ABSTRACTS OF THESES

I. Felegi, Sampling without replacement with probabilities proportional to size. Carleton University, October 1961 (Supervisory Committee: J. W. Hopkins, D. K. Dale and M. S. Macphail).

The sampling method investigated is that of sampling without replacement from a finite population with probabilities proportional to a measure of size. The estimator of the total or mean depends on π_i, the probability that the i-th unit of the population is in the sample of n units. The calculation of the π_i, which are functions of the measures of size, may be extremely laborious. The dual aim of the investigation is to (1) develop approximate values of the measures of size which give rise to optimal values of π_i and (2) to develop approximate values of π_i as functions of the measures of size. For small values of N (the number of units in the population) the approximations as well as the reduction of the variance due to the solution of problem (1) are empirically studied. For large values of N the order of the error term of the approximation of π_i is found to be $0(\frac{1}{N^3})$ if $n = 2$, and $o(\frac{1}{N})$ for other values of n. As a side result, it is shown that if $n = 2$ sampling with pps without replacement is preferable to the corresponding design with replacement.

John D. Dixon, On general group extensions. McGill University, April 1961 (Supervisor, Hans Schwerdtfeger).

The problem of normal extensions of groups has received considerable attention and has reached the point of reasonable solution. The aim of this thesis is to generalise some of the results obtained in the normal case to the less tractable case of general extensions.

Let G be an arbitrary subgroup of a group H and let

$$H = G_1 + G_\alpha + G_\beta + \dots$$

be the decomposition of H into right cosets modulo G. We can introduce a structure into the set of such cosets (which following

A. Loewy and R. Baer (1928) we call a mixed group) by defining a product $G_\alpha \circ G_\beta$ if and only if the set product $G_\alpha G_\beta$ is a single coset modulo G. In the latter case $G_\alpha \circ G_\beta = G_\alpha G_\beta$.

The mixed group may be defined abstractly and we call the set of cosets of H modulo G a representation of the corresponding mixed group Γ and write it H/G. Γ is a group if and only if G is normal in H. H is called an extension of the group G by the mixed group Γ. Two extensions of G by Γ are called equivalent if they are G-isomorphic.

For a group G and a mixed group Γ the skew product (G, Γ) is defined as the set $\{(a, \alpha) \mid a \varepsilon G, \alpha \varepsilon \Gamma\}$ with the binary operation $(a, \alpha)(b, \beta) = (af(\alpha, b, \beta), \varphi(\alpha, b, \beta))$ for some functions f and φ. Necessary and sufficient conditions are found on f and φ such that (G, Γ) is an extension of G by Γ.

When the mixed group Γ is a group and the extension is a normal extension then the mapping $\alpha\theta : a \to f(\alpha, a, 1)$ is an automorphism of G and the mapping $\alpha \to \alpha\theta$ is an antihomomorphism of Γ into A(G) (the group of automorphisms of G). For given Γ certain pairs $\langle G, \theta \rangle$ are called kernels and Eilenberg and Maclane (1947) in their examination of normal extensions proved that these form an abelian semigroup under a properly defined product. Not all $\langle G, \theta \rangle$ have an extension H but those kernels which are extendible form a subsemigroup.

These concepts are generalised as follows. We take a particular representation B/A of a mixed group Γ and a particular semigroup of kernels $\langle K, \theta \rangle$ for (normal) extensions by the group B and consider

(a) u-groups $\langle G, K, \theta \rangle$, where $\langle K, \theta \rangle$ is a kernel for normal extension by B and G is an extension of $\langle K, \theta \rangle$ by the subgroup A;

(b) o-groups H, corresponding to $\langle G, K, \theta \rangle$, which are normal extensions of $\langle K, \theta \rangle$ by B such that H contains G. Thus H is an extension of G by B/A.

Many of the theorems on normal extensions generalise. Thus:

All u-groups $\langle G, K, \theta \rangle$ which have any corresponding o-group have the same number of inequivalent o-groups;

If the centre of K is the unit group then every $\langle G, K, \theta \rangle$ has exactly one inequivalent o-group.

André Barbeau, <u>Sur la structure de certaines classes d'anneaux.</u> l'Université de Montréal, Mai 1961 (Directeur, G. Thierrin).

Dans un mémoire paru en 1959, G. Thierrin a déterminé la structure des anneaux bipotents à droite, un anneau A étant bipotent à droite si l'on a $aA = a^2A$ pour tout $a \in A$. La thèse que nous avons présentée constitue dans une certaine mesure une suite à cette étude sur les anneaux bipotents à droite.

En premier lieu, nous avons étudié les anneaux complètement bipotents à droite, c'est-à-dire les anneaux dans lesquels tout sous-anneau est bipotent à droite. En particulier, nous avons montré qu'un anneau A est un corps, dont tous les éléments sont d'ordre fini pour la multiplication, si et seulement si A est primitif et complètement bipotent à droite.

En second lieu, nous avons étudié les anneaux c-bipotents à droite et d-bipotents à droite, ces anneaux étant respectivement définis par les relations $aA = aAa$ et $aA = Aa^2$ pour tout $a \in A$. Ces anneaux sont isomorphes à une somme sous-directe d'anneaux de carré nul et de corps.

Puis, s'est présentée l'étude des anneaux complètement c-bipotents à droite et des anneaux complètement d-bipotents à droite. On retrouve pour ces anneaux des résultats analogues à ceux qui ont été formulés pour les anneaux completement bipotents à droite.

Nous avons ensuite ajouté un chapitre sur les idéaux d-bipotents à droite d'un anneau quelconque A, un idéal I de A étant d-bipotent à droite, si l'anneau quotient A/I est d-bipotent à droite. Nous avons montré en particulier que tout idéal d-bipotent à droite d'un anneau vérifiant la condition de chaîne ascendante pour les idéaux d-bipotents à droite, est l'intersection d'un nombre fini d'idéaux d-bipotents à droite e-primaires.

Les derniers chapitres de notre thèse sont consacrés à l'étude des anneaux absorbants, des anneaux de triple nul et des anneaux complètement absorbants.

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W. J. Kotzé, On infinitely many algorithms for the solution of an analytic equation. McGill University, October 1961 (Supervisor, H. Schwerdtfeger).

The object of this study is the consideration of the iterative solution of an arbitrary analytic equation $f(z) = 0$. In other words we wish to find an iteration formula of the type

$$z_n = F(z_{n-1}) = G(z_{n-1}, f(z_{n-1}), f'(z_{n-1}), \ldots, f^{(s)}(z_{n-1}))$$

$$s \geqq 1, \quad n = 1, 2, \ldots$$

which gives an approximation z_n of a zero ξ of $f(z)$ after n applications. An algorithm of this type is said to converge towards a root ξ of $f(z) = 0$ for all initial approximations $z = z_0$ in a vicinity of ξ, of order $k > 0$, when

$$F(z) - \xi = 0(|z - \xi|^k), \quad z \to \xi .$$

Special attention was given to the second and third order algorithms which are modifications of the well-known Newton-Raphson method. Attention was especially directed at the convergence of the different modifications and an error estimate of each. These modifications include those proposed by A. M. Ostrowski, J. S. Frame and others. The author proposed five more modifications. Four of these are probably of theoretical interest only. The fifth one however might turn out to be of some interest to the practical computor. This modification is for the approximation of a root ξ of $f(z) = 0$, where ξ is of known multiplicity p.

$$\left.\begin{aligned} z_{n+1} &= z_n - \frac{f^{(p-1)}(z_n)}{f^{(p)}(z_n)} \\ z_{n+2} &= z_{n+1} - \frac{f^{(p-1)}(z_{n+1}) \cdot (z_{n+1} - z_n)}{2f^{(p-1)}(z_{n+1}) - f^{(p-1)}(z_n)} \end{aligned}\right\} \quad (1)$$

$$n = 0, 1, 2, \ldots$$

This thesis was accepted for the M. Sc. degree. It is not customary to abstract Master's theses for the Bulletin, but the Editors feel that there is sufficient originality here to justify a departure from the usual policy.

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It was shown analytically that the convergence speed of this algorithm is indeed comparable with (if not superior to) that of other algorithms used for finding a root of known multiplicity, e. g.

$$z_{n+1} = z_n - p \frac{f(z_n)}{f'(z_n)} \qquad n = 0, 1, 2, \ldots \tag{2}$$

Factually, two successive applications of (2) will roughly give the same degree of approximation as one application of (1). Then we have the added advantage that in case (2), two values of both f(z) and f'(z) must be calculated, whilst in the case of (1) we have to calculate (though to twice the degree of accuracy) only two values of $f^{(p-1)}(z)$ and one of $f^{(p)}(z)$.

The construction and error estimation of two types of higher order algorithms are discussed. These constructions (due to E. Schröder and H. Ehrmann respectively) are very often quite laborious, and in practice it was found that in most cases not much is gained in the use of algorithms of order higher than three. However, there does exist a need for a means of choosing the most expedient algorithm for a specific function. An attempt, involving three original theorems, was made to comply with this demand.

A new acceleration method for an algorithm of order higher than two was also proposed. The thesis is concluded with a short résumé of well-known and some lesser-known theorems and methods (especially those based on the theory of continued fractions) which might be of some assistance in determining the approximate location of the roots of an analytic equation. The knowledge of such locations is of importance to the often arduous task of choosing an initial approximation z_0 to the desired root.