A NUMERICAL STUDY OF THE FISSION HYPOTHESIS FOR ROTATING POLYTROPES

R. H. Durisen
Indiana University

and

J. E. Tohline
Los Alamos Scientific Laboratory

1. INTRODUCTION

Analytic stability analyses (Lyttleton, 1953; Chandresekhar, 1969) have shown that Maclaurin spheroids are dynamically unstable to non-axisymmetric perturbations if

$$t \equiv T/|W| \geq 0.274,$$

where T is the rotational kinetic energy of the spheroid and W is its total gravitational potential energy. If appropriate dissipative processes are present, the spheroids are unstable on a secular time scale if

$$t \geq 0.138.$$

Maclaurin spheroids are highly idealized objects, being uniform in density and having solid body rotation; but in recent years it has been realized that a wide range of centrally condensed, non-uniformly rotating objects have critical values of t (points of instability) very nearly the same as those quoted above for the Maclaurin spheroids (e.g., Ostriker and Tassoul, 1969; Ostriker and Bodenheimer, 1973; Durisen and Imamura, 1981). If, then, any star can become dynamically unstable to non-axisymmetric perturbations when it evolves to a configuration with $t \geq 0.27$, it is of considerable astronomical interest to know what the eventual outcome of such an instability is. It has been suggested that such a global non-axisymmetric instability leads to fission and is the mechanism by which close binary star systems are formed (Jeans, 1919; Stoeckly, 1965; Roxburgh, 1966; Lebovitz, 1972, 1974). This "fission hypothesis" cannot be confirmed or denied through linear stability analyses alone, since fission is inherently a non-linear process. By using a 3-D hydrodynamic computer code, we are studying the non-linear growth of non-axisymmetric perturbations in

Space Science Reviews **27** (1980) 267–273. 0038–6308/80/0273–267 $01.05.

rapidly rotating polytropes in an attempt to understand the fate of dynamically unstable stars. In the long run, we expect to investigate the stability of a wide range of values of t in order to test the general validity of the classically determined stability limit and in hopes of testing the fission hypothesis. To date, we have studied polytropes of index $n = 3/2$ and $n = 1/2$ over only a limited range of t, and have carried only a few of the evolutions far enough in time to be able to examine fully developed non-axisymmetric structures. A brief summary of the results of this initial phase of our investigation is presented in this paper.

2. THE NUMERICAL TECHNIQUES

Equilibrium axisymmetric models of rotating polytropes were constructed using the Ostriker-Mark-Bodenheimer (Ostriker and Mark, 1968; Bodenheimer and Ostriker, 1973) self-consistent field code as initial models for the 3-D hydrodynamic investigation. Constructing initial models in this manner ensures that we are investigating the stability of well-defined structures and enables us to directly compare our results with their linear-analysis counterparts. Calculations initiating from these equilibrium initial models, we believe, will give a clearer test of the fission hypothesis than have previous works that have used initial states of rapid contraction or expansion (Lucy, 1977; Gingold and Monaghan, 1978, 1979).

The 3-D hydrodynamic computer code that was used in this study is described in detail by Tohline (1980). Briefly, the code gives an explicit, first-order-accurate time-integration of the three-dimensional equation of motion and continuity equation on a moving-Eulerian, cylindrical computational grid. A solution of the three-dimensional Poisson equation at each time step permits the analysis of self-gravitating gas flows. A polytropic equation of state replaces an energy equation in this investigation. In the model evolutions discussed here, a grid resolution of (32, 16, 16) in (R, Θ, Z) was used. The Z-axis was the rotation axis of the star. The 16 zones in the Z-direction resolved only the "northern" hemisphere of the star, as reflection symmetry through the equatorial plane was assumed in all cases. The 16 azimuthal zones were distributed over only π radians and a periodic boundary condition was used in order to get adequate angular resolution; as a result, only "even" non-axisymmetric modes have been examined in this phase of the investigation. No viscous dissipative forces have been explicitly introduced into the components of the equation of motion, so we expect to find only points of dynamic instability in this investigation. As will be discussed later, there is some dissipation intrinsic to the code due to the finite differencing techniques used.

Using the hydrodynamic code, some of our initial models have been evolved in the absence of non-axisymmetric distortions for a couple of rotation periods and have shown negligible readjustment in the meridional, R-Z plane. One such evolution (the same n and t as the model

https://doi.org/10.1017/S0252921100081641 Published online by Cambridge University Press

discussed is §4) remained well-behaved as we ran it through eight rotations. This test assures us that our initial models are satisfactory equilibrium axisymmetric configurations.

3. SUMMARY OF THE NON-AXISYMMETRIC EVOLUTIONS

In order to examine the growth of non-axisymmetric features, the density structure of most of the initial models was "perturbed" following the prescription:

$$\rho(R,\Theta,Z) = \rho_0(R,Z) \cdot [1.0 + A_0 \mathrm{COS}(2\Theta)] \tag{1}$$

where $\rho_0(R,Z)$ is the equilibrium axisymmetric structure and A_0, the amplitude of the perturbation, was either 1/10 (10%) or 1/3 (33%). In most evolutions, this uniform amplitude perturbation evolved, within a single rotation period, to what we will call a Maclaurin bar-mode shape (Hunter, 1977). That is, the amplitude of the perturbation $\delta\rho/\rho$ developed a radial dependence of the form,

$$\frac{\delta\rho}{\rho} = - \frac{\partial \ \ell n \ \rho}{\partial \ \ell n \ R} \cos(2\Theta + \Psi(R)) \quad . \tag{2}$$

A phase lag in the azimuthal structure as a function of radius developed for high t models and created a two-armed, trailing spiral pattern.

In models with $t \lesssim 0.30$, the average amplitude of the non- axisymmetric perturbations actually damped during the course of an evolution. Typically, within two rotation periods, the amplitudes $\delta\rho/\rho$ damped below 1% throughout the entire structure of the star. The damping indicates that there is some finite dissipation in the numerical code which acts to make structures axisymmetric. This means that it will be difficult for us to identify marginally unstable modes and hence to pinpoint the exact value of t at which a given polytrope becomes unstable.

In models with sufficiently high t, the average amplitude of the non-axisymmetric perturbations grew with time. Identifying this growth with an "instability," our results for both n = 3/2 and n = 1/2 polytropes are consistent with the following statement: "Initial models with $t \gtrsim 0.30$ are dynamically unstable to non-axisymmetric perturbations." The fate of such a star is illustrated by describing one extended evolutionary sequence in the following section.

4. AN UNSTABLE POLYTROPE

Figure 1(a-h) illustrates the non-axisymmetric evolution of an n = 3/2 polytrope with an initial t = 0.33. The figure shows density

https://doi.org/10.1017/S0252921100081641 Published online by Cambridge University Press

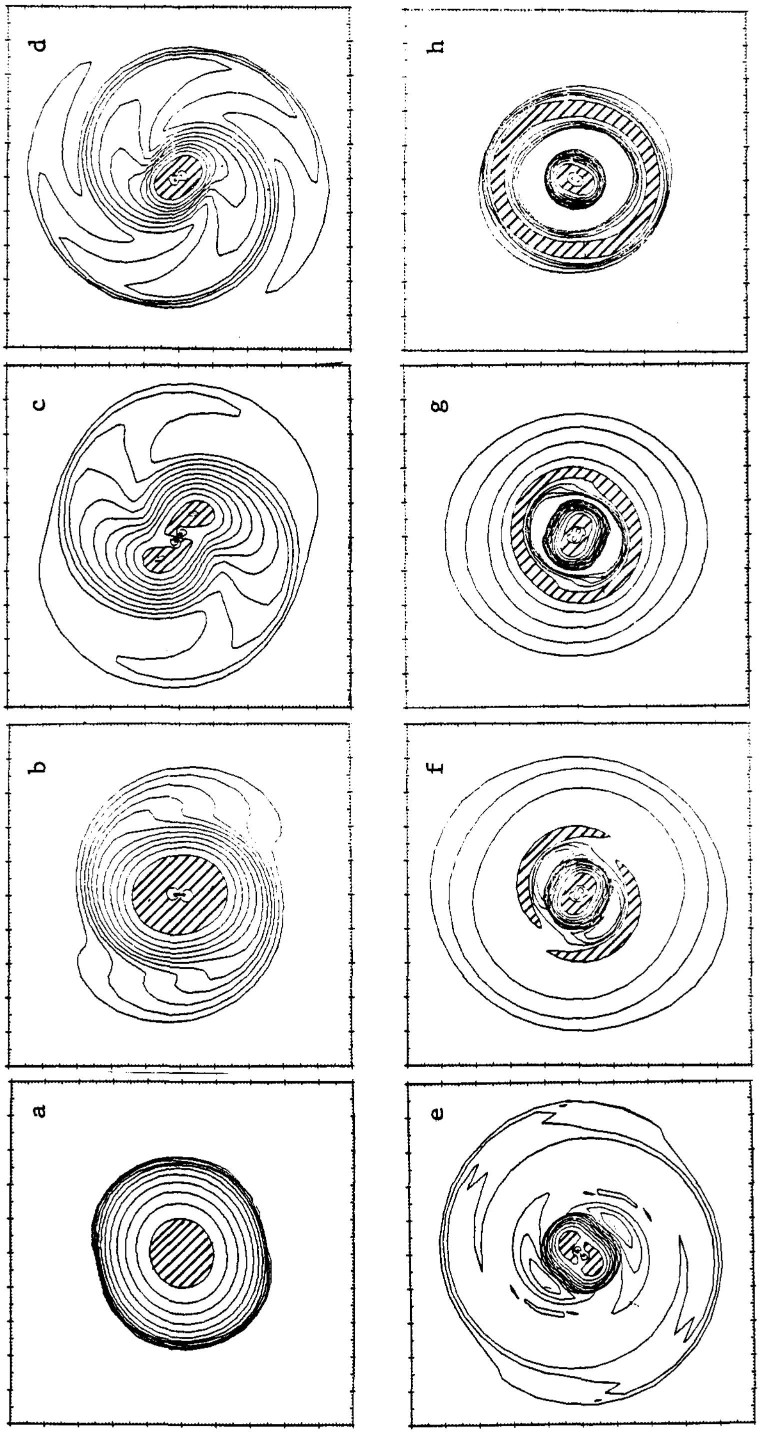

Figure 1.

https://doi.org/10.1017/S0252921100081641 Published online by Cambridge University Press

contours in the equatorial plane of the model; cross-hatched areas are regions of locally maximum density. Due to grid expansion, the edges of frames e-h represent a length 23% larger than frames a-d. The initial perturbation of the axisymmetric equilibrium model (shown as figure 1a) was a pure Maclaurin bar-mode (eq. 2) with $\Psi(R) = 0$; the amplitude of $\delta\rho/\rho$ half way out from the rotation axis was 10%. A velocity perturbation corresponding to a pure bar-mode was also imposed on this initial model. The evolution proceeded as follows (the time unit given is such that $\tau = 1$ is one initial central rotation period):

i. The inner region formed an "elliptical ring" (Fig. 1b, $\tau = 1.9$) with a slight density minimum at the star's center.

ii. Two "blobs" formed from the ring (Fig. 1c, $\tau = 4.7$), but had only a modest factor of 2 density enhancement over the central density--this turned into a bone-shaped configuration that remained throughout the evolution.

iii. Two dramatic, material spiral arms formed (Fig. 1d, $\tau = 5.5$).

iv. The arms "wrapped up" (Figs. 1e-1f, $\tau = 6.6$-7.6) to form what can loosely be described as an exponential disk outside of the central bone.

v. The disk separated from the central bone (Fig. 1g, $\tau = 8.3$) at a radial distance approximately equal to "corotation"--this gap remained.

vi. The disk, compressing axisymmetrically, became a distinct "ring" surrounding, and being well separated from the central bone (Fig. 1h, $\tau = 9.1$).

At the end of the calculation (2000 integration time steps), the central bone contained $\sim$ 85% of the total mass of the system and had a ratio of rotational to gravitational potential energy, $t_{final} \sim 0.19$. The external ring contained more than one-half of the total angular momentum of the system. There was a low amplitude non-axisymmetric structure visible in the external ring, but when the calculations were stopped, it wasn't clear whether the ring would eventually fragment or not. The central bone did not appear to be headed toward fission, but appeared instead to be a stable triaxial star! Fluid was circulating dynamically and stably from one knob of the bone to the other.

We feel that this evolution is representative of the non-linear growth of a dynamic instability in polytropes, rather than being a unique example which is dependent on initial conditions, because an $n = 1/2$ polytrope with $t = 0.33$ showed a similar behavior and because the $n = 3/2$, $t = 0.33$ model run with a different type and amplitude initial perturbation evolved in the manner described above.

5. CONCLUSIONS

Our results show that n = 3/2 and n = 1/2 polytropes are dynamically unstable to non-axisymmetric perturbations if $t \gtrsim 0.30$, in rough agreement with linear theory. Instead of fission as the direct end product of dynamic instability in rapidly rotating, centrally condensed stars, it appears that a rapidly rotating star can, through gravitational torques, eject some high angular momentum material in its equatorial plane and settle down into a dynamically stable (lower t) configuration. The central object, at least in the detailed evolution described above, ends up as a stable traixial star! The assumption of reflection symmetry through the rotation axis needs to be relaxed before this evolutionary picture can be considered realistic and before meaningful comparisons can be made with the results of Lucy (1977) and Gingold and Monaghan (1978, 1979), who found "odd" modes to be of crucial importance.

We cannot escape mentioning that the ejected disk/ring of material in our models may have some connection with the formation of planetary systems.

This research was supported by National Science Foundation Grant No. AST-7821449. We thank NASA-Ames Research Center and Goddard Institute for Space Studies for use of their computing facilities during part of this work.

REFERENCES

Bodenheimer, P., and Ostriker, J. P.: 1973, Astrophys. J. 180, p. 159.
Chandrasekhar, S.: 1969, Ellipsoidal Figures of Equilibrium, Yale Univ. Press, New Haven.
Durisen, R. H., and Imamura, J. N.: 1981, Astrophys. J. 243, in press.
Gingold, R. A., and Monaghan, J. J.: 1978, Monthly Notices Roy. Astron. Soc. 184, p. 481.
Gingold, R. A., and Monaghan, J. J.: 1979, Monthly Notices Roy. Astron. Soc. 188, p. 39.
Hunter, C.: 1977, Astrophys. J. 213, p. 497.
Jeans, J. H.: 1919, Problems of Cosmology and Stellar Dynamics, Cambridge Univ. Press, Cambridge.
Lebovitz, N. R.: 1972, Astrophys. J. 175, p. 171.
Lebovitz, N. R.: 1974, Astrophys. J. 190, p. 121.
Lucy, L. B.: 1977, Astron. J. 82, p. 1013.
Lyttleton, R. A.: 1953, The Stability of Rotating Liquid Masses, Cambridge Univ. Press, Cambridge.
Ostriker, J. P., and Bodenheimer, P.: 1973, Astrophys. J. 180, p. 171.
Ostriker, J. P., and Mark, J. W-K: 1968, Astrophys. J. 151, p. 1075.
Ostriker, J. P., and Tassoul, J. L.: 1969, Astrophys. J. 155, p. 987.
Roxburgh, I. W.: 1966, Astrophys. J. 143, p. 111.
Stoeckly, R.: 1965, Astrophys. J. 142, p. 208.
Tohline, J. E.: 1980, Astrophys. J. 235, p. 866.

DISCUSSION

SIMON: All your perturbations are greater than 10% and sometimes as high as 50%. That isn't a random fluctuation. If you are going to do that by, let's say, a supernova blast wave, shouldn't you perturb them from one side? Would that make a difference?

TOHLINE: One model was run with no initial perturbation, only machine noise, and after 1500 time steps it developed a significant amplitude, though less than 1%, in modes up to m = 4. Due to the large amount of computing time we don't start with such small perturbations. Also, our course grid in the azimuthal direction gives intrinsic damping of nonaxisymmetric features. With improved resolution, we can go to smaller amplitude kicks. A 10% amplitude is not unrealistic; observations indicate that interstellar clouds are lumpy.

TSCHARNUTER: Do you think that at low $T/|W|$ the numerical damping will prevent your finding the stability limit?

TOHLINE: At high $T/|W|$ I don't think the intrinsic numerical damping is very significant. At better resolution we still get comparable results. At low $T/|W|$ we are going to have to worry about this.

YOUNG: Is it hopelessly complex to use a more realistic equation of state?

TOHLINE: An equation of state with the specific heats ratios as a function of density can be used in a 3D code. In order to do any more you need the very time consuming radiative transfer.

https://doi.org/10.1017/S0252921100081641 Published online by Cambridge University Press