adding to each "Stichwort" the corresponding word in English, French, Italian, Russian.

H. Schwerdtfeger, McGill University

Infinitistic methods, Proceedings of the symposium on foundations of mathematics, Warsaw 1959. Pergamon Press. 362 pages. £5 net.

Under the influence of Hilbert's program to provide a sound basis for mathematics by purely finitary consistency proofs, logicians have worked in metamathematics for many years with one hand tied behind their backs. To mark the recognition of a new era, we have here the proceedings of a symposium on the foundations of mathematics with no holds barred: Everything goes, from Zorn's lemma to formulas of infinite length. The results are tremendously interesting and illuminate many different branches of mathematics.

The volume under review contains research papers that were presented at the 1959 Warsaw Symposium by the following authors:

J. Łoś, S. MacLane, R. Montague, C. Spector, G. Kreisel, A. Mostowski, L. Henkin, A. Heyting, Dana Scott, A. Robinson, R. L. Vaught, L. Kalmar, all in English.

P. Bernays, G. H. Müller, P. Lorenzen, R. MacDowell and E. Specker, R. Péter, all in German.

A. S. Ésénine-Volpine, L. Rieger, R. Fraissé, Gr. C. Moisil, all in French.

P. S. Novikov, in Russian.

It is clearly impractical to review all these papers or even to list their titles. Instead, I shall make some remarks about three of the papers that caught my fancy on first browsing through the volume.

(1) S. MacLane, Locally small categories and the foundations of set theory. There are several tentative proposals, for avoiding the paradoxes of set theory, the one accepted by most mathematicians being due to Gödel. One distinguishes between sets and classes, all sets are classes, but only sets can be members of classes. Many recent constructions in homological algebra have violated this principle; for example, people have considered the category of all categories, in spite of the fact that categories are usually too large to be sets. MacLane proposes a uniform method for overcoming these difficulties, and incidentally gives a very readable and concise account of the major concepts and recent developments in homological algebra.

(2) L. Henkin, Some remarks on infinitely long formulas. The paper deals with three ways of creating formulas of infinite length: First by allowing infinitary predicates, secondly by allowing infinite

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conjunctions and disjunctions, and thirdly by allowing infinite sequences of quantifiers. Concerning this last problem, it turns out that one need not restrict oneself to sequences, other partial orderings of the quantifiers also make sense. For example, Henkin considers the following pseudo-formula:

$$\left.\begin{array}{l}(\forall x)\,(\exists v)\\ (\forall y)\,(\exists w)\end{array}\right\}\; R(x, y;\, v, w)\ ,$$

the idea being that v depends on x but not on y and w depends on y but not on x . This makes sense, for it is obviously equivalent to the second order formula

$$(\exists g)\,(\exists h)\,(\forall x)\,(\forall y)\; R\,(x, y; g(x),\ h(y))\ .$$

However, it is not equivalent to an ordinary linear formula in the first order predicate calculus, as was shown by A. Ehrenfeucht. One might be tempted to think that such a pseudo-formula is excluded from the usual predicate calculus for typographical reasons only, but actually Ehrenfeucht showed that if it were admitted into the predicate calculus, the enriched calculus would no longer be axiomatizable.

(3) Dana Scott, On constructing modes 1 for arithmetic. This paper contains some fascinating applications of logic to algebra, but it can be read without any knowledge of formal logic. If Q is the field of rationals, I an infinite index set, M a non-principal maximal ideal of Q , then it follows from the work of S. Kochen that Q^I/M is a field which is not isomorphic to Q , but which is elementarily equivalent to Q , in the sense that every elementary (first order) statement valid for the latter is valid for the former. The present author shows that if Z is the domain of integers, P a non-principal minimal prime ideal in Z^I , then Z^I/P is an integral domain which is not isomorphic to Z , but which is elementarily equivalent to Z . In fact Z^I/P is related to Skolem's original non-standard model of arithmetic. It can be described as an ordered integral domain which is inductive (in a certain technical sense) and non-Archimedean. (The reader will easily convince himself that the Archimedean property of a ring is not an elementary statement; one must quantify over something else than the elements of the ring.) The paper also contains several purely algebraic results of considerable interest.

Let it be emphasized again that the selection of the three papers reviewed here is not intended to reflect in any way on the quality or importance of the remaining papers.

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