had expected at a first glance. The treatment of these problems given in this account is rather algebraic and the original geometry is sometimes hard to find. The main interest of Sah's account is that he constantly tries to make connections with other topics in mathematics. Some of these connections may turn out to be fruitful. Among the subjects that arise are group representations, Hopf algebras and the cohomology of groups. As an example of how a simple geometric problem can be generalised and abstracted this book could hardly be beaten.

ELMER REES

STRĂTILĂ, S. and ZSIDÓ, L., *Lectures on von Neumann algebras* (Abacus Press, 1979), pp. 478, £29·95.

Although the study of operator algebras commenced in the 1920's, it was not until 1957 that the first comprehensive monograph on the subject, J. Dixmier's "Les algèbres de'opérateurs dans l'espace hilbertien", appeared, to be followed in 1964 by the same author's "Les C^*-algèbres et leurs représentations" (both Gauthier-Villars, Paris). In 1971 a unified, if idiosyncratic, treatment was given by Sakai in his "C^*-algebras and W^*-algebras" (Springer-Verlag, Berlin). Since these works appeared there has been considerable and spectacular progress in understanding the structure of von Neumann algebras, largely as a result of the formulation in the late 1960's of the Tomita-Takesaki theory of modular Hilbert algebras. Until recently there was no readily accessible exposition of the theory of von Neumann algebras which included the Tomita-Takesaki theory. The situation has now changed dramatically with the publication, in the last year or so, of works by Bratteli and Robinson, Pedersen, Takesaki, and the volume under review.

In their book Strătilă and Zsidó, unlike some of the other authors just mentioned, concentrate on von Neumann algebras; their aim is to give a clear and self-contained exposition of the theory up to and including the Tomita-Takesaki results. They have, in many ways, been successful in this. The first eight chapters are devoted to a careful presentation of the classical, i.e. pre-Tomita-Takesaki, theory, the approach being in the Murray–von Neumann tradition: underlying Hilbert spaces are always clearly in view, and comparison of projections is to the fore. This approach is, in my opinion, easier for a newcomer to the subject than the elegant, but less transparent, approach of Dixmier using traces, or the non-spatial formulation of Sakai. The authors write clearly and include concise alternative proofs of certain important results (notably Yeadon's proof of the existence of a trace in a finite von Neumann algebra).

Chapter 9 prepares the ground for the Tomita-Takesaki theory with a systematic study of unbounded linear operators on Hilbert space. The chapter contains much useful material in a very accessible form. As well as presenting the standard results, such as polar decomposition and Stone's theorem, the authors discuss a certain operator equation and its solution in the form of an integral. It is interesting to see this result, the key to the fundamental results of the Tomita-Takesaki theory, set in this wider context.

The lengthy final chapter, entitled "the theory of standard von Neumann algebras", is devoted to an exposition of the Tomita-Takesaki theory. The basic objects in the theory, such as Hilbert algebras and the modular operator, are defined and their properties established. It is shown that a normal, faithful, semifinite weight on a von Neumann algebra satisfies the KMS condition with respect to a suitable modular automorphism group. Finally, Conne's remarkable unitary cocycle result is proved, and some of its applications given. At the end of each chapter there are exercises and comments amplifying earlier points. The book ends with a short appendix on fixed point theorems, followed by a very up to date bibliography of some 120 pages.

It is a pity that such a substantial portion of the book is given over to the bibliography, as many of the references do not relate directly to matters in the text, and moreover some important topics have been omitted. There is, for example, no treatment of direct integral decomposition. I am also disturbed at the lack of concrete examples of von Neumann algebras of types II and III. Surely any comprehensive treatment of the subject should include a proof, at least as an exercise, that the objects under consideration actually exist (to be fair to the authors, they do give several references). A minor criticism I would make is of the English, which, though always clear, is sometimes not idiomatic. Also, some of the terminology is unconventional, for example "of

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countable type" is used in place of the more usual "countably decomposable" or "σ-finite". Notwithstanding these reservations, I consider this book valuable both as a reference source and, in conjunction with examples of the sort given by Dixmier and Sakai, as a clear and readable introduction to von Neumann algebras and Tomita-Takesaki theory.

A. S. WASSERMANN

KOSTRIKIN, A. I., *Introduction to algebra,* translated by N. Koblitz (Universitext, Springer-Verlag, Berlin-Heidelberg-New York, 1982), 575 pp., £16·50.

This book is far too expensive to be realistically recommended for purchase to today's overdrawn undergraduates, but should certainly be acquired by university libraries. Like many books for undergraduates it has grown out of a course of lectures, or rather in this case, out of two courses of lectures, for the book is formally divided into two roughly equal parts corresponding respectively to a first and a third semester course at Moscow University. This is a source of strength, in that the material is thoroughly class tested and the associated exercises are interesting, varied and apposite; but it gives rise to a weakness in that the elementary real vector space theory of Part 1 is an inadequate preparation for the material on group representations and modules in Part 2. The Moscow students are well catered for, since they receive a second semester course on linear algebra and geometry, but the reader of this book enjoys no such advantage and the attempts that are made to plug the gap are not entirely successful.

It is certainly interesting to see what is taught in an important (and presumably not too unrepresentative) Soviet institution and to realise that their traditions in algebra teaching are not very different from our own. Part 1 begins with "concrete" linear algebra (with vectors as n-tuples of real numbers) and includes an unusually thorough chapter on determinants. From there it proceeds to a fairly typical first course on groups, polynomials, rings and fields, with perhaps a greater emphasis than usual on polynomials as such. Part 2 begins with a long chapter (64 pages) of graphs followed by an even longer chapter (86 pages) on representations going as far as character theory and tensor products. This leaves relatively little space for further developments in rings and fields, and so although many interesting aspects (such as finite fields and ruler and compass construction) are discussed in the final chapter there is no systematic exposition of the Galois group, and the insolubility by radicals of the quintic, heralded in the introduction as one of the motivating problems of abstract algebra, is not in fact discussed in detail.

The exposition is precise, careful and thorough and the translation reads so smoothly that it is hardly ever noticeable that English was not the original language. The photographically reduced typescript has been produced very skilfully and deserves to be supplemented by less amateurish diagrams.

In summary, this book will be a useful source for both teacher and student and should be valued by both, not least for its wide-ranging and striking examples of the application of algebraic ideas.

J. M. HOWIE

ROBINSON, D. J. S., *A course in the theory of groups* (Graduate Texts in Mathematics Vol. 80, Springer-Verlag, Berlin-Heidelberg-New York, 1982) xvii+481 pp., DM 98.

This is a lovely book, whose stated aim and intention (successfully accomplished) is to give an introduction to the general theory of groups. The reader is expected to have the maturity of a year's graduate study in a British or American university, with a good basic knowledge of rings, fields, modules and the like. The writing is meticuously clear and concise. While not leisurely, it is not terse and indeed it is done with such enthusiasm and erudition as to carry the reader happily along. Many examples of groups are given, the life-blood of the subject, of course; in addition there is a truly vast set of exercises (some 650 in all!), varying from the elementary to the really quite challenging. Some of them are required for the subsequent development, and are noted as