Mathematical Structures in Computer Science (2021), **31**, pp. 361–362
doi:10.1017/S0960129521000451

CAMBRIDGE
UNIVERSITY PRESS

PREFACE

Preface to the MSCS Issue 31.1 (2021) Homotopy Type Theory and Univalent Foundations – Part II

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(Received 13 November 2021; revised 13 November 2021; accepted 13 November 2021)

This issue of *Mathematical Structures in Computer Science* is Part II of a Special Issue dedicated to the emerging field of *Homotopy Type Theory and Univalent Foundations*. Part I of the Special Issue was published as Volume 31, Issue 1 of Mathematical Structures in Computer Science. In the preface to that issue,[1] we give a brief overview of the history of the workshop series "Homotopy Type Theory and Univalent Foundations (HoTT/UF)" from which this Special Issue arose.

This issue comprises articles covering a range of topics in Homotopy Type Theory – from the formulation and formalization of mathematics within Univalent Foundations to the study of the meta-theory of type theory using category theory.

Modalities allow one to extend type theories by additional type and term constructions in a well-controlled way. Felix Cherubini and Egbert Rijke's *Modal descent* studies the factorization systems generated by a modality, focusing on the *modal reflective* factorization system defined in this work. In one of the main results of this work, the authors characterize the right maps of this factorization system via the *modal descent theorem*.

Nilpotency is an important property of spaces (or homotopy types) in classical homotopy theory. Luis Scoccola's *Nilpotent types and fracture squares in homotopy type theory* develops these notions synthetically in Homotopy Type Theory. Several important results about nilpotency are proved, including different characterizations of nilpotency. Scoccola also shows that cohomology isomorphisms between nilpotent types induce isomorphisms in all homotopy groups. Finally, he also proves a fracture theorem for a localization of truncated nilpotent types.

Simon Boulier and Nicolas Tabareau's *Model structure on the universe of all types in interval type theory* introduces a type theory with an interval type, that is, a form of *cubical* type theory. Building on the Orton–Pitts axioms for modeling cubical type theory in a topos, they then construct a model structure on the universe of – not necessarily fibrant – types of that type theory, using, crucially, an operation of "fibrant replacement" defined via a quotient-inductive type. Many of the results presented in this contribution are mechanically checked in the computer proof assistant `Coq`; the source files are available in a public `Git` repository.

In *Syntax and Models of Cartesian Cubical Type Theory*, Carlo Angiuli, Guillaume Brunerie, Thierry Coquand, Kuen-Bang Hou (Favonia), Robert Harper, and Daniel R. Licata define a cubical type theory based on Cartesian cubical sets. They also develop axioms, in the style of Orton and Pitts, which provide sufficient criteria for constructing a model of the type theory. This construction is computer checked using `Agda` as an internal language extended with these axioms. The obtained cubical set model requires less structure on the cube category than previous structural cubical set models. To make up for the lack of structure on the cube category, the notion of fibration had to be modified, and the proof that fibrancy is preserved by all type formers, in particular the universe, relies on the key step of adding the diagonal map of the interval as cofibration.

During the preparation of this special issue, three pillars of the community have passed away prematurely.

- In September 2017, Vladimir Voevodsky passed away from an aneurysm aged 51. Voevodsky founded the research field of Homotopy Type Theory and designed the Univalent Foundations of mathematics.
- In January 2018, Martin Hofmann passed away during a snowstorm aged 53. Hofmann's work with Thomas Streicher on the groupoid model of type theory paved the way toward Homotopy Type Theory. Furthermore, Hofmann contributed several analyses of type theory, in particular, of extensionality principles in intensional type theory.
- In November 2019, Erik Palmgren passed away due to illness, aged 56. Palmgren contributed to type theory in many different ways; we only mention a few. Firstly, he studied in depth the formalization of mathematics, in particular, of category theory, in intensional type theory. Furthermore, he analyzed the relation between type theory and constructive set theory, with an eye toward predicativity.

The work presented in this special issue would not have been possible without the groundbreaking achievements by Voevodsky, Hofmann, and Palmgren. We hope that their mathematical legacy will continue to inspire researchers to work on, and in, Homotopy Type Theory and Univalent Foundations.

Note

1 doi:10.1017/S0960129521000244.