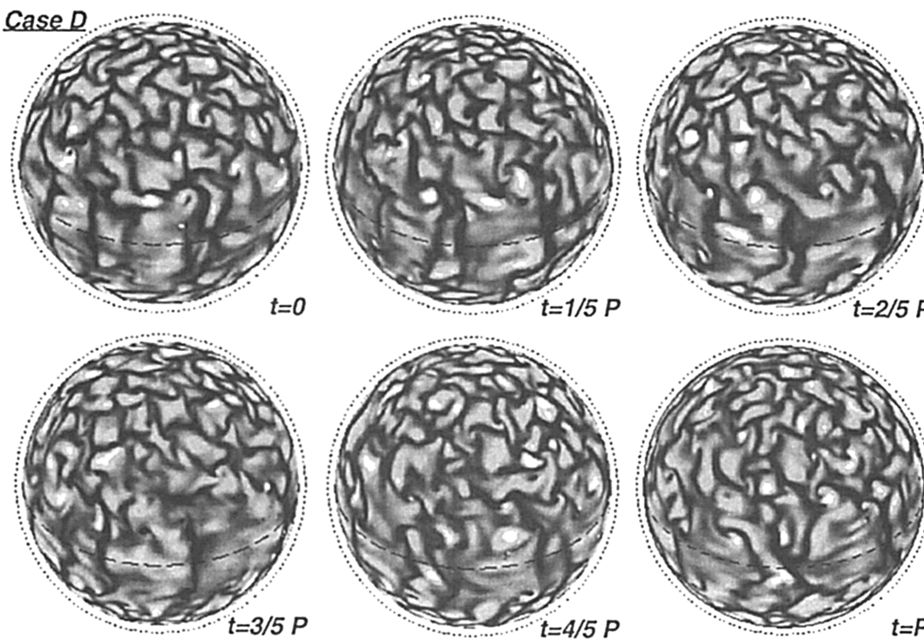
Session II

Convection Zone and Local Area Helioseismology



Evolution of the convection over one solar rotation of period P, showing the radial velocity in case D near the top of the domain as seen from a uniformly rotating frame. The time interval between each successive image is about 5 days (adapted from Toomre, p. 131).

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Turbulent Convection and Subtleties of Differential Rotation Within the Sun

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Abstract. The solar convection zone exhibits a differential rotation with radius and latitude that poses major theoretical challenges. Helioseismology has revealed that a smoothly varying pattern of decreasing angular velocity Ω with latitude long evident at the surface largely prints through much of the convection zone, encountering a region of strong shear called the tachocline at its base, below which the radiative interior is nearly in uniform solid body rotation. Helioseismic observations with MDI on SOHO and with GONG have also led to the detection of significant variations in Ω with 1.3 yr period in the vicinity of the tachocline. There is another shearing layer just below the solar surface, and that region exhibits propagating bands of zonal flow. Such rich dynamical behavior requires theoretical explanations, some of which are beginning to emerge from detailed 3–D simulations of turbulent convection in rotating spherical shells. We discuss some of the properties exhibited by such numerical models. Although these simulations are highly simplified representations of much of the complex physics occurring within the convection zone, they are providing a very promising path for understanding the solar differential rotation and its temporal variations.

1. Introduction

The sun possesses a deep shell of intensely turbulent convection just below its surface which is observed to have striking dynamical properties that continue to challenge basic theory. The most fundamental questions concern its rotation profile with latitude and depth (the 'solar differential rotation problem'), and the manner in which the sun achieves its 22-year cycles of magnetic activity (the 'solar dynamo problem'). These two issues are intimately linked, for the global dynamo action is likely very sensitive to the angular velocity Ω profiles realized within the sun. Both global issues touch on the remarkable property that turbulence can be both highly intermittent and chaotic on smaller spatial

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and temporal scales, and yet achieve a large-scale order that is sustained (cf. Brummell, Cattaneo & Toomre 1995). The solar convection zone possesses a wide range of scales, ranging from granules ($\sim 10^3$ km or 1 Mm in horizontal size), to supergranules ($\sim$30 Mm), to possible patterns of giant cells comparable to the overall depth of that zone ($\sim$200 Mm, or nearly 30% by radius). There are similar contrasts in temporal scales. Small-scale magnetic fields rekindle with the turnover of the granulation ($\sim$10 minutes), and yet the sun also possesses 22-year cycles of global-scale magnetic activity. That activity is quite orderly, involving sunspot eruptions with very well defined rules for field parity and emergence latitudes as the cycle evolves. The wide range of dynamical scales of turbulence in the solar convection zone presents severe challenges to both theory and simulation: these scales range from about 10^5 km (the overall depth of the zone) to 0.1 km or smaller (estimated dissipation scales), encompassing at least *six* orders of magnitude for each of the three physical dimensions. The largest current 3–D turbulence simulations can resolve about *three* orders of magnitude in each dimension. Yet despite the vast difference in the range of scales dynamically active in the sun and those accessible by simulations, the latter have begun to reveal basic *self-ordering dynamical processes yielding coherent structures* that appear to play a crucial role in the global differential rotation and magnetic dynamo activity realized in the sun. We shall discuss here some of our general findings based on 3–D simulations of turbulent convection in full spherical shells, contrasting those results with what has been deduced recently from helioseismology about the remarkable differential rotation achieved within the sun.

2. Solar Differential Rotation: Fast Equator, Slow Poles

The surface of the sun rotates differentially: it has long been known from tracking surface features that there is a smooth poleward decline in the angular velocity Ω, the rotation period being about 25 days in equatorial regions and about 33 days near the poles. More recently, the subject of helioseismology (e.g. Gough & Toomre 1991), which involves the study of the acoustic p-mode oscillations of the solar interior that are observable at the surface, has revealed how Ω varies with both radius and latitude throughout much of the sun. The differential rotation profiles deduced from helioseismology have turned out to be *revolutionary*, for they are unlike any anticipated by theory prior to such probing of the interior of a star. For instance, early numerical simulations of rotating convection in spherical shells (e.g. Gilman & Miller 1981, Glatzmaier 1987) yielded columnar convection cells (or 'banana cells') oriented in the north-south direction, the tilting of which produced Reynolds stresses that sustained the differential rotation. Such convection models suggested that Ω should be nearly *constant on cylinders*, aligned with the rotation axis and decreasing with depth in the equatorial plane. Such properties for Ω had also been embraced by early mean-field models for the solar dynamo, and thus the helioseismic findings forced major reconsideration.

Helioseismology has shown that the rotation profiles obtained by inversion of frequency splittings of the p modes (e.g. Libbrecht 1989, Tomczyk et al. 1995, Thompson et al. 1996; Schou et al. 1998; Howe et al. 2000b, Toomre et al. 2000)

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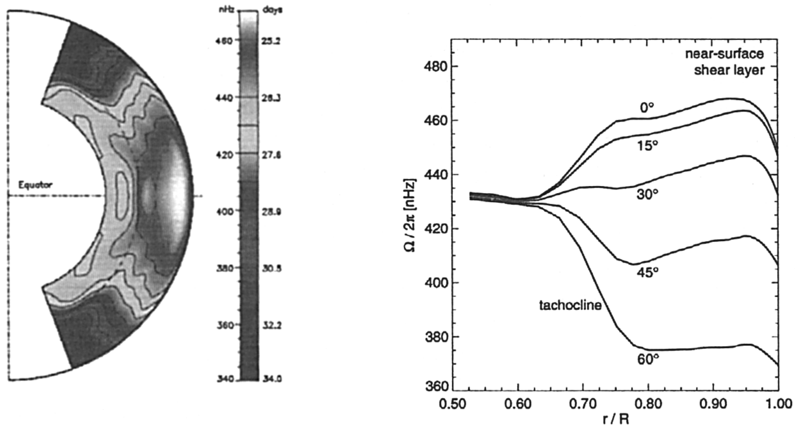

Figure 1. A (left). Solar angular velocity profile Ω with radius and latitude using GONG helioseismic data, adapted from Thompson et al. (1996). B (right). Time-averaged rotation rates from five years of GONG data, plotted against radius at different latitudes, adapted from Howe et al. (2000a). The variable shear just below the surface is clearly evident, as is the tachocline near the base of the convection zone, determined to be at radius 0.713 R.

have a decidedly different character than the Ω predicted by early models: the angular velocity variation observed near the surface, where the rotation is faster at the equator than near the poles, extends through much of the convection zone with little radial dependence. As shown in Figure 1, the angular velocity Ω at high latitudes in the convection zone appears to increase slowly with depth, at mid-latitudes it is *nearly constant on radial lines*, and near the equator it first increases and then gently decreases with depth. Another striking feature is the region of strong shear at the base of the convection zone, known as the *tachocline*, where Ω adjusts to apparent solid body rotation in the deeper radiative interior. Thus whereas the convection zone exhibits prominent differential rotation, the deeper radiative interior does not; these two regions are joined by the complex shear of the tachocline. There is further a thin *shear boundary layer near the surface* in which Ω increases with depth at intermediate and low latitudes. More detailed deductions about subsurface solar flows and rotation, and changes as the sun enters active phases of its magnetic cycle, are continuing to become available from the nearly continuous helioseismic observations with the Michelson Doppler Imager (MDI) experiment (Scherrer et al. 1995) on the SOHO spacecraft and from the ground-based six-station Global Oscillation Network Group (GONG) project (Harvey et al. 1996).

3. Tachocline of Intense Shear

We should emphasize that the presence of a tachocline of strong shear, separating the differential rotation of the convection zone from the solid body rotation

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of the deeper radiative interior, has been one of the most *surprising discoveries* of helioseismology. Such a tachocline had not been anticipated, and current theoretical approaches to explain its presence are still only innovative sketches: the boundary layer physics is likely to variously involve intermittent penetrative plumes of downflow, shear instabilities within a very stable stratification, possible angular momentum transport by internal gravity waves, magnetic shearing structures and MHD waves. In the absence of such processes, the differential rotation established by necessity within the convection zone by Coriolis effects must gradually imprint itself by diffusive means into the radiative interior. Although this would be a very slow process, the 5 billion years that the sun has existed provides ample time for differential rotation to be imprinted – yet we find that the radiative interior is instead rotating uniformly. We should note that it is not clear from the inversions of helioseismic data whether the tachocline of shear in Ω overlaps with the base of the convection zone or not. Helioseismic estimates place the midpoint of the tachocline at radius $0.692R_o$, with a thickness estimated to be of order 0.02 to $0.05R_o$ (e.g. Kosovichev 1996; Elliott & Gough 1999), compared to the placement of the base of the convection zone at a radius of $0.713R_o$ deduced from both MDI and GONG data. Thus the tachocline seems to be largely embedded in a region of very stable stratification; active helioseismic research is underway to better characterize its radial positioning and probable variation with latitude.

The nearly continuous helioseismic observations during the past five years with MDI on SOHO and with GONG have recently enabled the detection of prominent variations in the rotation rate near the base of this convective envelope, with a period of 1.3 yr evident at low latitudes (Howe et al. 2000a, 2001). Temporal changes in the angular velocity Ω are of order 6 nHz and occur out of phase above and below the tachocline; these represent substantial variations compared to the 30 nHz difference in Ω with radius across the tachocline at the equator (Figure 1*b*). These results are potentially very important since they are the first indications of detectable changes in rotation rate close to the presumed site of the global solar magnetic dynamo as the cycle is advancing.

Spiegel & Zahn (1992) gave the tachocline its name and argued that the inward diffusive spreading could be short-circuited by stably stratified shear turbulence leading to very effective mixing in the horizontal compared to that in the vertical. There would be a very slow meridional circulation to balance anisotropic diffusive stresses within their boundary layer; the solutions were designed to transmit no net torques to the deep interior. Gough & McIntyre (1998) took another tack, arguing that even a feeble remnant of a primordial magnetic field in the radiative interior would force it to rotate uniformly, given enough time, as had been also appreciated by Mestel & Weiss (1987). They then worked out properties of the magnetic skin layer that would be formed at the base of the tachocline of shear, with the primordial field excluded from diffusing upward by another weak meridional circulation. Gilman (2000) has argued that such slow dynamical processes within a tachocline (with overturning times of order one million years, comparable to the Kelvin-Helmholtz time there) may have trouble competing with *fast dynamical processes*, such as internal gravity waves triggered by combined shear and magnetic instabilities in that vicinity (e.g. Charbonneau, Dikpati & Gilman 1999). Even more vigorous are likely to be intermittent penetrative plumes of downflow near the base of the convection

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zone, aspects of which we have studied at high resolution with local f-plane modeling discussed by Brummell, Clune & Toomre (2001). Adding further complication is that the tachocline region is likely to be the site of the interface magnetic dynamo (e.g. Parker 1993; Charbonneau & MacGregor 1997). Such global dynamos are thought to operate by using the rotational shear in the tachocline to stretch magnetic fields that have been pumped downward from their generation within the convection zone itself (e.g. Tobias et al. 1998), thereby yielding strong toroidal fields that become unstable to magnetic buoyancy instabilities. Thus the tachocline is likely to be many different things to a selection of outstanding problems, and they cannot be readily separated.

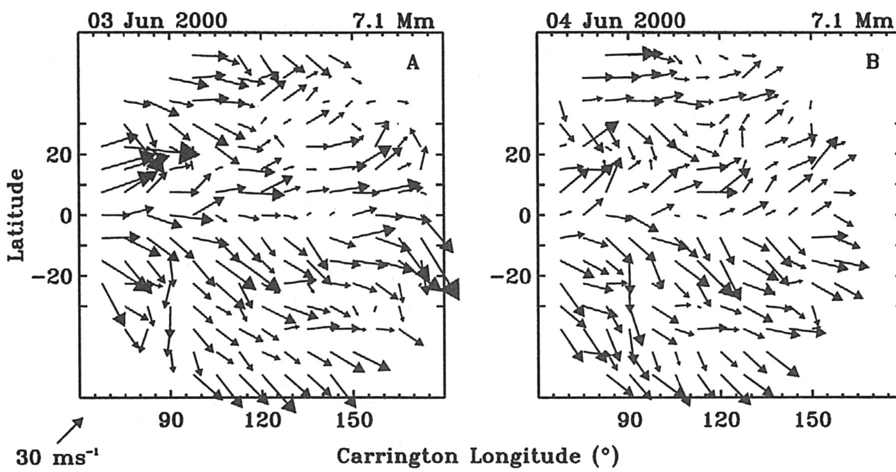

Figure 2. Dense-pack ring diagram sampling of horizontal flows averaged over 15° tiles at a depth of 7.1 Mm covering much of the solar disk on two successive days. Adapted from Haber et al. (2001b).

4. Complex Evolving Flows Near Top of Convection Zone

Helioseismic probing has also revealed systematic temporal changes in Ω with the advancing solar cycle in the upper portions of the convection zone. Inversion of the frequency splittings of global modes determined over successive two- or three-month intervals reveal propagating bands of weak zonal flow speedup (superposed on the strong differential rotation with latitude) that extend from the surface to at least a depth of 60 Mm (e.g. Schou 1999, Howe et al. 2000b), and complex speedups and slowdowns in the bulk of the convection zone (Toomre et al. 2000). Other approaches using properties of the acoustic wave fields in local domains, such as time-distance and ring-diagram analyses, have shown that the larger scale flows within the convection zone possess complex structures that can evolve over intervals of a few days. Figure 2 shows a mapping of the mean horizontal flows at a depth of 7.1 Mm below the surface from inversion of ring diagram analyses of MDI data carried out over a dense-pack of overlapping 15° tiles sampling much of the solar disk on two successive days (e.g. Haber et

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al. 2000, 2001a,b). There are noticeable and systematic changes from day to day, with the mean flows possessing amplitudes of order 30 ms^{-1} at that depth when viewed in a frame relative to the rotation rate of the surface layers at each latitude. The averaging of such data over a full Carrington rotation in two successive years is presented in Figure 3, revealing that the banding in the zonal flows at about 15° latitudes in the two hemispheres in 1999 has shifted closer to the equator in 2000. The associated meridional flow is poleward in both these years in the southern hemisphere, but has an evolving two-cell structure in the northern hemisphere; in earlier years the northern hemisphere also possessed only a single-cell circulation in its near-surface layers (Haber et al. 2001a). The changing pattern of meridional circulation cells with broken symmetries in the two hemispheres can only be seen using local helioseismic techniques, for the global modes provide symmetric averages about the equator. Such ring diagram analyses are providing a remarkable new perspective on large-scale meandering flows in the near-surface shear layer.

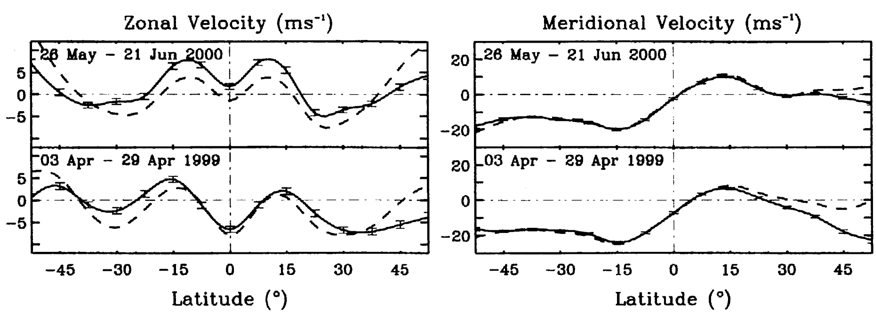

Figure 3. Mean zonal and meridional flows deduced from temporal averaging over a full solar rotation of ring-diagram dense pack maps as shown in Fig. 2. Shown are results of inversion at a depth of 2.0 Mm (dashed) and 7.1 Mm (solid). Adapted from Haber et al. (2001b).

5. Simulations of Rotating Compressible Convection in Spherical Shells with ASH

We have used our new anelastic spherical harmonic (ASH) code (Clune et al. 1999) to conduct major numerical simulations of highly turbulent compressible convection influenced by rotation within deep spherical shells, thereby studying the classes of differential rotation that can be established within a deep convection zone like that of the sun. The larger scales of convection are influenced most strongly by the Coriolis forces associated with rotation, and this leads to both nonlinear Reynolds stresses and mean circulations that redistribute the angular momentum and yield a significant differential rotation within the convection zone. We will describe the attributes of some of the richly time-dependent solutions that are beginning to make reasonable contact in the bulk of the convection zone with the angular velocity Ω profiles being deduced from helioseismology.

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The ASH code solves the 3–D anelastic equations of motion in a rotating spherical shell using a pseudo-spectral method, and is optimized to run efficiently on massively parallel computers. The mass flux and the magnetic field remain divergence-free by using a poloidal-toroidal representation for each. Their poloidal and toroidal components, along with the thermodynamic variables, are projected onto spherical harmonics $Y_\ell^m(\theta, \phi)$ to resolve their horizontal structures and onto Chebyshev polynomials $T_n(r)$ to resolve their radial structures. Such spectral expansions have the distinct advantage that the spatial resolution is uniform everywhere on a sphere when a complete set of spherical harmonics is used up to some maximum in degree ℓ (retaining all azimuthal orders $|m| \leq \ell$). The anelastic approximation is used to capture the effects of density stratification without having to resolve sound waves. Temporal discretization is accomplished using a semi-implicit Crank-Nicholson time-stepping scheme for linear terms and an explicit Adams-Bashforth scheme for nonlinear terms. Subgrid-scale eddy viscosities and conductivities are used to account for transport by unresolved turbulence. A stably stratified region is included below the model convection zone in some of our studies in order to simulate penetrative convection. Enhanced resolution near the base of the convection zone is provided by a stacked Chebyshev representation in the radial dimension, which consists of two separate expansion domains.

Figure 4. Simultaneous snapshots of radial velocity on spherical surfaces located near the top, middle and bottom of the deep shell of rotating turbulent convection being studied as Case D. Dark tones denote downflows, and light shades upflows; the dashed curves indicate the equator, and the dotted circles the solar radius.

The latest 3–D simulations with the ASH code are used to study differential rotation profiles established within full spherical shells of rotating turbulent convection, typically using spherical harmonic Y_ℓ^m expansions up to degree $\ell \sim 340$ (Elliott et al. 2000; Miesch et al. 2000; Brun & Toomre 2001a,b). These models are intended to be a faithful if highly simplified description of the solar convection zone. Solar values are taken for the heat flux, rotation rate, mass and radius, and a perfect gas is assumed since the upper boundary of the shell lies well below the H and He ionization zones; contact is made with a real solar structure model for the radial stratification being considered. The computational domain extends typically from about $0.72R_o$ to $0.96R_o$, where R_o is solar radius. In some of our studies the lower boundary is positioned deeper at $0.63R_o$, thereby

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including a region of stable stratification of thickness $\sim 0.07R_o$ below the primary unstable zone in which effects of penetrative convection can also be studied. The unstable shells have an overall density contrast in radius of about 30, and thus compressibility effects are substantial. We have softened the effects of the very steep entropy gradient close to the surface that would otherwise favor the driving of very small granular and mesogranular scales of convection, with these requiring a spatial resolution at least ten times greater than presently available. The flux of enthalpy by the unresolved eddies near the surface is explicitly taken into account with subgrid scale (SGS) terms, and enhanced eddy diffusivities are used in these large eddy simulations (LES).

6. Richly Time-Dependent Convection

The convection realized in our simulations is highly time dependent and complex. The tendency of convection to be organized into 'banana cells' aligned with the rotation axis, as realized in many laminar solutions, is progressively disrupted by increasing the level of turbulence. The convection is now characterized by strong downflows whose patterns become increasingly intricate. There is lessened north-south alignment in the equatorial regions, and intricate flow evolution is realized. Turning to one of our more turbulent simulations identified as Case D for which the Prandtl number $P_r = 1/4$ (Brun & Toomre 2001a), we can see in Figure 4 that many downflows extend over the full depth of the zone, as is evident in comparing the patterns in the three shell cuts. These downflows represent *coherent structures* that can be realized in turbulent motions with symmetries broken by rotation and stratification. The presence of these structures has a pivotal role in changing the character of the Reynolds stresses that serve to redistribute angular momentum and thus drive the differential rotation in such shells.

The convection is intrinsically rich in its temporal response. The downflow patterns evolve quite rapidly, and are also displaced and sheared by the associated differential rotation with latitude, as shown in the sequence of images in Figure 5 sampling a complete rotation period. Yet even as the downflow lanes are sheared and deformed, they are still recognizable on a time scale close to a half month when comparing sites such as near the equator in that solution. Frequent cyclonic swirling features are evident at the interstices of the downflow network in the turbulent cases, as was also realized in the high resolution studies of localized f-plane convection (Brummell et al. 1998).

These simulations of solar convection with ASH are now making serious contact with helioseismic findings about differential rotation within the deep interior of the sun. In particular, Figure 6 shows the time-averaged mean angular velocity Ω obtained from the Case D simulation. The Ω in this model is nearly constant on radial lines throughout much of the convection zone at mid latitudes, and there is a systematic decrease of rotation rate with latitude in going from the equator to the poles. We can understand many aspects of the resulting Ω profiles in terms of the modified mean Reynolds stresses established by the turbulent convection as contrasted to that of more laminar flows. Such differential rotation is accompanied by distinctive banded temperature variations with latitude near the top of the zone, with the poles consistently warmer by a few K than the

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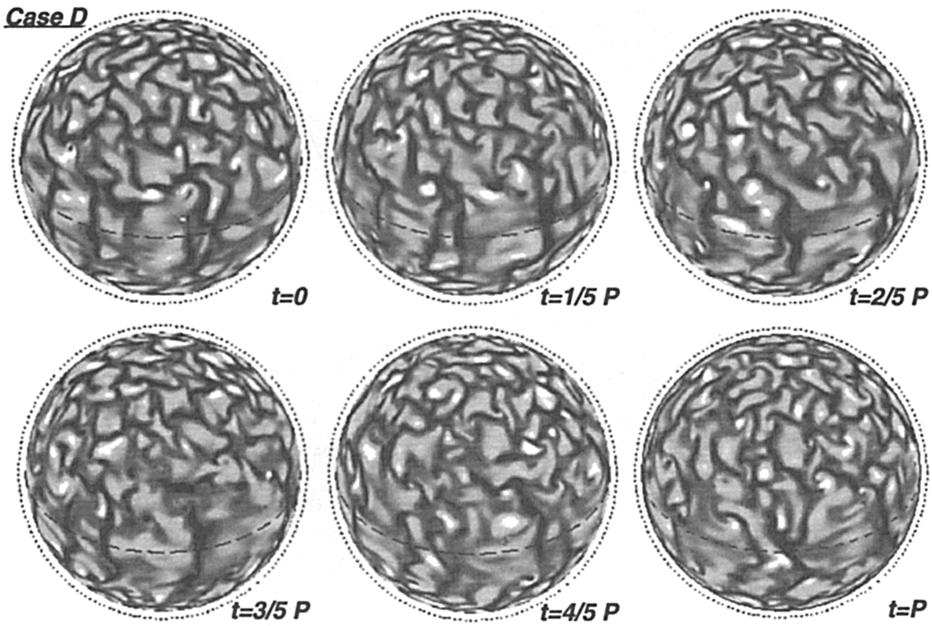

Figure 5. Evolution of the convection over one solar rotation of period P, showing the radial velocity in case D near the top of the domain as seen from a uniformly rotating frame. The time interval between each successive image is about 5 days.

equator. Further, the Ω profiles in Figure 7 determined over shorter intervals for Case C (Brun & Toomre 2001a) serve to emphasize that not only is the convection time dependent, but so is the differential rotation, readily showing variations of order 20 nHz in radial cuts at the equator and at 30° latitude. This makes contact with the temporal variations in Ω within the convection zone deduced from extended analysis of MDI data (Toomre et al. 2000), though the changes are more rapid in our simulations.

The Ω profiles with radius and latitude realized in our latest simulations are most encouraging, but there are still aspects that give us concern. The helioseismic observations suggest that Ω decreases rather more rapidly with latitude near the poles than in most of our simulations, which have much of the latitudinal variation occurring from the equator to about 60° latitude and more modest changes at higher latitudes. Helioseismology reveals a contrast of about 21% in rotation rate near the surface between equator and 60°, and a contrast of 29% in going to the higher latitude of 75° (Schou et al. 1998). It is comforting that we have achieved contrasts of about 30% in the simulation shown in Figure 6. However, we find that the angular velocity contrasts are rather sensitive to the typical Prandtl number P_r and Peclet number P_e achieved within a simulation. We can vary the intensity of the convective turbulence by modifying the eddy diffusivities and thus the Reynolds number R_e, along with P_r and P_e. Doing so we find that increasing the turbulence level beyond a certain point leads to a

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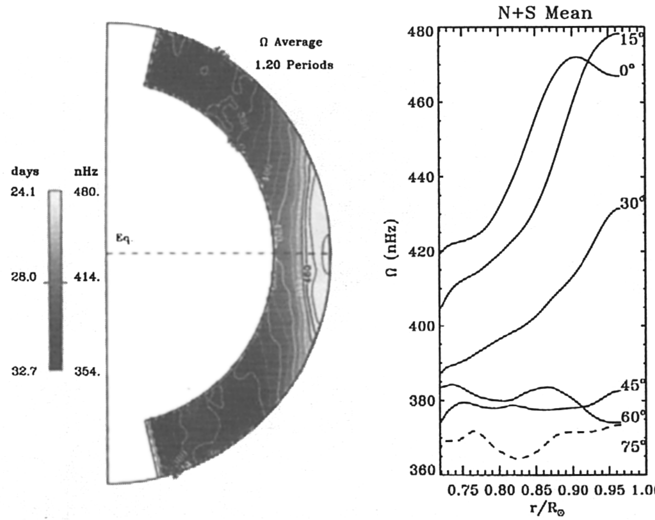

Figure 6. Time-averaged angular velocity profile obtained in case D, yielding Ω profiles that possess substantial variations with radius and latitude in the bulk of the convection zone being studied here.

gradual decrease in the Ω contrast between equator and pole. This is probably a consequence of the decorrelation of various velocity components leading to reduced Reynolds stresses that have to compete with overly strong viscous fluxes in redistributing angular momentum; reducing those viscous fluxes may require more complex functional forms for the SGS terms than we have utilized to date. Although our solutions have many attributes of turbulent flows, we have not yet experienced the full effects of coherent structures amidst the turbulence that we think should have a dominant role in angular momentum redistribution. Resolving such structures may require two- to four-fold higher spatial resolution in all dimensions than we have attained so far, and may also require dealing with explicit effects of scales such as supergranulation close to the surface.

7. Shearing Layers and Supergranulation

Another issue to be addressed concerns the shear boundary layers in Ω both near the surface and particularly in the tachocline at the base of the convection zone. Our solutions discussed in Miesch et al. (2000) admit a stable zone below the primary unstable convection shell, but such penetrative convection has not yet yielded a tachocline of shear close to that interface. This probably comes about because our simulations still operate at Reynolds numbers R_e where viscous coupling can imprint the local rotation rate of the unstable zone upon the stable region. The development of a tachocline is likely to involve effective anisotropic diffusivities and confining magnetic fields.

The shear layer of angular velocity speedup just below the surface at low to mid latitudes is not realized in our primary simulations, most likely a consequence of supergranulation effects which are deliberately suppressed by our choice of eddy viscosity functions near the surface in most of our runs. As

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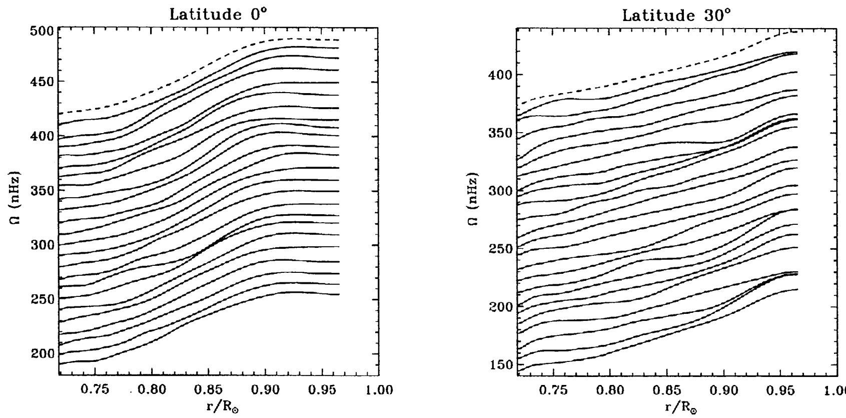

Figure 7. Stacked differential rotation radial cuts for Case C at the equator (left) and 30° (right). Each curve is a 7 day time average for Ω and is shifted down by 10 nHz. The dashed lines represent the cumulative average and are correctly positioned on the ordinate axis.

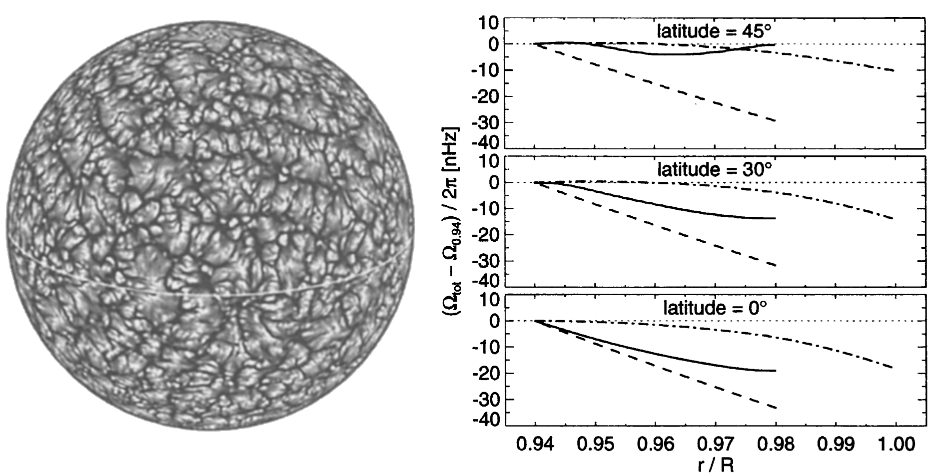

Figure 8. Snapshot of radial velocity in simulation with ASH of supergranular scales of convection in a shallow spherical shell, being used to study the formation of the near surface shear layer. The variation of angular velocity with radius at selected latitudes is shown on the right: solid lines denote mean angular velocity attained in simulation, dash-dot lines variation of Ω from GONG data, and dashed lines the angular velocity of a fluid parcel conserving angular momentum.

shown in Figure 8, we have begun preliminary studies of near-surface effects in which we admit supergranular scales of convection using a relatively shallow shell ($0.94R_o$ to $0.98R_o$). These supergranular scales with a convective overturn time of about a day compared to a rotation period of about 25 days do

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not in themselves sense the Coriolis forces very much. However, the motions toward and away from the rotation axis do seek to conserve angular momentum to some extent, as suggested by Gilman & Foukal (1979), and the predominance of faster downflows does serve to build Ω gradients with radius at lower latitudes with largely the same sense as deduced from helioseismic observations. Further, movie sequences show that the supergranular scales are advected by the more global scales of convection toward their downflow networks (DeRosa & Toomre 2001), forming the first realization of supergranular pattern advection that is now being searched for in actual solar data from MDI.

8. Reflections

Helioseismology as enabled using the nearly continuous MDI and GONG observational data has revealed that the solar convection zone possesses a great richness in the evolving large-scale horizontal flows that can now be probed with both global and local acoustic wave fields. This is providing further stimulus to theoretical efforts to understand how the solar differential rotation is established and what controls its variations during the course of the solar cycle. The rapidly advancing computational technology is now admitting simulations of turbulent convection in full spherical shells that are beginning to make interesting contact with some of the helioseismic findings. Yet major challenges remain since the range of dynamical scale involved in the solar convection zone is formidable. However, such theoretical work provides very promising routes to be pursued in order to interpret the remarkable helioseismic finding of systematic variations deep within the sun as the cycle advances.

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