and Ideas of Fluid Dynamics in their Historical Development", *Archive for History of Exact Sciences* 2:52-86.

Newhouse, S.E., Ruelle, D., and Takens, F. (1978), "Occurrence of strange axiom A attractors near quasi-periodic flow on Tm (m=3 or more)", *Communications in Mathematical Physics* 64:35-43.

Packard, N., Crutchfield, J., Farmer, J., and Shaw, R. (1980), "Geometry From a Time Series", *Physical Review Letters* 45:712-716.

Porush, D. (1990) "Making Chaos: Two Views of a New Science", *New England Review and Bread Loaf Quarterly* 12:427-442.

Rapp, P.E., et al. (1990), "Dynamical Characterization of Brain Electrical Activity", in *The Ubiquity of Chaos*, S. Krasner (ed.). Washington: American Association for the Advancement of Science, pp. 10-22.

Rouse, J. (1987), *Knowledge and Power*. Ithaca: Cornell University Press.

Ruelle, D., and Takens, F. (1971), "On the nature of turbulence", *Communications in Mathematical Physics* 20:167-192.

Ruelle, D. (1991), *Chance and Chaos*. Princeton: Princeton University Press

Saffman, P.G. (1968), "Lectures on Homogeneous Turbulence", in *Topics in Nonlinear Physics*, N. J. Zabusky (ed.). New York: Springer Verlag, pp. 485-614.

Shaw, R. (1981), "Modeling Chaotic Systems", in *Chaos and Order in Nature*, H. Haken, (ed.). New York: Springer-Verlag, pp. 218-231.

Suppes, P. (1962), "Models of Data" in *Logic, Methodology, and Philosophy of Science*, E. Nagel, P. Suppes, and A. Tarski (eds.). Stanford: Stanford University Press, pp. 252-61.

Suppes, P. (1967), "What is a Scientific Theory?", in *Philosophy of Science Today*, S. Morganbesser (ed.). New York: Basic Books, pp. 55-67.

Takens, F. (1981), "Detecting Strange Attractors in Turbulence", in *Lecture Notes in Mathematics*, D. A. Rand and L.-S. Young (eds.). New York: Springer-Verlag, pp. 366-381.

Van Atta, C.W. (1990), "Fluid Dynamical Chaos in Vortex Wakes", in *The Ubiquity of Chaos*, S. Krasner (ed.). Washington: American Association for the Advancement of Science, pp. 64-74.

Van Fraassen, B.C. (1972), "A Formal Approach to the Philosophy of Science" in *Paradigms and Paradoxes*, R. G. Colodny (ed.). Pittsburgh: Pittsburgh University Press, pp. 303-66.

___________. (1980), *The Scientific Image*. Oxford: Clarendon Press.

Walker, G.H. and Ford, J. (1969), "Amplitude Instability and Ergodic Behavior for Conservative Nonlinear Oscillator Systems", *Physical Review* 188:416-432.

White, F.M. (1979), *Fluid Mechanics*. New York: McGraw-Hill.