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The Angular Momentum Problem in CDM Cosmolgies: The End of the Beginning?

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Abstract. We show, by adopting a plausible model for star formation and energetic feedback in cosmological simulations of galaxy formation, that we are able to alleviate the angular momentum problem which has bedeviled many previous attempts to generate realistic disc galaxies in Cold Dark Matter cosmogonies. This paper highlights the "cooling catastrophe" as manifest in numerical cosmology and describes a simple prescription for modelling the sub-resolution physics of star formation and feedback in Smoothed Particle Hydrodynamic simulations. We show results for angular momentum and disc scale length for simulations with and without feedback.

1. Introduction

Angular momentum loss during simulations of the formation of disc galaxies in Cold Dark Matter cosmologies has stood as one of the fundamental problems of post-recombination numerical cosmology. The pioneering work of Navarro & Benz (1991) showed a systematic loss of angular momentum by condensed baryon cores to the dark matter halo during the formation of discs, contrary to theoretical expectations (e.g., Fall & Efstathiou 1980).

The solution to this problem is widely believed to be that energetic feedback, primarily from supernovae, regulates the flow of gas into the cold phase. This limits the amount of material condensing into cores which would otherwise readily lose angular momentum to the halo via dynamical friction. So far there has been limited success in constructing internally consistent models of the star formation–energetic feedback loop which preserve angular momentum. Schemes, which by construction, prevent cooling before a specified condensation epoch are substantially more successful at reproducing observed discs (e.g., Weil, Eke, & Efstathiou 1998).

In this paper we describe the cooling catastrophe as it is manifest in numerical simulations of Cold Dark Matter on galaxy scales and describe a simple star formation and feedback scheme which has been implemented in an SPH code. We show with this prescription that self-regulated star formation is an important player in the construction of the model galaxies and may have a crucial rôle to play in cosmologies with Cold Dark Matter.

2. The Cooling Catastrophe in Numerical CDM

Whilst collisionless simulations of CDM cosmologies have matured to the point where many features of the models are well established, there are many aspects of hydrodynamic simulations of two-component models (baryons and dark matter) which are much less well established. In particular, simulations which attempt to model the cooling of gas in the cosmological context in CDM models suffer from a number of difficulties which have greatly limited the predictive power of the numerical programme.

A convenient way of understanding some of the key issues facing numerical galaxy formation is via the Rees–Ostriker (1977) picture for baryon cooling and condensation as illustrated in Fig. 1. The figure shows, for simple top-hat perturbations in the mass–redshift-of-collapse plane, the regions within which the gas within a dark matter halo will be able to cool on a dynamical time. This is of central importance in hierarchical models as gas which cannot cool on a dynamical time will remain in quasi-static equilibrium within the halo and be rapidly absorbed into the medium of larger haloes as these build through merging. As is well known, galaxies lie on the high-mass edge of the region which can cool efficiently. The particular cosmological model and initial spectrum define the path of structure formation through this plane. For standard CDM, the 1 and 3 σ overdensity curves are shown indicating the masses of the typical and rare haloes collapsing at a given epoch.

The so-called cooling catastrophe in numerical simulations can be understood by conducting the following thought experiment. Consider simulating the formation of a single disc galaxy in a cosmological box large enough to properly generate the correct tidal torques, 40 Mpc say. As the resolution is increased the first objects to appear in the simulation will do so at higher densities and will include a greater fraction of the mass. Thus, as the effective resolution moves left in Fig. 1 into the cooling region, the cooling becomes more effective and an increasing fraction of mass would be found in condensed clumps. This process would reach a limit only when the resolution was high enough to permit the modelling of haloes to the left of the cooling region in the figure.

The cooling catastrophe is a physical result of hierarchical structure formation. In reality the fraction of cold material observed is very much less than would be predicted if all haloes which could cool did so. It is believed that this cooling runaway is alleviated by a negative feedback cycle, whereby energetic feedback from stars which form as the result of cooling and condensation in haloes limits the flow of gas into the cool phase. Numerically, this effect leads to extreme numerical sensitivity and an over concentration of baryons in galaxy cores with the result that scale lengths and angular momenta are substantially smaller than are observed. A numerical prescription is needed which can model the sub-resolution physics of star formation and feedback in a resolution-independent manner. This paper provides a simple prescription for modelling this feedback cycle.

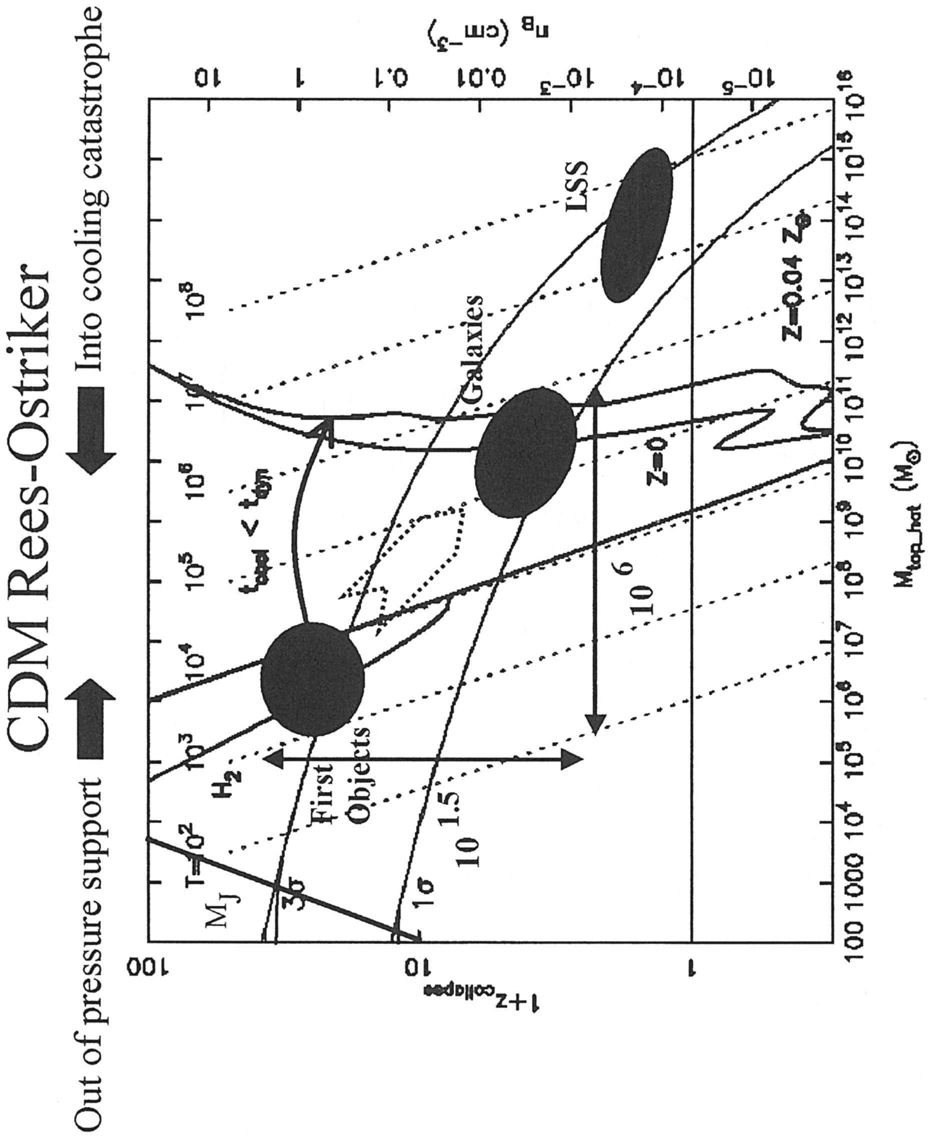

Figure 1. An illustration of the Rees-Ostriker picture for CDM. The figure shows the locus of mass and collapse redshift for dark matter haloes which can and cannot cool in a dynamical time. Diagonal dashed lines show the virial temperature for a given halo. The 1 and 3σ curves correspond to the mass fluctuation spectrum of standard CDM. Also shown are the approximate regions occupied by the first objects, galaxies and large-scale structure. Full explanatory details may be found in the text.

3. Numerical Simulation of Galaxy Formation

It is instructive to estimate the requirements of an ideal simulation. We will concentrate here on SPH simulations although most of these arguments are applicable to other hydrodynamic simulations of cooling gas in CDM cosmologies.

3.1. Ideal Mass Resolution

Star formation requires gas condensation, likely in bound haloes. This clearly cannot occur in haloes of mass less than the resolution limit of the simulation. Since star formation is a pre-requisite to feedback, it follows immediately—and obviously—that the details of structural evolution on scales close to the resolution limit will depend sensitively on the resolution. The hope is that structure on scales much later in the merger hierarchy, when self regulated star formation has been operating for several dynamical timescales, will be converged and largely independent of resolution.

The alternative would be to conduct experiments with a dynamic range which spans the "cooling gap" in Fig. 1. In this way we would be modelling the formation of the actual first cosmic objects and, since condensation under these conditions requires a mass which exceeds a threshold dictated by H_2 cooling (see Fig. 1), sensitivity to resolution at greater resolution would be removed (as has been shown in several numerical investigations of first object formation, for example, Fuller & Couchman 2001).

It is worthwhile considering the particle number which would be necessary to achieve simultaneous simulation of galaxies in the cosmic context and first objects. The mass difference between these two scales is roughly 10^6, requiring a further factor of 10 for the cosmic environment (possibly with fewer, larger mass particles modelling large scales). Supposing that the first objects can be reliably identified with 100 particles, this would suggest a minimum of 10^9 particles. This number is not possible with current hydrodynamic simulations. Proper evolution would require a correspondingly large number of timesteps to properly resolve the range of dynamical times and the hydrodynamic flow (shocks etc.), amounting to a temporal range of perhaps 10^6. Such a simulation will likely remain a prodigious feat for several years yet.

3.2. Modelling Star Formation and Feedback

Even supposing that adequate resolution is available, the task of reliably modelling star formation and feedback is daunting. Massive star formation occurs in molecular cloud cores of several hundred solar masses, virtually impossible to model on cosmological scales. Most numerical work uses a Schmidt Law in some form to predict star formation rates, motivated in part by observed correlations linking dense clouds and star formation such as that between molecular density and Hα. We take the star formation rate $dM/dt = c_* M_{\rm gas}/t_{\rm freefall}$, where $c_* = 0.06$, motivated by observations of rates in the local group. We limit star formation to regions which are marginally self-gravitating and in which the flow is locally convergent.

The second component is to return energy to the model ISM. The key difficulty here is the profound mismatch between the characteristic timescale of a simulation, which is set by the resolution scale and which is roughly 10^6 years for current simulations, and the time over which a real supernova would heat the ISM, roughly 10^{3-5} years. The effect in the simulation is that any energy injected as heat will be radiated at high gas densities before the simulation can respond to reduce the density. In order for the simulation to respond appropriately would require modelling changes which are occurring far below the resolution scale of the simulation.

The approach we have taken here is to smooth the energy resulting from a model supernova onto the SPH neighbour particles and to let the energy persist in an adiabatic state for 30 Myr, roughly the lifetime of an OB Association. This allows the energy to effectively couple to the model ISM and regulate star formation. A full description of this and other methods is described in Thacker & Couchman (2000). Although the model is crude, it has been shown to successfully produce an internally consistent feedback and regulation which is not highly sensitive to resolution. For a more sophisticated model, which attempts to model the multiphase nature of the ISM directly, see Springel's contribution in this volume.

4. The simulations

We performed three standard CDM simulations starting from identical initial conditions:

1. ES: star formation and feedback,
2. NF: star formation but no feedback,
3. NSF: no star formation (or feedback).

Each simulation consisted of four mass hierarchies in a 48 Mpc cube, with the central high resolution region occupying a sphere of comoving diameter 6 Mpc. The central region contains gas as well as dark matter particles: the surrounding shells of lower number density, higher mass particles model just the dark matter. The details of the central region are as follows: 65,454 particles of dark matter (each mass $1.1 \times 10^8\,M_\odot$) and 65,454 particles gas (each mass $1.2 \times 10^7\,M_\odot$); 3 kpc softening; 1% $Z_\odot$ metallicity. The simulations were run to a redshift of 0.52 at which point discs are well established in all runs. It was not possible to run the simulations further without the collapse of the innermost low resolution shell contaminating the high resolution region. Table 1 shows further details of the three simulations.

5. Results

The three simulations showed strikingly different outcomes. Comparing the qualitative features of the NF and ES runs, it is clear that feedback provides a mechanism for maintaining a reservoir of diffuse gas which forms the bulk of an extended disc between redshift 1 and 0.5. In the runs without feedback, dense knots of gas rapidly spiral into the centre of the potential leading to much smaller discs. We will quantify these results by focussing, below, on the disc scale lengths and specific angular momenta of the model discs.

Table 1. Comparison of simulations

Run	Time steps to z=0.52	Mean time step (Myr)[a]	Wallclock (days)[b]	Particles $< R_{200}$[c] Dark Matter	Gas	Star
ES	15,930	0.44	9	14,546	8,350	13,193
NF	15,900	0.44	10	14,466	5,805	17,264
NSF	17,440	0.40	14	14,403	12,713	0

[a] $\Delta t_{\rm min} = 0.12$ Myr (all simulations)
[b] OpenMP implementation of *Hydra* on a 4-processor Alpha ES40
[c] $R_{200} = 203$ kpc

5.1. Star Formation Rate

In the ES run, star formation begins at z=5.6 and the first feedback event occurs at z=3.95 with a peak star formation rate (SFR) of $27\,{\rm M}_\odot\,{\rm yr}^{-1}$. In the NF run, the SFR peaks at $51\,{\rm M}_\odot\,{\rm yr}^{-1}$ showing that a substantial degree of regulation is operating in the feedback model. At $z = 0.52$, the star formation rate has dropped to $1.7\,{\rm M}_\odot\,{\rm yr}^{-1}$, with the rate in the NF run being much lower due to significant gas exhaustion. The star formation rate for the ES run is plotted in Fig. 2.

5.2. Disc Scale Lengths

We quantify the effect of feedback by comparing the disc scale-lengths in the three runs. To begin, Fig. 3 shows the projected surface density of the disc in the ES run together with exponential fits to the bulge and disc. Table 2 compares the properties of the disc systems forming in each of the three simulations. Disc edges were identified at a surface density threshold of $7 \times 10^{12}\,{\rm M}_\odot\,{\rm Mpc}^{-2}$.

Table 2. Disc, bulge and halo properties in the ES, NF, NSF runs.

Run	Scale length (kpc)	Disc Edge (kpc)	Mo et al. (1998)[a]	$M_{\rm halo}$ $10^{12}\,{\rm M}_\odot$	$M_{\rm bulge}$ $10^{10}\,{\rm M}_\odot$	$M_{\rm disc}$ $10^{10}\,{\rm M}_\odot$	Bulge/ Disc
ES	7.6	29	6.5	1.56	6.69	3.94	1.7
NF	4.7	19	2.5	1.55	6.54	3.26	2.0
NSF	3.6	13	2.5	1.55	7.98	2.15	3.7

[a] Theoretical prediction following from Mo et al. 1998 as described in subsection 5.3.

It is clear that the inclusion of feedback leads to substantially larger discs than either the No Feedback (NF) or No Star Formation (NSF) runs. Feedback in the ES run produces a disc with scale-length in close accord with those observed and similar to theoretical predictions.

5.3. Specific Angular Momentum

Navarro and Steinmetz (2000), following work of Mo et al. (1998), constructed an empirically-motivated theoretical argument predicting the specific angular momentum of a disc which would form in a dark matter halo of given circular velocity. For the properties of our model halo, this would predict a specific angular momentum for the disc which is 55% of the halo specific angular momentum.

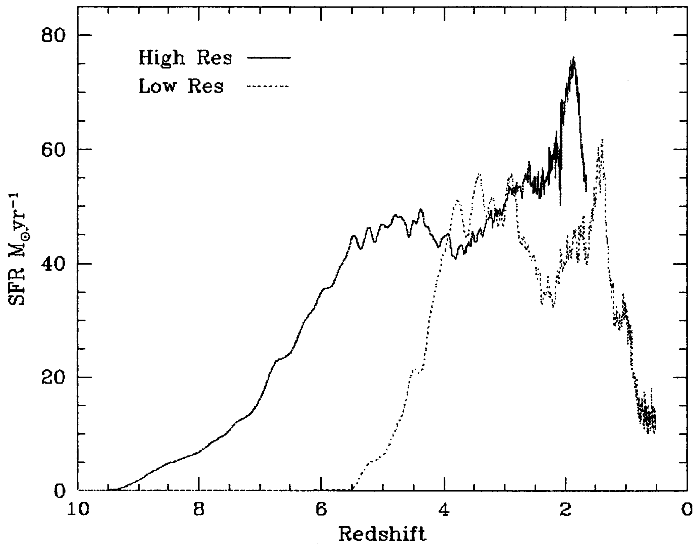

Figure 2. The star formation rate for the ES model (dashed). The solid curve show the results of a simulation with identical cosmological parameters, star formation and feedback algorithm, but with eight times the particle density.

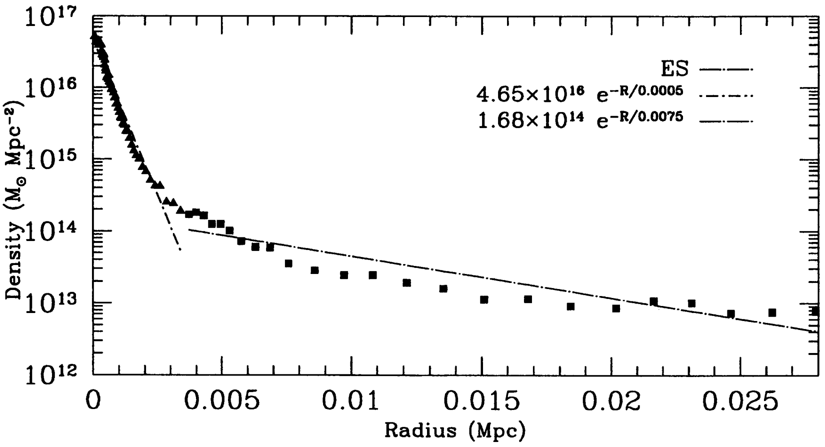

Figure 3. The projected surface density in the ES galaxy simulation.

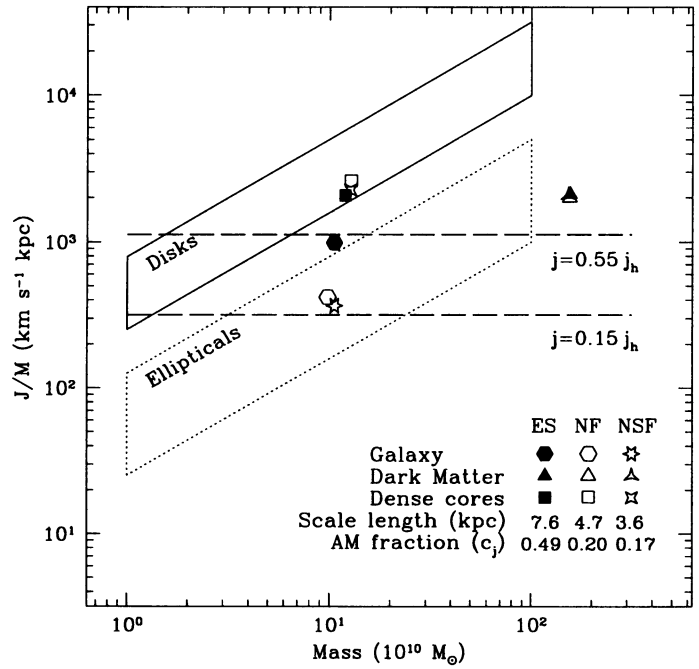

Figure 4. The specific angular momenta of various components of the three simulations at z=0.52. The ES simulation achieves a specific angular momentum value close to the predicted value for the dark matter halo modelled. The boxes are from observational data assembled by Fall (1983).

This value is shown as a horizontal line in Fig. 4 It is clear that the ES run produces a value only 10% below that required, whereas the NF and NSF suffer a loss of over 80% of their specific angular momenta. It is apparent that all of the models have a large reservoir of angular momentum available at large radii as measured by all material within the virial radius at overdensities > 2000 (dense cores). This material, if kept in a diffuse state by feedback, can contribute the required specific angular momentum. Condensed baryons occupying these dense core will rapidly spiral to the centres of the proto-galactic potential well.

5.4. Rotation Curves

Finally, we show the rotation curve for the ES galaxy. The large bulge component of our model galaxy makes it difficult to make a good determination of the stellar disc. Instead we concentrate on the gas disc rotation curve. This is plotted in Fig. 5 and is compared with the derived circular velocity curve for this halo.

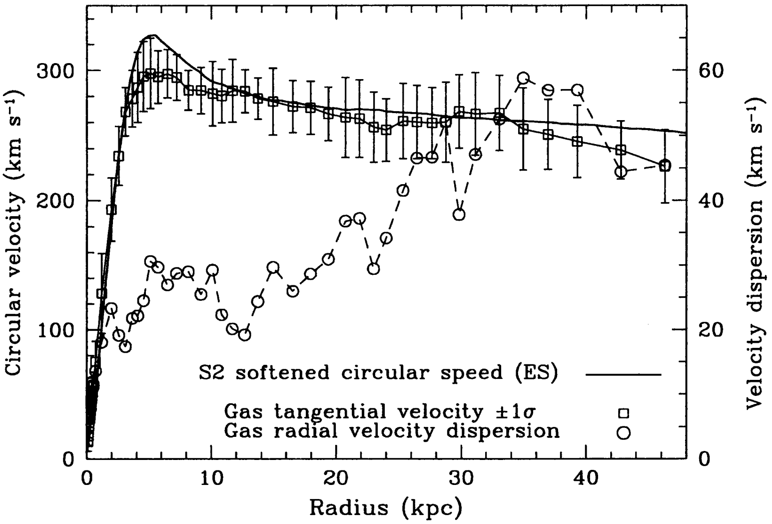

Figure 5. The rotation curve for the ES model disc.

6. Conclusions

We have introduced a simple, yet physically motivated, model for star formation and feedback in disc galaxy simulations. By comparing simulations started from identical initial conditions with star formation and feedback, with star formation but no feedback, and with no star formation we are able to judge the efficacy of our simple model for gas regulation in our model galactic ISM.

The model produces discs with scale lengths close to those observed and preserves the angular momentum content of the baryons to with 10% of the value required to reproduce observed discs. In contrast, simulations without feedback produce small discs which have suffered a catastrophic loss of 80% of the initial angular momentum. The inclusion of star formation, but not feedback, does not alleviate the angular momentum problem and runs without feedback suffer rapid gas exhaustion by the end of the simulation.

This model has been run at lower resolution in Thacker & Couchman (2000) but with slightly different parameters. We are now running the same halo with the ES feedback model with eight times the particle resolution of the ES run described here. The SFR up to $z = 1.9$ is plotted in Fig. 2. As expected, the high resolution run shows an earlier star-formation epoch and epoch of first feedback. What is pleasing is that the peak SFR is broadly similar in the two models, suggesting a useful convergence in the physical model.

The galaxy that we have simulated is large and would represent a relatively rare event, it is thus unwise to suggest that the details of our model have specific bearing on galaxy formation in CDM cosmologies. The fundamental result of this work is that we have demonstrated that hierarchical structure formation does not preclude the formation of extended discs: the inclusion of a physically plausible feedback model has indeed successfully regulated star formation in a forming galaxy and lead to disc angular momenta and scale lengths consistent with those observed.

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