2.1.5 FLUX OF HYPERBOLIC METEOROIDS

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Abstract

The production of hyperbolic meteoroids by inelastic collisions between meteoroids is estimated. It is found that, under reasonable assumptions, the calculated flux of hyperbolic meteoroids agrees with satellite data and with lunar microcrater distributions.

We have therefore obtained independent theoretical support for Zook and Berg's (1975) ß-meteoroid hypothesis and for Fechtig et al. (1974) suggestion that submicron lunar microcraters are produced by ß-meteoroids.

I. Introduction

Recent measurements (cf. Fechtig, 1976, for a review) by the Pioneer 8 and 9 satellites led Berg and Grün (1973) to conclude that a substantial flux of micrometeoroids in hyperbolic orbits originate in the region of space between the sun and earth's orbit (also cf. McDonnell, Berg and Richardson, 1975 and Grün, Berg and Dohnanyi, 1973). Zook and Berg (1975) and Zook (1975) have discussed the origin of these particles which they named ß-meteoroids. Whipple (1976) has discussed the occurrence of these particles and given further support to the hypothesis that these particles are being expelled from the solar system by radiation pressure.

We shall, in this paper, quantitatively examine the dynamics of the population of these particles as they are produced by inelastic collisions between larger "parent" particles. It will be found that with reasonable assumptions, Zook and Berg's (1975) hypothesis is in good quantitative agreement with predictions from an analysis of meteoroidal collisions.

II. ß-Meteoroids

When two meteoroids, in elliptic orbits, collide with each other, a number of fragments will be produced. Many of these fragments will be so small that the force of radiation pressure will be significant compared with the gravitational attraction of the sun. The particle velocity will, in almost every case, be comparable to the velocity of the larger parent object (cf. Gault et al., 1963 and Eichhorn, 1976,

) On leave from Bell Laboratories, Holmdel, N.J./USA for a discussion of the ejecta velocity distribution). Diminishing the effective attractive force of the sun by the repulsive radiation force will also diminish the magnitude of the solar escape velocity of the effected particles. In the extreme case, for example, when the repulsive force of radiation pressure equals the attractive force of gravity, no net force at all is acting on the particle and, in that case, even the slightest speed relative the sun would cause it to escape from the solar system. It then follows that, for sufficiently small fragments, the velocity of the parent object exceeds the solar escape velocity of the fragments and the latter will be expelled from the solar system (cf. Dohnanyi, 1970, 1972). This process has originally been suggested by Harwit (1963) and the resulting fragments in hyperbolic orbits have been named ß-meteoroids by Zook and Berg (1975).

III. Distribution of Parent Objects

In order to estimate the flux of ß-meteoroids, it is necessary to estimate the space density and velocity distribution of the parent objects.

Inclinations: The distribution in inclinations of meteoroids will be taken to be similar to that of photographic meteors reduced by McCrosby and Posen (1961).

The following rough approximation to the density of inclination will be adequate for our purposes:

$$(1) \qquad f_1(i)di = 2.36 \exp[-7.2\, i/\pi]di, \quad 0 \leq i \leq \pi/2$$
$$= 0 \qquad , \quad i \geq \pi/2$$

where $f_1(i)di$ is the relative number of meteoroids having an inclination in the range of i to $i + di$ radians; $f_1(i)$ is normalized to 1.

Impact Speed: The relative speed V of a particle having a heliocentric velocity V relative to another particle with a heliocentric velocity W is

$$(2) \qquad V^2 = |\vec{U} - \vec{W}|^2 = |\vec{U}|^2 + |\vec{W}|^2 - 2\, \vec{U}\cdot\vec{W}$$

where the last term on the right-hand side of eq. 2 is the ordinary inner product. For sun axes we shall use the radial, transverse and z-coordinate axes where the z-direction is perpendicular to the ecliptic. Using well-known relationships (cf. e.g. Handbook of Chemistry and Physics), V^2 can be expressed in terms of the orbital elements of the

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colliding particles and the distance from the sun.

We then assume that most collisions between 1 AU and the sun occur near perihelion and expand V^2 in the neighbourhood of the particles' perihelion passage. The resultant formula is then weighted and averaged using the McCrosky and Posen (1961) meteors. The result for the radial dependence for the root mean square (RMS) value of relative velocity is

$$\sqrt{\langle V^2 \rangle} = V_o \quad r^{-.559} \tag{3}$$

where V_o is the RMS value of the relative velocity at 1 AU from the sun and r is the distance from the sun in AU.

Spacial Distribution: In order to estimate the number density of particles as a function of distance from the sun we shall assume that

$$\begin{aligned} f(m, r)dm &= f(m)\ dm\ r^{-b} && R_o \leq r \\ &= 0 && r \leq R_o \end{aligned} \tag{4}$$

where $f(m, r)dm$ is the number density of particles in the mass range m to m + dm, at a distance r AU from the sun, b is a parameter and R_o is the cut-off distance from the sun; within a distance R_o from the sun all particles are assumed vaporized or otherwise destroyed by heat so that the meteoroid population is taken to be negligible in that region.

For f(m)dm, we take (Dohnanyi, 1973)

$$\begin{aligned} f(m)dm &= A_o\ m^{-11/6}\ dm && m \quad 10^{-10}\ kg \\ &= A_1\ m^{-3/2}\ dm && m \quad 10^{-10}\ kg \end{aligned} \tag{5}$$

where

$$\begin{aligned} A_o &= 1.36 \times 10^{-15} \\ A_1 &= 10^{20/3}\ A_o. \end{aligned} \tag{6}$$

IV. Collision Dynamics

In order to estimate the number density of fragments produced during collisions, we shall use a method based on experimental results by Gault et al. (1963) and discussed earlier by Dohnanyi (1969). Accordingly, we take for the ejecta spectrum

$$g_M(m)dm = H(M)m^{-\eta}dm \tag{7}$$

where $g_M(m)dm$ is the number density of fragments produced when an object having a mass M is catastrophically disrupted. H(M) is obtained from the conservation of mass requirement:

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(8) $M = H(M) \int_{\mu'}^{M_b} m^{-\eta+1}\, dm$

where M_b is the mass of the largest and μ' is that of the smallest fragment and η is a constant. Eq. 8 can then be solved for $H(M)$ in terms of M, M_b, μ' and η.

Following Dohnanyi (1969,10,12) we take

(9) $M = \Lambda M$

where

(10) $\Lambda = .5\, V^2$

where the collision speed V is in km/sec.

The mass of the smallest projectile, M_p, capable of catastrophically disrupting the target mass M is taken to be (Dohnanyi, 1970)

(11) $M_p = M/\gamma$

where

(12) $\gamma = 250\, V^2$, where V is in km/sec.

With this notation, the production rate (per second and per cubic meter) of fragments in the mass range of m to m + dm, due to catastrophic collisions between larger masses (M and M_2) is:

(13) $$g(m, r)dm = K(2-\eta)\, m^{-\eta} \Lambda^{\eta-2}\, dm \cdot$$
$$\cdot \int_{m/\Lambda}^{M_\infty/\gamma} dM \int_{M}^{\gamma M} dM_2 (M+M_2)(M^{1/3}+M_2^{1/3})^2 M^{\eta-2} f(M) f(M_2) r^{-2b}$$

where $f(M)$ is given by Eq. 4, 5 and 6 and where (in standard units)

(14) $K = (3\sqrt{\pi}/4\rho)^{2/3}\, V$

where ρ is the average material density of the colliding objects and V is the average impact speed. For fragment mass ranges of our interest, the contribution to $g(m)dm$ of erosive collisions can be shown to be minor (cf. Dohnanyi, 1969, 1970) for distributions of parent objects given by Eq. 5.

When the integral in Eq. 13 is properly evaluated, and only the dominating terms retained, we obtain for the number of fragments produced per unit volume and unit time in the mass range m to m+dm and a distance r from the sun,

$$(15) \quad g(m, r)dm \simeq K(2-\eta)\Lambda^{\eta-2} A_o^2 \mu^{\eta-4/3} m^{-\eta} dm \times (r^{-2b})$$

where $\cdot \left\{ \gamma^{2.5-\eta}\left[\frac{-1.14}{2.5-\eta} + \frac{2}{2-\eta} + \frac{6/7}{\eta-4/3}\right] - \frac{(6/7)\,\gamma^{1/6}}{\eta-4/3}\left(\frac{m}{\mu\Lambda}\right)^{\eta-4/3} \right\}$

$$(16) \quad \mu = 10^{-10} \text{ kg}$$

is the mass corresponding to the point where a break occurs on a double logarithmic plot of the mass-flux curve (cf. Eq. 5). The dominating contribution to this expression is the production of fragments when a projectile with a mass smaller than 10^{-10} kg catastrophically collides with a target object having a mass greater than 10^{-10}kg.

V. Flux of Fragments

When the fragment production per unit volume is integrated over a sphere of radius r with the sun at the centre, we obtain the total number of these particles, produced in this volume, every second. When these fragments are ß-meteoroids, then division of this production rate by a surface element at r would also give the flux of these ß-meteoroids through the surface element.

It was shown (Dohnanyi, 1970, 1972) that fragments having a mass of about 10^{-9} kg will become ß-meteoroids if the orbit of the parent body is highly eccentric ($e > .9$). For parent orbits with smaller eccentricities the limiting fragment size for hyperbolic orbits (ß-meteoroids) becomes smaller; for parent objects in circular orbits the maximum fragment mass of the ß-meteoroids is about 10^{-14} kg for a material density of 10^3 kg/m^3 and 10^{-15} kg for a material density of 3×10^3 kg/m^3. We may therefore assume that most fragments having a mass smaller than 10^{-15} kg will become ß-meteoroids. We shall, in what follows, assume that all fragments under consideration are ß-meteoroids and estimate their flux using our model.

We now proceed to calculate the flux of fragments. We use the expression g(m, r)dm, Eq. 15, for the production rate, per unit volume, of ß-meteoroids in the mass range m to m+dm at a distance r from the sun. We substitute the expression Eq. 14 for K, expression Eq. 12 for γ, expression Eq. 10 for Λ and thus we obtain the explicit functional dependence of g(m, r)dm on the average impact velocity, V. We now employ the relationship between V and r: we estimate V with the formula Eq. 3, i.e. we let

$$(17) \quad V \approx \sqrt{\langle V^2 \rangle} = V_o\, r^{-.559}$$

as given by Eq. 3 and hence obtain the explicit dependence of $g(m,r)dm$ on the distance from the sun, r.

The total number of fragments produced per second in a spherical volume with radius r and in a mass range m to m+dm is

$$(18)\quad h(m,r)dm = dm \int_{R_o}^{r} dR \int_{o}^{\pi} di \int_{o}^{2\pi} d\phi\; g(m,R)\; R^2\; f_i(i) \sin i\; di$$

where R_o is defined by Eq. 4.

The flux of particle fragments in the mass range dm, at a distance r from the sun and per unit area in the ecliptic (i = 0) is then

$$(19)\qquad g(m,r)dm = \frac{h(m,r)dm}{4\pi\; r^2} \qquad f_i(o)$$

where $f_i(i)$ is defined in Eq. 1.

The cumulative flux G of these particles having a mass m or greater is then readily obtained

$$(20)\qquad G(M,r) = \int_{m}^{\infty} g(M,r)dm$$

The result for the flux at 1 AU and = 5/3 is:

$$(21)\qquad G(m,1\ AU) = \frac{2 \times 10^{-18}\ Vo^2}{1.882 - 2b} \left[1 - \left(\frac{R_o}{AU}\right)^{1.882-2b}\right] m^{-2/3},\ M.K.S.$$

where we have expressed the cumulative flux of fragments per m^2sec having a mass of m kg or greater as a function of the parameters V_o, R_o and b. This expression is also an upper limit for the production of ß-meteoroids because this flux also includes the contribution to the flux by meteoroids in bound orbits.

In Table 1 we list the numerical value of G(m, 1AU) for $m = 10^{-15}$ kg and for a number of values of the parameters R_o, b and V_o. It can be seen, from Table 1, that the largest fluxes of ß-meteoroids are obtained when b is large and R_o small. This happens because for large b the concentration of particles near R_o is large compared with smaller values of b and furthermore, the closer we are to the sun, the faster are the collision speeds and hence, the more destructive are the collisions.

The expression for the flux of ß-meteoroids Eq. 21 can naturally be evaluated for a large number of other reasonable values of the various parameters. Table 1, however, provides an adequate indication of the

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Table 1. Values of the flux of fragments G(m,1 AU) for m = 10^{-15} kg in units of particles per (meter^2 sec 2π sterad)

R_o = .02

b \ V_o	10	20	40
0	5.5×10^{-7}	2.2×10^{-6}	8.7×10^{-6}
1.0	5.1×10^{-6}	2.0×10^{-5}	8.2×10^{-5}
2.0	1.9×10^{-3}	7.7×10^{-3}	3.1×10^{-1}

R_o = .04

b \ V_o	10	20	40
0	5.5×10^{-7}	2.2×10^{-6}	8.7×10^{-6}
1.0	4.0×10^{-6}	1.6×10^{-5}	6.4×10^{-5}
2.0	4.5×10^{-4}	1.7×10^{-3}	7.1×10^{-3}

R_o = .1

b \ V_o	10	20	40
0	5.4×10^{-7}	2.1×10^{-6}	8.6×10^{-6}
1.0	3.9×10^{-6}	1.5×10^{-5}	6.2×10^{-5}
2.0	6.3×10^{-5}	2.5×10^{-4}	1.0×10^{-3}

sensitivity of the flux to the values of the parameters used. We shall employ, in what follows, a value of $\eta = 5/3$ because it is within the range of values obtained experimentally and because it will provide a slightly better fit to the lunar data than the slightly higher value of $\eta = 1.8$.

VI. Distribution of Lunar Microcraters

We shall now attempt to express lunar microcrater frequency data as a function of the mass of the projectile objects and compare it with our results for the flux of β-meteoroids Eq. 21. This will then enable us to verify, in some detail, the suggestion of Fechtig et al. (1974) that lunar craters smaller than 1 micron are produced, mainly, by β-meteoroids.

We use the results of Fechtig et al. (1975) for the composite relative crater cumulation frequency, Fig. 1. Using Mandeville and Vedder's (1971) calibration, (also cf. Fechtig et al., 1974) we convert crater sizes to projectile masses using the relation

(22) $M = 1.88 \times 10^4 \; D^3/V^2$

where M is the mass of the projectile, in kg, D is the crater diameter in meters and V is the impact speed in km/sec.

Taking an impact speed of 20 km/sec in Eq. 22 we can compute the projectile masses that correspond to given crater diameters and we identify the point, (at a crater diameter of 173 microns) in Fig. 1, with the similar break in the meteoroid flux curve at 10^{-10} kg; (cf. Eq. 5) we can then compare the lunar microcrater data with satellite flux data and with the results of our calculation. The result is Fig. 2 where we have plotted the model incoming flux, the cumulative frequency of lunar microcrater producing particles and our calculated results for the flux of ß-meteoroids.

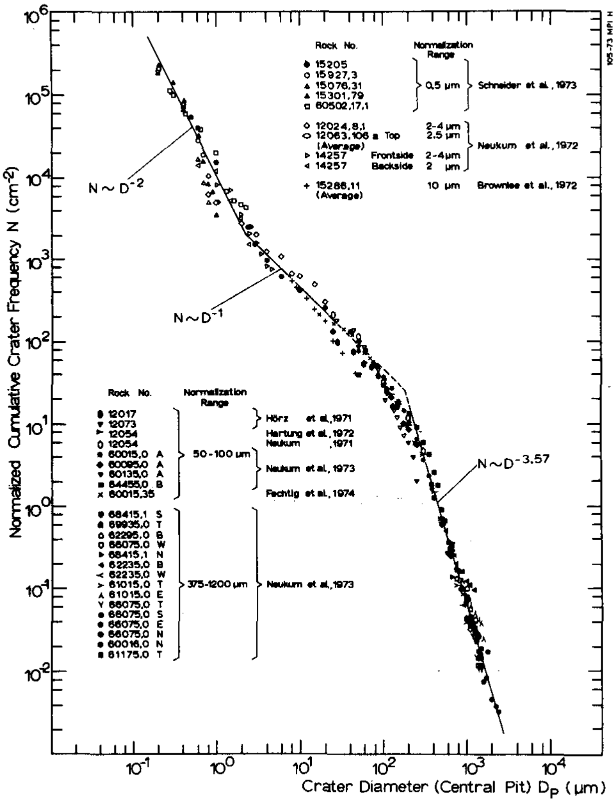

Fig. 1

Compilation of microcrater measurements from Fechtig et al. (1975). Frequencies are normalized; absolute frequency contains an arbitrary factor.

It can be seen from Fig. 2 that reasonable values for the parameters in Eq. 21 will provide estimates of ß-meteoroids in good agreement with satellite data and with the flux curve derived from lunar micro-craters. We have therefore provided independent theoretical support for Fechtig et al. (1974) suggestion.

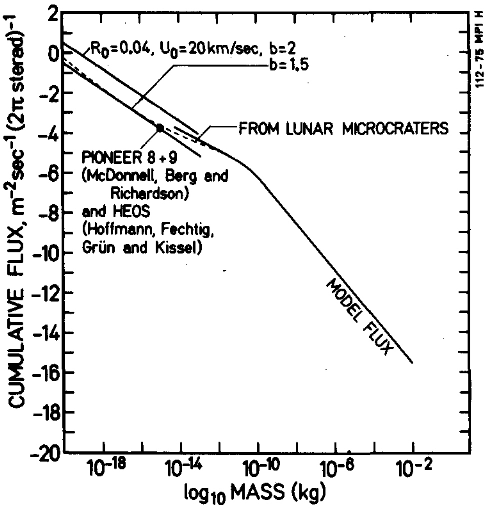

Fig. 2

Comparison of calculated and observed microparticle fluxes. Solid lines represent the calculated flux frequencies for the indicated values of the parameters R_o U_o and b (see text).

VII. Conclusion

The present study shows that using reasonable assumptions regarding the values of the physical parameters employed, disruptive collisions between meteoroids accompanied by fragmentation will produce a flux of β-meteoroids in agreement with satellite and lunar microcrater data. We have therefore provided independent theoretical support for Zook and Berg's (1975) hypothesis of β-meteoroids and to Fechtig et al. (1974) suggestion that lunar microcraters smaller than 1 micron are produced mostly by β-meteoroids.

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References:

Berg, O.E., and Grün, E. (1973), "Evidence of hyperbolic cosmic dust particles", Space Research XIII, pp. 1046-1055.

Brownlee, D.E., Hörz, F., Hartung, J.B., and Gault, D.E. (1972), "Micrometeoroid craters smaller than 100 microns", in: The Apollo 15 Lunar Samples, pp. 407-409. The Lunar Science Institute, Houston.

Dohnanyi, J.S. (1969), "Collisional model of asteroids and their debris", J. Geophys. Res. 74, 2531-2554.

Dohnanyi, J.S. (1970), "On the origin and distribution of meteoroids", J. Geophys. Res. 75, 3468-3493.

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Dohnanyi, J.S. (1972), "Interplanetary objects in review: statistics of their masses and dynamics", Icarus 17, 1-48.

Dohnanyi, J.S. (1973), "Current evolution of meteoroids", Proc. of the IAU Colloquium No. 13, NASA SP-319, pp. 363-374.

Eichhorn, G. (1976), "Impact light flash studies: Temperature, ejecta and vaporization", this Volume.

Fechtig, H. (1976), "In situ records of interplanetary dust particles - methods and results", this Volume.

Fechtig, H., Gentner, W., Hartung, J.B., Nagel, K., Neukum, G., Schneider, E., and Storzer, D. (1975), "Microcraters on lunar samples", Proc. of the Soviet-American Conference on Cosmochemistry, Pergamon Press (in press).

Fechtig, H., Hartung, J.B., Nagel, K., Neukum, G., and Storzer, D. (1974), "Microcrater Studies, Derived Meteoroid Fluxes and Comparison with Satellite-Borne Experiments", Lunar Science V, Abstract Vol. pp. 22-224 Proc. Fifth Lunar Sci. Conf. Geochim. Cosmochim. Acta Suppl. 5, Vol. 3, 4, pp. 2463-2474.

Gault, D.E., Shoemaker, E.M., and Moore, H.J. (1963), "Spray ejected from the lunar surface by meteoroid impact", NASA Rept. TND-1767.

Grün, E., Berg, O.E., and Dohnanyi, J.S. (1973), "Reliability of cosmic dust data from Pioneer 8 and 9", Space Research XIII, pp. 1057-1062.

Hartung, J.B., Hörz, F., and Gault, D.E. (1972), "Lunar microcraters and interplanetary dust", Proc. Third Lunar Sci. Conf., Geochim. Cosmochim. Acta, Suppl. 3, Vol. 3, pp. 2735-2753. MIT Press.

Harwit, M. (1963), "Origins of the zodiacal dust cloud", J. Geophys. Res. 68, 2171-2180.

Hörz, F., Hartung, J.B., and Gault, D.E. (1971), "Micrometeorite craters and lunar rock surfaces", J. Geophys. Res. 76, 5770-5798.

Hoffmann, H.J., Fechtig, H., Grün, E., and Kissel, J. (1975), "First results of the micrometeoroid experiment S-215 on the HEOS 2 satellite", Planet. Space Sci. 23, 215-224.

Mandeville, J.C., and Vedder, J.F. (1971), "Microcraters formed in glass by low density projectiles", Earth Planet. Sci. Letters 11, 297.

McCrosky, R.E., and Posen, A. (1961), "Optical elements of photographic meteors", Smithson. Contrib. Astrophys. 4, 15-84.

McDonnell, J.A.M., Berg, O.E., and Richardson, F.F. (1975), "Spatial and time variations of the interplanetary microparticle flux analysed from deep space probes Pioneers 8 and 9", Planet. Space Sci. 23, 205-214.

Neukum, G. (1971), "Untersuchungen über Einschlagskrater auf dem Mond", Ph.D. Thesis, Universität Heidelberg.

Neukum, G., Schneider, E., Mehl, A., Storzer, D., Wagner, G.A., Fechtig, H., and Bloch, M.R. (1972), "Lunar craters and exposure ages derived from crater statistics and solar flare tracks", Proc. Third Lunar Sci. Conf., Geochim. Cosmochim. Acta, Suppl. 3, Vol. 3, pp. 2793-2810. MIT Press.

Schneider, E., Storzer, D., Hartung, J.B., Fechtig, H., and Gentner, W. (1973), "Microcraters on Apollo 15 and 16 samples and corresponding Cosmic Dust Fluxes", Proc. Fourth Lunar Sci. Conf., Geochim. Cosmochim. Acta, Suppl. 4, Vol. 3, pp. 3277-3290.

Southworth, R.B., and Sekanina, Z. (1973), "Physical and dynamical studies of meteor", NASA CR-2316.

Whipple, F.L. (1976), "Sources of interplanetary dust", this Volume.

Zook, H.A. (1975), "Hyperbolic cosmic dust: its origin and its astrophysical significance", Planet. Space Sci. 23, 1391-1397.

Zook, H.A., and Berg, O.E. (1975), "A source for hyperbolic cosmic dust particles", Planet. Space Sci. 23, 183-203.