PART I.

Questions of General Background and Methodology Relating to Aerodynamic Phenomena in Stellar Atmospheres.

Summary-Introduction

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1. – Introduction.

This paper is an introduction to the astronomical material underlying the Varenna Symposium on Aerodynamical Phenomena in Stellar Atmospheres. The term « aerodynamical phenomena » rather than simply « velocity fields » is used in the title of the symposium to imply that primary concern centers as much on the physical phenomena and consequences associated with the presence of velocity fields as it does simply on the velocity fields themselves. To fully appreciate this distinction between aerodynamical phenomena and velocity fields from the astronomer's viewpoint, one must consider it against the background of the *classical theory* (*) of stellar atmospheres, which assumes that all the properties of the atmosphere are strictly controlled by the radiation field. The thermodynamic state of the classical atmosphere is fixed by the three conditions of radiative equilibrium (no energy transport other than by radiation), hydrostatic equilibrium (no mechanical momentum transport), and local thermodynamic equilibrium at a temperature fixed by the local energy-density of the radiation field (complete coupling between radiation field and atomic degrees of freedom). Analyses of stellar spectra under the framework of this classical atmospheric model take account of the presence of velocity fields (other than thermal) only in their effect upon the atomic

(*) It is necessary to distinguish between what astronomers call *empirical* models of a stellar atmosphere and *theoretical* models, when discussing models of stellar atmospheres and the assumptions underlying their construction. An empirical model has as its basis an empirical determination of the distribution of T_e through the atmosphere. (There are, of course, assumptions underlying the particular empirical determination of T_e, which also may be questioned, as we discuss in the following.) The theoretical model is based wholly on a set of assumptions which suffice to determine the temperature structure. In the following, we hold literally to these definitions.

absorption coefficient, not in their energetic or momentum coupling to the thermodynamic state of the atmosphere. Thus, if we become interested in aerodynamic phenomena in stellar atmospheres, we must investigate the possible perturbation these velocity fields may have upon the thermodynamic state of the atmosphere. We develop a primary concern with differential motions, velocity gradients, and dissipation mechanisms — all quantities which may produce a local non-relative energy source — rather than directing our attention only at stellar rotation and uniform expansion of an atmosphere. Thus, what we call aerodynamic phenomena embraces not only velocity fields but also their influence upon the thermodynamic state of the atmosphere.

Granted such a primary concern with aerodynamic aspects, rather than simply with velocity fields as such, one must still recognize that the only *direct* observational approach to the existence of these aerodynamical aspects lies in empirical studies of velocity fields. The inadequacy of astrophysical exploitation of such empirical studies lies, for the most part, in failure to ask the physical consequences attending the existence of such observed velocity fields. The physical consequences are of two types, returning to the distinction between empirical and theoretical atmospheric models made earlier. First, on the side of empirical models, taking both velocity fields and thermal structure as known empirically, can we construct a self-consistent atmospheric model? Second, on the side of the theoretical models, taking the velocity fields as known empirically, what can we say about the modification of the assumptions, introduced to construct a theoretical model atmosphere and temperature distribution, over those assumptions used in the absence of the velocity fields? The whole class of mechanical energy dissipative problems, and their coupling with thermal structure, enters here.

These last considerations lead to the possibility of an *indirect* observational approach to the existence of aerodynamic phenomena, which would act to modify the thermodynamic state of the atmosphere over that predicted by the classical model, by looking for phenomena that would be anomalous under the classical model predictions.

The whole class of variable stars forms an immediate example, but there a primary source of information is observation of the velocity fields often associated with such variation. Magnetic and spectral variability suggest, but do not directly establish, the presence of aerodynamic (including hydromagnetic) effects. Evidence for mass-ejection, usually but not always based on associated velocity measures, is another example. Possibly the most interesting kind of indirect evidence — from the standpoint of current interest in high-energy aerodynamics — is that based upon spectroscopic evidence for the existence of effects of non-equilibrium thermodynamics in the stellar atmospheres, of the kind that would be associated with a non-radiative energy supply perturbing the radiative control upon which the classical model is based. The most detailed investigations of such evidence have thus far been carried out on the solar chromosphere and corona, and they are summarized in detail by THOMAS and ATHAY (1961). A number of the phenomena characterizing wide classes of stars have not been subjected to detailed analysis, but offer promise of providing the same kind of indirect evidence on aerodynamic phenomena. Examples are the presence of emission cores in the absorption lines of Ca^+; and the presence of emission lines themselves in certain cases.

In the following, we limit ourselves to a summary of the methodology by which direct astrophysical knowledge of velocity fields in stellar atmospheres has been obtained. We stress both the conceptual basis upon which the methodology in current use rests, and the conceptual problems which have either been set aside for simplicity or have been ignored. These latter may raise non-trivial question on the interpretation of the results of existing analyses. We neither present nor discuss particular observational results, nor their interpretation; these points are covered in the individual summary-introductory papers of the Varenna program. We do not include here a discussion of the kind of non-equilibrium-thermodynamic effects upon which *indirect* inference on aerodynamic phenomena rests. Our aim is simply to present for the aerodynamicist-physicist participants some background in the conceptual basis upon which rest the astronomical inference of velocity fields; and to raise for the astrophysicist a critical commentary upon these matters.

We distinguish four kinds of inferential procedures upon which astrophysical knowledge of velocity fields is based: 1) and 2) refer to the empirical approach defined earlier; 4) to the theoretical; and 3) to something intermediate.

1) *Direct observation of motion at some angle to the line of sight.* – Such observations are possible only in a few cases. One is in the case of solar phenomena observed on the solar limb, such as prominences. Another is the case of shells in novae and supernovae. The primary problem in interpretation lies in separating the motion of an excitation front from the actual motion of material. The spectroscopic aspects of this problem are similar to those in 2) below. Aside from this complication, however, there is nothing particularly novel to the astronomical, as contrasted to any other, situation. There is no particular question of general methodology; so we do not consider this procedure further.

2) *Interpretation of a spectrum to infer line-of-sight motion.* – Such interpretation requires a detailed theory of the formation of a spectral line in a gaseous atmosphere, with and without the presence of velocity fields. Two extreme situations are particularly easy to interpret — one, a line symmetrically broadened by random motions in an optically thin gas; the

other, displacement of the line as a whole due to uniform motion of the entire body of gas. In general, however, the observed line profile is a composite of random and non-random motions which both displace and broaden the line in a complicated manner. We concentrate our discussion in this paper upon this alternative 2), from which comes by far the most of the astrophysical information.

3) *Partly-empirical, partly-theoretical inference.* – One observes some phenomenon, and from it infers the existence of some velocity field. For example, one observes the differential rotation of a star (the sun), and infers from it as a necessary physical consequence the existence of vertical currents. The first suggestion of the existence of turbulent motions in stellar atmospheres came from such reasoning. ROSSELAND (1929) commented that stellar rotation exists; computed a Reynolds' number based upon the stellar radius as a characteristic length, and the observed rotational velocity, finding the Reynolds' number to be very large; then suggested that on the basis of laboratory experience such a Reynolds' number requires turbulent motion in the stellar atmosphere. Such a first-approximation procedure must be refined for a detailed quantitative treatment; we are not aware of such a more refined discussion; such must be carried out before much can be said about the properties of any turbulent motion to be expected to accompany stellar rotation.

A similar background for the introduction of an empirical « astronomical turbulence » must be mentioned. The concept arose in considerations of the state of the solar chromosphere, although similar arguments have been applied to the atmospheres of certain eclipsing stars. In the solar chromosphere, the change of intensity with height of many emission lines is observed to lie an order of magnitude lower than the isothermal density gradient corresponding to a temperature associated with the continuous distribution of energy in the optical spectrum. Under the classical stellar model, the temperature of the atmosphere decreases monotonically outward. Thus, these emission gradients were identified with atmospheric density gradients, and an « astronomical turbulence » postulated, which maintained this low density gradient without, however, coupling energetically to the thermodynamic state of the atmosphere (MCCREA, 1933). Since the velocity of such « turbulence » is highly superthermic, the suggestion is hardly tenable from a standpoint of physical consistency (CHANDRASEKHAR, 1934; THOMAS, 1948). The necessity for such a construction has now been removed, by applying an analysis of the observational material from the standpoint of non-equilibrium-thermodynamics. The emission gradients have been shown to differ from the density gradient, and the actual density gradient has been inferred and shown to satisfy hydrostatic equilibrium without introducing any « turbulence », astronomical or otherwise (cf., the summary of work on this point by THOMAS and ATHAY, 1961).

Similar analyses must be applied to other cases in which this concept of « astronomical turbulence » has been invoked to explain apparently anomalous density gradients, before such suggestions can be taken as serious evidence for the existence of this kind of « astronomical turbulence » in a stellar atmosphere.

4) *Wholly theoretical inference, for which there is apparently some indirect observational confirmation.* – The best example is the inferred existence of vertical convection in the lower solar photosphere. One computes the existence of a zone of radiative instability arising from the ionization of hydrogen and helium (UNSÖLD, 1930), (C. PECKER, 1953), (also cf. the recent summary by J-C. PECKER, 1959) thus infers the onset of convection. The *existence* of solar granulation *appears* to confirm the theoretical result. The problem is to obtain sufficiently detailed astronomical observations, and a sufficiently complete physical theory of such convection in the stellar-atmospheric environment, that the two may be compared in detail. The comparison involves not only velocity fields, but spectral, and angular (over the solar disk), distributions of radiation. We defer consideration of such phenomena to the detailed treatment in the Symposium proceedings Part IV-A.

Of these four kinds of inferential procedures, 2) has provided by far the greatest body of astrophysical results. Thus, we concentrate our discussion upon it.

2. – A quick look at the conventional astronomical approach to the analysis of spectral lines.

In discussing the analysis of a stellar spectral line, we will at times emphasize questions relating to a difference of only a few percent in the intensity at some point on the line-profile, insisting on the critical nature of this apparently-small difference for a determination of velocity fields. The velocity we have to determine is obtained by comparison of « *a* » theoretical line-profile and « *the* » observed one. Because the differences between almost any theoretical line-profile and the observed one are usually not gross, it is essentially the situation that a *first-order* approximation on the determination of the velocity requires a *second-order* approximation on the theory of the line-profile. Such a second-order approximation requires a knowledge of the chemical composition of the atmosphere, and of the distribution through the atmosphere of temperature and occupation numbers of energy levels (as given by thermodynamic equilibrium distribution functions or not). Very often, astrophysical

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work has been limited to the first-order approximation (constant temperature and root-mean-square value of velocity, thermodynamic equilibrium relations), which has given valuable information for the investigation of differences of chemical composition between different kinds of stars, and of stellar evolution.

Put this another way. We can try to distinguish the interest of three kinds of people, in these questions of velocity fields, at this symposium. One group are those astrophysicists whose primary concern lies in questions of stellar composition and evolution, and who wish to eliminate as far as possible a consideration of the details of physical processes entering line-formation except insofar as they make a large difference in results on composition. A second group, on the other extreme, are aerodynamicists who would like the fullest possible information on aerodynamic phenomena occurring in stellar atmospheres, as a possible extension of experience from terrestrial laboratories, and with not much concern for details of stellar evolution except insofar as they bear on aerodynamic questions. Finally, there is an intermediate group, astrophysicists-physicists, whose primary interest lies simply in the physical problems associated with the interaction between radiation and velocity fields, and thus interested in the fullest possible attention to details of line-formation. For the first category of interests, it often appears sufficient to deal with the total energy in the spectral line, ignoring details of the line-profile, particularly since they wish to deal with large numbers of stars, some quite faint, for which detailed spectra are unavailable. The other two categories require a detailed consideration of line-structure, particularly in the very central regions of the line, where velocity fields have their maximum influence upon the absorption coefficient. Clearly, our discussion here must aim primarily at the two latter categories, and to a large extent deal with problems of the kind of analysis which should be applied, even though only few of the necessary data may be presently available. However, we introduce the discussion by a glossary of standard astrophysical terminology, and an introductory first-order physical exposition, which essentially reflects the viewpoint of the first category above.

2·1. *Glossary of terminology.*

1) The spectral line represents a transition between energy levels, whose occupation numbers we denote by n_2 (upper level) and n_1 (lower). The spontaneous transition probability between levels is A_{21}. We use subscript «c» to denote continuum, «s» to denote selective process in the line, and «ν» to represent the combined line and continuum, which is the observed quantity. n_k denotes a concentration; N_k denotes an integral over some path length, thus number/cm².

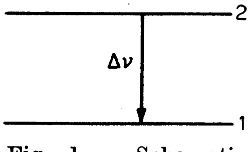

Fig. 1. – Schematic transition.

2) I_ν = specific intensity (erg cm^{-2} s^{-1} ν^{-1} solid angle^{-1}) at frequency ν.

I_c = specific intensity in the continuum immediately adjacent to the line.

$R_\nu = I_c - I_\nu$.

j_ν = emission coefficient per atom in upper level (erg s^{-1} ν^{-1}).

α_ν = absorption coefficient per atom in lower level (cm^{-2}).

φ_ν = profile of absorption coefficient such that $\int \varphi_\nu \, d\nu = 1$.

$\alpha'_\nu = \alpha_\nu$ including stimulated emission (similar definitions with the index λ).

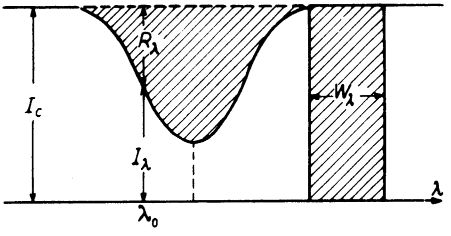

Fig. 2. – Notation for the spectral line (indices λ can be replaced by indices ν).

3) For a Doppler profile of absorption coefficient (only velocity broadening)

$$\alpha_\nu = \frac{\pi e^2}{mc} f_{12} F(V)\, dV\,, \qquad f_{12} = \text{oscillator strength}\,, \qquad \int F\, dV = 1\,,$$

$$\nu = \nu_0(1 + V/c)\,, \qquad \text{subscript 0 refers to line center}\,,$$

$F(V)$ is the local velocity distribution. If $F(V)$ is Gaussian — *i.e.* random velocity distribution:

$$\alpha_\nu = \frac{\pi e^2}{m_e c} f_{12} \frac{1}{\sqrt{\pi}} \frac{d\nu}{\Delta\nu_D} \exp\left[-(\Delta\nu/\Delta\nu_D)^2\right],$$

$$\Delta\nu_D = (\nu_0/c)(2\overline{V^2})^{\frac{1}{2}}; \text{ thermal motion—}\overline{V^2} = kT/m_a\,.$$

4) The time-independent equation of radiative transfer in an atmosphere where curvature effects may be neglected is

$$\mu \frac{dI_\nu}{d\tau_\nu} = I_\nu - S_\nu\,,$$

where τ_ν is called the optical depth, and defined by

$$\tau_\nu = \tau_s + \tau_c\,, \qquad d\tau_s = n_2 \alpha'_\nu\, dx;$$

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x is distance measured along stellar radius; $\mu = \cos\theta$ where θ is the angle between the outward radius and the direction of propagation of the beam; thus as observations on the stellar disk move from center to edge, the value of μ for the emergent beam of radiation varies from 1 to 0.

S is called the source-function and defined by

$$S_\nu = \frac{S_s + r_\nu S_c}{1 + r_\nu} = \frac{\eta_\nu S_s + S_c}{\eta_\nu + 1},$$

$$S_s = N_2 j_\nu / N_1 \alpha'_\nu; \qquad r_\nu = \eta_\nu^{-1} = \mathrm{d}\tau_c/\mathrm{d}\tau_s .$$

5) Equivalent width — measures « total absorption » in a line:

$$W_\lambda = \int \frac{(I_c - I_\lambda)\,\mathrm{d}\lambda}{I_c} = \int \frac{R_\lambda\,\mathrm{d}\lambda}{I_c},$$

$$R_\lambda = I_c - I_\lambda,$$

subscripts λ and ν on I and R denote quantities per unit wave-length or unit frequency, respectively.

6) $B_\nu(T_e)$ = Planck function for electron temperature, T_e.

7) ε = ratio of rates of collisional to radiative de-excitation evaluated at the local values of n_e and T_e under conditions of local thermodynamic equilibrium.

ηB^* = ratio of rates of radiative ionization from the lower level to spontaneous downward transition in the line.

η = ratio of rates of radiative ionization from upper level to spontaneous downward transition in the line.

8) $B(\Delta\lambda)$ = broadening function used by Huang and Struve due to macroscopic mass motion.

$R'(\lambda)$ = physical Doppler broadening function used by Huang and Struve.

$R(\lambda)$ = geometrical Doppler broadening function used by Huang and Struve.

9) Note that the following pairs of quantities should not be confused; they have nothing to do with each other:

$$\eta \text{ and } \eta_\nu; \qquad B_\nu(T_e),\ B^*, \text{ and } B(\Delta\lambda); \qquad R_\lambda \text{ and } R(\lambda).$$

We have simply followed notation used in the astronomical literature for ease of supplementing this article with references. We have changed one

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item of notation — S_s replacing S_L used in the literature for the source-function in the line.

2·2. *Total energy in the line — rough approach.* – In a rough, semi-quantitative way, we distinguish the manner in which three major effects enter to determine the total energy carried by the line. In Fig. 3, we plot

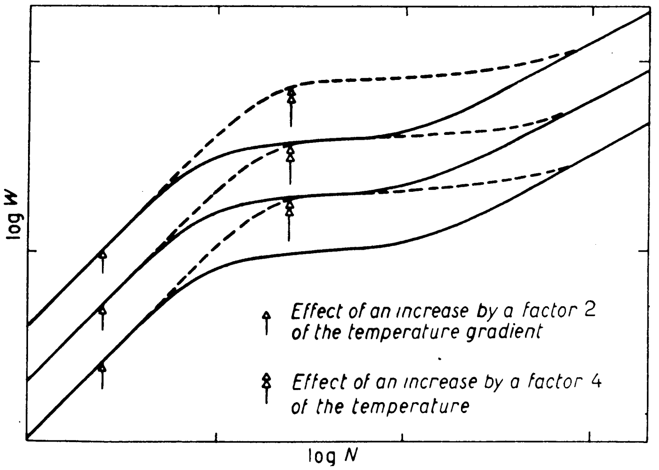

Fig. 3. – Influence of T, $\mathrm{d}T/\mathrm{d}h$ on total absorption using LTE relations.

the behavior of the total integrated absorption (ordinate) as a function of the abundance of atoms in the ground level (abscissa), with differing curves corresponding to differing values of the temperature gradient in the atmosphere, and to differing kinetic temperature (or other very small-scale random motion). We see from the figure that an increase in $\mathrm{d}T/\mathrm{d}h$ by a factor 2 increases the equivalent width, W, by the same factor; and that this same effect requires an increase by a factor 4 in the kinetic temperature. We make these computations on the basis of the assumed applicability of thermodinamic equilibrium relations.

Generally speaking, measurements of the distribution of energy in the continuous spectrum have given relatively fair knowledge of the temperature of the atmosphere, using classical equilibrium thermodynamics. These temperatures have been used to compute the theoretical intensity of the line, and the differences between this theory and observations have been laid to velocity fields without questioning the underlying hypotheses. (We will be primarily concerned in Section **3** with the validity of these hypotheses.)

2·3. *Profile of a line — rough approach.* – Consider the first integral of the equation of transfer, in a semi-infinite atmosphere, under the assumption

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that S_ν does not increase inward as fast as $\exp[\tau_\nu]$. This integral gives the emergent specific intensity at some position, μ, on the stellar disk as

$$(1) \qquad I_\nu(\tau_\nu = 0, \mu) = \int_0^\infty S_\nu \exp[-\tau_\nu/\mu]\, d\tau_\nu/\mu \,.$$

(The restriction on the inward rate of increase of S_ν is not a serious one, but we do not discuss its justification here.) Consider now the situation where local thermodynamic equilibrium, LTE, is satisfied so that

$$(2) \qquad S_\nu = B_\nu(T_e) \,,$$

Then, because of the exponential term in eq. (1), we have, roughly

$$(3) \qquad I_\nu(0, \mu) \sim S_\nu(\tau_\nu \sim \mu) \sim B_\nu(T_e[\tau_\nu \sim \mu]) \,.$$

Using this rough relation, we see the structure underlying the analysis of the total absorption in the line, summarized in Section **2**; and also see how the more extensive analysis permitted by the line-profile gives us more detailed information about the atmosphere.

First, the relation (3) demonstrates that the form of the line-profile simply reflects $dT_e/d\tau$—for the line-profile simply corresponds to looking to different depths in the atmosphere in the line and the continuum, deeper in the continuum than in the line. Thus, T_e decreasing outward gives an absorption line; increasing outward, an emission line (cf. Fig. 4). A line with a greater ratio τ_s/τ_c gives a more pronounced absorption, a *deeper* line, greater R_ν (for $dT_e/d\tau_\nu > 0$). Thus, we see the effect of both model (change in value

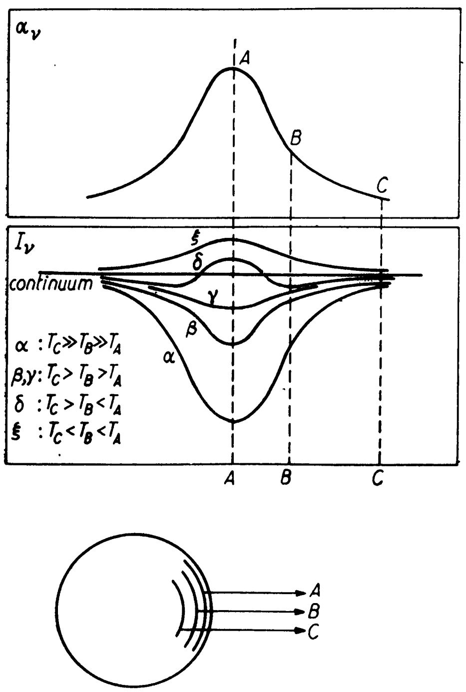

Fig. 4. – The influence of the temperature gradient on the profile of a line (schematic).

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of $dT_e/d\tau$) and of abundance (value of τ_s/τ_c), as already illustrated in Section **2** for total absorption in the line.

Second, the relation (3) gives a method for obtaining an empirical model, $T_e(\tau)$, for the atmosphere. It gives the mapping of $T_e(\tau_\nu)$ from $I_\nu(\mu)$ for every point, or value of ν, on the line. Each point covers a range in τ_ν equal to the range in μ, or about a factor 10, since astronomical observations generally cover the range $1 \geqslant \mu > 0.1$. Observations of the continuum let us map the deeper layers; of the lines, the higher layers. Thus, we obtain a series of line-segments, each giving $T_e(\tau)$ over a limited portion of the atmosphere. If we know, a priori, $d\tau_c/d\tau_s$, or $d\tau_{\nu 1}/d\tau_{\nu 2}$, we can directly relate the segments determined from observations at the several ν_1. Such an *a priori* knowledge requires two kinds of information: knowledge of the ν-dependence of the absorption coefficient; knowledge of the relative concentrations in the lower levels of the transitions considered. The relative concentrations depend upon both abundance of atom considered, and distribution over excited energy states; in the LTE case, the latter is specified by the LTE distribution functions, leaving only abundance as a free parameter. Thus, for the sun, the only star whose disk we can resolve, we can use line-profiles to get abundance by forcing agreement of the $T_e(\tau_\nu)$ segments determined from different ions.

Third, because we can use both $I_\nu(\mu)$, and $I(\nu)$ for a fixed μ, to infer T_e, at different depths, we have an empirical method of investigating the ν-dependence of τ_ν, and thus the ν-dependence of α_ν. In the central regions of the line, α_ν has a frequency-dependence fixed by the velocity field; thus, an empirical result for the ν-dependence of α_ν gives an empirical measure of the velocity field.

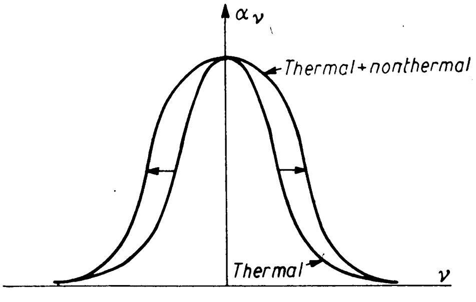

Fig. 5. – Effect of a microscopic velocity field on the absorption coefficient.

In Section **3**, we comment in more detail on each of these points, together with their validity, which rests upon equation (2). Here, however, we have tried only to give a rough picture of the usual general approach to the astrophysical analysis of a spectral line.

3. – Critical look at the general astronomical methodology.

In the structure of this discussion, we want to do two things. On the one hand, we want to make clear the difference between the factors influencing the shape of the line-profile in the usual astrophysical situation, where one generally studies an optically-thick gas, and those factors in the usual laboratory situation, where one generally studies an optically-thin gas. On the other hand, we want to make clear the physics underlying the analytical procedure for the interpretation of line-profiles *as it has thus far evolved* in astrophysics. In such a way, we hope to make clear the basis upon which astrophysical knowledge of velocity fields rests, and also the problems faced in applying the analytical techniques developed in astrophysics to the situations encountered in high-energy aerodynamics and laboratory plasma physics.

We proceed to this analysis in four stages: *A*) a comparison of the laboratory and astrophysical situations with respect to line-profile formation; *B*) analysis of a line-profile formed in an atmosphere where the velocity fields are wholly thermal; *C*) analysis of a spectral line formed in an atmosphere where the velocity fields existing are those in which the *variations* of the velocity, over any scale larger than some dimension much smaller than one photon free path, are completely uncorrelated; *D*) analysis of a spectral line formed in an atmosphere where the velocity fields are of any other type. Because we concentrate on the methodology, deferring results to the several presentations at Varenna, the reader may wish to consult more specific references. We give these in the text on detailed points. As more general summary-type references, we suggest the articles by K. O. WRIGHT (1955), C. DE JAGER (1959), S-S. HUANG and O. STRUVE (1960). The first summarizes detailed numerical results on random velocities inferred from total line-absorption; the second, summarizes solar material; the last gives a methodological critique and bibliography on empirical results on « stellar turbulence ».

3'1. *Astrophysical vs. laboratory situations.* – Consider a small sample of radiating gas in the laboratory, whose optical thickness is negligible, and within which those quantities fixing the frequency-profile of the emission coefficient are independent of position. Then it will produce an emission line, whose specific intensity will be

$$I_\nu = N_2 j_\nu A_{21}/4\pi \,. \tag{4}$$

That is, the frequency-profile of the line will be given by the frequency-profile of the emission coefficient, j_ν. If we know from theoretical considerations the dependence of j_ν upon velocity field, then the profile gives us directly the velocity. Indeed, if we consider a thin atmosphere, the profile of j_ν is just the Doppler profile, so the first-order dependence of the line-profile is upon the velocity field. Thus, the analysis of an observed profile for velocity of emitting atoms is reasonably straightforward.

Precisely the same situation holds, if we consider an absorption line formed by passing a light beam through a thin layer of gas. In this case, the absorption coefficient α_ν replaces the emission coefficient j_ν, and its profile is fixed by the velocity field. The observed spectral line-profile is then the profile of α_ν.

Consider, by contrast, the astronomical situation, where except in unusual circumstances the expression for the line-profile comes from the first-integral of the equation of radiative transfer, in an atmosphere where curvature effects can be neglected and where the optical opacity along the line of sight is large; viz.,

$$(5) \qquad I_\nu(0, \mu) = \int_0^{\tau_{\nu\,(\max)}} S_\nu \exp\left[-\tau_\nu/\mu\right] \mathrm{d}\tau_\nu/\mu \,,$$

assuming either no incident ratiation at $\tau_{\nu\,(\max)}$ or, in the most usual case of direct observations of the stellar disk, where $\tau_{\nu\,(\max)} = \infty$, that S_ν does not increase inward as fast as $\exp[\tau_\nu]$, and eq. (5) becomes eq. (1). There are exceptional cases in astrophysics where the situation reduces to the laboratory case of the thin atmosphere, in which case the integral of eq. (5) becomes

$$(6) \qquad I_\nu = \int_0^{\tau_\nu} S_\nu \exp\left[-\tau_\nu/\mu\right] \mathrm{d}\tau_\nu/\mu \xrightarrow[\tau\to\text{small}]{} \int_0^{\tau_\nu} S_\nu \, \mathrm{d}\tau_\nu = A_{21} \int_0^{y\,(\max)} j_\nu n_2 \, \mathrm{d}y/4\pi \,,$$

which is eq. (4) in an atmosphere where j_ν and n_2 vary with position. Examples are gaseous nebulae, atmospheres viewed tangentially as at a solar eclipse, etc. We consider here, however, the most usual case where $\tau \lll 1$.

Then, we see the essential character of the problem that plagues the working astronomer—the line-profile represents an integration over the optical depth variation of the source-function, S_ν, and S_ν is in general not constant even if the quantities fixing the frequency profiles of absorption and emission coefficients are constant throughout the atmosphere. That is, the first-order dependence of the observed line-profile is not upon $\alpha(\nu)$—or upon $j(\nu)$—but upon $S_\nu(\tau, \nu)$. Thus, there are two kinds of dependence of the line-profile upon thermodynamic quantities in the atmosphere: (i) the dependence of profile upon *gradient* of S_ν in the atmosphere through the two relations $S(\tau)$ and $\tau(\nu)$—the last being equivalent to $\alpha(\nu)$ and (ii) the dependence of profile upon any ν-dependence of S (at a particular point in the atmosphere). If we could invert the integral in eq. (5) to obtain directly $S(\tau)$ and $\alpha(\nu)$, knowledge of $\alpha(\nu)$ would give us directly the total velocity field, precisely as in the laboratory case corresponding to eq. (4) and the simplified astrophysical case corresponding to eq. (6). Then, in both laboratory and astrophysical cases, we must separate thermal from non-thermal components. The possibility of inferring T_e in the laboratory case lies in a discussion of N_2; in the astrophysical

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case, of S_ν. Thus, in the astrophysical case, the problem is to analyse the line-profile in such a way as to invert eq. (3) to obtain both $S_\nu(\tau_\nu)$ and $\alpha(\nu)$. $\alpha(\nu)$ gives total velocity field, and we must ask the relation between the value of S_ν and the value of T_e.

A precise parallel to the laboratory case would arise if we were able to by-pass the determination of $S(\tau)$ and determine directly $\alpha(\nu)$. We first summarize an approach to such a procedure, which has been used in astrophysics but is actually only valid under certain highly restrictive assumptions, then consider the more general case where $\alpha(\nu)$ cannot be determined without prior or simultaneous determination of $S(\tau)$.

3·1.1. Direct determination of $\alpha(\nu)$. The following method has been used by several authors (cf. DE JAGER, 1952; ATHAY and THOMAS, 1957, 1958, who critically examined its use; GOLDBERG, 1958; UNNO, 1959). One observes several lines originating on a common lower level, then proceeds to analyse the observations under the two assumptions that S_ν is ν-independent over a given line, and is the same for all the lines considered. Observations of the line-profiles are compared at the same point on the solar disk. Find two points of equal intensity, one on each line. We have $I_{\nu_i} = I'_{\nu_j}$ (distinguishing the lines simply by primed and unprimed notation cf. Fig. 6). Since S_ν is the same for both lines, consider its functional form expressed in terms of τ_{ν_i}, say. Further define the ratios, at a given geometric point in the atmosphere:

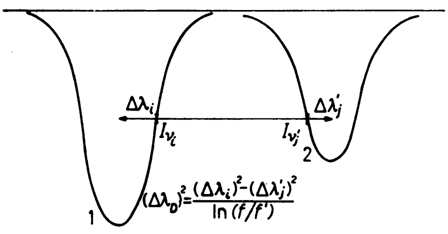

Fig. 6. – Intercomparison of two spectral lines to obtain $\Delta\lambda_D$, assuming a common ν-independent S_ν; Gaussian distribution of velocities.

$$(7) \qquad \mathrm{d}_{\nu_i}\tau/\mathrm{d}\tau'_{\nu_j} = \alpha_{\nu_i}/\alpha'_{\nu_j} = \delta; \qquad \tau_{\nu_i}/\tau'_{\nu_j} = \Delta\,,$$

Δ and δ are generally functions of position, of course. Then eq. (1) gives

$$(8) \qquad 0 = I_{\nu_i} - I'_{\nu_j} = \int_0^\infty \{S_\nu(\tau_{\nu_i}) \exp[-\tau_{\nu_i}]\,\mathrm{d}\tau_{\nu_i} - S_\nu(\tau'_{\nu_j}\Delta) \exp[-\tau'_{\nu_j}]\,\mathrm{d}\tau'_{\nu_j}\} =$$

$$= \int_0^\infty S_\nu(\tau_{\nu_i}) \exp[-\tau_{\nu_i}]\{1 - \delta^{-1} \exp[\tau_{\nu_i}(1 - \Delta^{-1})]\}\,\mathrm{d}\tau_{\nu_i}\,.$$

One solution corresponds to $\Delta = \delta = 1$, thus fixes empirically the ν-dependence of α. This requires constancy through the atmosphere of α_ν—which

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requires constancy of $\Delta\nu_D$ or total velocity. This solution is not the only one, but it is the only one that has been used thus far in astrophysical practice, as some kind of first approximation. Other solutions correspond to variable velocity fields, and can be obtained only iteratively.

This procedure depends upon the assumption that S_ν is both ν-dependent, and the same for several lines. The assumption of LTE, eq. (2), satisfies this criterion, and is sometimes adopted (cf. NEVEN and DE JAGER, 1954). Other authors have not insisted upon the applicability of LTE, but have taken it as obvious that lines originating on a common lower level, and having not too great an energetic separation of upper levels, will satisfy the condition of a common S_ν (cf. GOLDBERG, MOHLER, MUELLER, 1959; UNNO, 1959). We remark only that the validity of these assumptions remains to be investigated in each case. We turn in Sect. 3·2 to the question of what form S_ν should have in reality.

3·1.2. Joint determination of $S(\tau)$ and $\alpha(\nu)$. In the general case, we ask first whether we may not avoid the uncertainty of assumption on relation between S_ν for several lines by working at several points on one line, looking at the variation of line-profile across the disk to obtain $S_\nu(\tau_\nu)$. The major uncertainty accompanying such a procedure arises in the possibility of departure of the atmosphere from spherical symmetry. We set this question aside, for the moment, and proceed under the assumption of spherical symmetry.

The difficulty in inverting eq. (5) to obtain $S_\nu(\tau_\nu)$ from an observed $I_\nu(\mu)$ is often so great as to divert the astrophysical analysis from the recognition that any physical interpretation of the results can *only* be made on the basis of a set of *local* values—of S_ν, of absorption coefficient, of occupation numbers—values referring to a particular point in the atmosphere. In the case of the solar atmosphere, and modern equipment, one has reasonably-good success in actually observing $I_\nu(\mu)$ with good resolution in both ν and μ, then using $I_\nu(\mu)$ to obtain $S_\nu(\tau_\nu)$. In the stellar case, however, one always treats observations referring to the stellar disk as a whole—integration over μ—and often results must be based only on total absorption in the line—integration over ν—because of the poor spectral resolution associated with a faint object. In such cases, passage from the observed integrated quantities to *local* quantities is quite difficult (physical interpretation of numerical values of *local* quantities can only come later).

To interpret local values of S_ν, one asks two questions: (i) how much does the numerical value of S_ν depend upon the local value of T_e, and how much directly upon the velocity field present; (ii) how can we use the empirical set of $S_\nu(\tau_\nu)$ to infer the ν-dependence of α_ν, for this last fixes the velocity field just as in the laboratory case. We consider first the second question, then consider the first question in Section 3·2.

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We ask now what procedure we may follow to interpret the set of empirical values, $S_\nu(\tau_\nu)$, to find thermal and non-thermal components of any atmospheric velocity fields. The procedure to be adopted depends very much upon the way S_ν depends upon the thermodynamic parameters characterizing the state of the atmosphere. Since in this subsection we are trying to compare astrophysical and laboratory situations, we introduce a second idealization, the very specialized form of S_ν holding under conditions of LTE; viz., S_ν given by eq. (2). From this specialized example, we try to make clear what characteristics of S_ν enter which part of the analysis, then in Sections **3**·2-**3**·4 we may ask into the form expected for S_ν from physical considerations, thus the relation of S_ν to T_e and velocity fields. It should be noted that the results of this specialized form are not only of interest for illustrative purposes. We have already remarked that astronomical analyses have often been so highly preoccupied with the difficult problem of inverting the integration problem to obtain $S_\nu(\tau_\nu)$, that they make oversimplified assumption on the interpretation of S_ν. This assumption usually takes the LTE form of eq. (2). Consider, then, how we obtain thermal and non-thermal velocity fields under this assumption.

Again to eliminate consideration of details of inverting eq. (5) at this point, and to permit us to focus attention on the essential characteristics of the joint analysis for $S(\tau)$ and $\alpha(\nu)$, we introduce another specialized assumption, that permits us to pass directly from a numerical value of $I_\nu(\mu)$ to a numerical value of $S(\tau)$, and links our systematic discussion to the rough analysis of Section **2**·3. We assume

$$S_\nu(\tau_\nu) = a_\nu + b_\nu \tau_\nu \, . \tag{9}$$

Then for $\tau_{\nu(\max)} \gg 1$, eq. (3) leads immediately to

$$I_\nu(\mu) = a_\nu + b_\nu \mu = S_\nu(\tau_\nu = \mu) \, . \tag{10}$$

In such a case, as noted in Section **2**·3, an observed set of values $I_\nu(\mu)$ suffices to map out $S_\nu(\tau_\nu)$ over the range $1 > \tau_\nu > \mu\,(\min)$. There is no a priori reason to expect such linearity for $S_\nu(\tau_\nu)$, but over a limited range in τ_ν it is often nearly true. As mentioned, we use the assumption here simply for illustrative purposes, then comment on the problems its use introduces.

Given $S_\nu = B_{\nu_0}(T_e)$, eq. (10) permits a mapping of the distribution of T_e in the atmosphere as a series of segments, $T_e(\tau_\nu)$, one segment for each point on the line-profile, over the permitted range in τ_ν. *If* one knew the relation between τ_{ν_i} and τ_{ν_j} (*i.e.* the ν-dependence of α_ν), where ν_i and ν_j are two points on the line-profile, the several segments could be combined to give $T_e(\tau_{\nu_0})$, say, over a large range in τ_{ν_0}. Thus, one would map out the *thermal* velocity

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field over the atmosphere, provided the assumptions of eq. (2) and (9) are satisfied. ATHAY and THOMAS (1955) have discussed some of the difficulties of such an analysis in the case of the Balmer lines of hydrogen; CURY, LEFEVRE and PECKER are currently carrying out a somewhat more extensive investigation of the problem for Ti. (*Ed. note*: cf. remarks by PECKER, Part I, Discussion).

A knowledge of the relation between τ_{ν_i} and τ_{ν_j} must come either from a theoretical calculation, or from some empirical procedure. Such a theoretical calculation depends upon an *a priori* knowledge of the general atmospheric velocity field, in order to compute the ν-variation of α_ν. Since we do not have this *a priori* knowledge, one requires an empirical relation between τ_{ν_i} and τ_{ν_j}, which is, of course, equivalent to an empirical investigation of the ν-variation of α_ν, this in turn being the basis for establishing the characteristics of any existing velocity field. Thus, by establishing the ν-variation of α_ν, we are able to join the segments to produce an overall $T_e(\tau)$, thus extending our knowledge of the thermal velocity field over that part of the atmosphere contributing to the observed line, and also measure the total velocity-field in the same atmospheric region.

Again, the LTE assumption permits an inference of the required ν-variation of α_ν directly from the empirical data in the following way. Since S_ν is, under this assumption, independent of ν over the line, the observed ν-variation of I_ν simply reflects a combination of the depth variation of S_ν and the ν-variation of α_ν. If we find points μ_1 and μ_2 on the solar disk such that $I_{\nu_i}(\mu_1) = I_{\nu_i}(\mu_2)$, the eq. (6) implies that $\tau_{\nu_i} = \mu_1$ and $\tau_{\nu_j} = \mu_2$ refer to the same geometrical point in the atmosphere. (Unless T_e does not vary monotonically with height, but this can be determined empirically, and does not introduce complication.) Thus, we have

$$(11) \qquad 0 = \tau_{\nu_i}/\mu_1 - \tau_{\nu_j}/\mu_2 = \int_z^\infty n_1(\alpha_{\nu_i}/\mu_1 - \alpha_{\nu_j}/\mu_2)\,\mathrm{d}x \rightarrow \alpha_{\nu_i}/\alpha_{\nu_j} = \mu_1/\mu_2 \,.$$

Provided we have a sufficiently-detailed and accurate set of measures of $I_\nu(\mu)$, over the line-profile, this procedure suffices to give an empirical evaluation of the ν-dependence of α_ν. Since we know how α_ν varies in the presence of a velocity field, we invert the empirical relation to infer the velocity field. Our knowledge of $T_e(\tau_\nu)$ permits us to separate out the thermal and non-thermal components.

Turn now to consider the effect of the simplifying assumptions represented by, respectively, eq. (2) and (9), upon the results just obtained. Eq. (2) is LTE; eq. (9) is the linearity of $S_\nu(\tau_\nu)$. We consider first the effect of eq. (9). Eq. (10) implies the highly restricted result that $\alpha_{\nu_i}/\alpha_{\nu_j}$ is constant throughout the atmosphere; viz, the effect of the velocity field on α_ν is constant throughout the atmosphere. This limitation is not particularly serious for our present purpose of comparison between thin gas in a laboratory, and stellar atmosphere; for it corresponds to the condition of eq. (4). We should, however, recognize the implication of the result in a discussion of astrophysical methodology. The result is a consequence of the assumption (9) applied to a ν-independent S_ν; it can readily be shown that these two conditions combine to require a constant value of $\alpha_{\nu_i}/\alpha_{\nu_j}$. Thus, when discussing a ν-independent S_ν, we recognize that only the non-linear terms in $S_\nu(\tau_\nu)$ contain information on the height-gradient of the atmospheric velocity field. We make this point, because the relation of eq. (8) is often extremely convenient to use in practice to make a first-estimate of $S_\nu(\tau_\nu)$. More refined analysis often appears to show only small change from this first-approximation result; *e.g.*, at $\mu = 1$, I_ν for a particular ν may correspond to S_ν at $\tau_\nu = 0.7$, rather than $\tau_\nu = 1$ as required by eq. (9). It is, however, these small differences that are important in specifying the detailed behavior of the velocity field. It is often necessary to discuss the concept of *effective depth of formation* of a line, particularly when one does not have available a complete set of $I_\nu(\mu)$ as data, but some integrated form of these. Under such circumstances, it is important to keep in mind the points we have made here.

Turn now to the second simplifying assumption, LTE given by eq. (2), and ask what it really does, in the way of fixing the analytical procedure. First, the assumption requires the *local* value of S_ν to depend *only* upon the *local* value of T_e. Thus, the empirical value of S_ν fixes immediately the thermal velocity field, but it has no direct connection with the non-thermal velocity field. Second, the assumption requires S_ν to be frequency-independent over the line. Consequently, the observed line-profile becomes immediately translated into the ν-dependence of α_ν, through the intermediary of the depth-dependence of S_ν. In a sense, the LTE assumption reduces the stellar-atmosphere case to an « equivalent » thin-atmosphere case. It accomplishes this by reducing S_ν to a quantity characteristic of the line as a whole, suppressing its ν-dependence, with magnitude fixed by the local thermal velocity; and by restricting the influence of the macroscopic velocity field wholly to α_ν, whose variation with ν translates the depth-dependence of S_ν into the observed profile of the line.

When, then, we turn to consider the actual physical expectation on the form of S_ν in the stellar atmosphere, these considerations suggest that our primary attention lie on two questions:

(i) To what extent can S_ν be considered to be a quantity characteristic of the line as a whole? That is, how strong is its ν-dependence? If the dependence is strong, we must expect some radically-different procedure than that outlined above to be necessary to obtain information on atmospheric macroscopic velocity field.

(ii) How strongly-controlled is the local value of S_ν by the local value of T_e in the atmosphere? If control is weak, we cannot expect a good determination of the thermal velocity field.

In Section 3.2, we summarize existing knowledge on S_ν from the standpoint of these two questions. Here, we have tried only to emphasize the reasons underlying a serious concern with the form of S_ν, and the question of the validity of the LTE assumption, in setting up the methodology for inferring properties of atmospheric velocity fields from observed line-profiles. We turn now to brief comments on the difficulties introduced in the analysis by problems of geometrical and instrumental resolution.

3.1.3. Problems of resolution—geometrical and spectral—in the astrophysical case.

a) Problem of geometrical resolution. We divide this question into two parts: that encountered in solar physics, because of lack of sufficient resolution to observe details of size less than some 1″ of arc, or about 700 km on the solar surface; and the stellar case, where no resolution of the disk is possible at all, and one observes F_ν rather than $I_\nu(\nu)$.

α) The solar case. A concern with the fine-structure of the solar case is quite recent, and provides much of the basis for direct inquiry into departure from spherical symmetry. One has concern that there may exist systematic velocity gradients over the solar surface, showing appreciable velocity differences over distances of the order 1″ or less. (That is, one is concerned with the existence of horizontal gradient in vertical velocity.) If indeed the line-of-sight velocity differences are large enough to introduce an observable shift in line-position, and the intensity variation over such a shift is comparable with the accuracy of measure of I_ν, then a serious systematic effect would be introduced into the interpretation of the $S_\nu(\tau_\nu)$ relation.

Suppose, for example, one had a situation where there was a system of rising and falling columns of gas, within each of which the only velocity field was thermal. If the instrumental resolution were such that only one column was observed, we could analyse its structure according to the procedures discussed in subsections 3.1.1 and 3.2.2, and in Section 3.2 following. If, however, several such columns were observed together, the observed profile would be the superposition of several profiles, each identical but displaced in wave-length. Clearly the analytical procedures discussed thus far would lead to erroneous results.

To investigate the errors arising from such lack of instrumental resolution, one must compare at the line-center the expected curvature of the theoretical profile for a line in a static atmosphere with the calculated curvature coming from the shift in line-position associated with the horizontal gradient in line-of-sight velocity. To our knowledge, such an investigation has not yet been made, for any of the lines where preliminary observations with the high resolution equipment have shown the fine-structure to exist. We regard this investigation as a fundamental one (cf. Fig. 7).

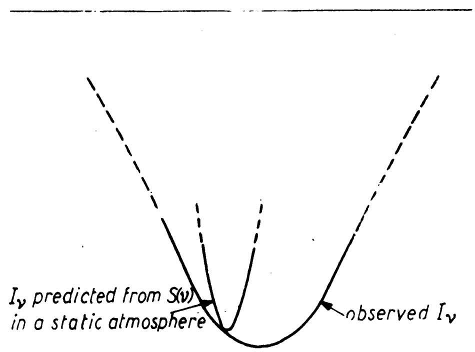

Fig. 7. – Line-profile as superposition of profiles from moving gas columns.

β) The stellar case. Dealing with observations of F_ν alone, rather than $I_\nu(\mu)$, makes more difficult the process of constructing $S_\nu(\tau_\nu)$ for each ν. Such construction could be accomplished only by the combination of several lines, such as outlined when discussing departures from spherical symmetry. What is usually done in practice, is to assume the validity of LTE, and a model of the atmosphere so that $T_e(\tau_c)$ is known, then compare the profile with one constructed assuming thermal velocities alone to be present. Usually, the first approximation to such an approach comes from comparing the total absorption in the line (cf. **3**·4 below) with that expected from thermal velocities alone, thus inferring a « turbulent » component of velocity. Then, one compares this « turbulent » velocity inferred from the total absorption with that required to give the observed width of the line-profile. We return to this subject in Sections **3**·3 and **3**·4, directing particular attention there to the methodology set up by HUANG and STRUVE. Actually few detailed analyses from even this LTE viewpoint have been carried out, mainly because a strong line would be required, and there is considerable uncertainty in the distribution of T_e in the upper atmospheric layers where such a line would be formed, even under this classical, LTE model. Nothing has been done, to our knowledge, from the standpoint of dropping the LTE assumption and treating the line from the complete non-LTE viewpoint.

b) Problem of spectral resolution. Again, as in Sect. **3**·1.2, we divide the question into two parts: the question of very-high resolution spectroscopy in solar work, *vs.* ordinary resolution; and the stellar case of weak lines from faint stars, where only integrated (over ν) profiles are available. These last are treated in terms of the so-called equivalent widths; viz., the width of a line of zero central intensity which would have the same total absorption as the observed line.

The high resolution solar spectroscopy offers the principal hope for detailed investigation of line-structure for really identifying the details of localized velocity fields. The same comment can be made as in **3**·1.3, essentially no detailed work has been based on this kind of material, primarily because little detailed observational material exists as yet.

Analysis in terms of equivalent widths gives information on depth variation of physical quantities only insofar as we can observe different lines having different « effective » depths of formation. Since a primary goal of most astrophysical analyses of stellar spectra lies in determination of abundances, and since the abundance of a given atmospheric constituent determines its depth of formation, one cannot approach the problem of depth-dependence of physical quantities, using equivalent widths alone, from a completely unambinguous viewpoint. This difficulty makes itself felt in a particularly obvious way when we attempt to ask how valid the LTE assumption may be, by comparing an empirically-determined S_ν with $B_{\nu_0}(T_e)$ in an atmospheric region where some independent measure of $T_e(\tau_c)$ exists. Use of equivalent widths alone does seem to permit some insight (cf. the recent summary of PECKER, 1959), but the uncertainties are very large compared with those encountered in analyses based on $I_\nu(\mu)$ (cf. Chapter 9, THOMAS and ATHAY, 1961). We would simply like to stress here that the question of the proper form for S_ν, the derived abundance, and the derived properties of the velocity field are all ultimately linked. When one has available only such triply integrated quantity as equivalent width of a stellar line, considerable *a priori* theoretical effort must be introduced to give meaningful results on the particular values of the physical quantities applying in the particular case analysed.

With these general comments on the astrophysical methodology providing a comparison with the laboratory situation, we proceed to more specific detail, breaking the summary into the idealized cases of **3**·2-**3**·4. The reason for such a breakdown is both historical and conceptual, this being the structure actually considered in astrophysical analyses; the reasons for this will become clear in the discussion of the methodology.

3·2 *Methods of analysis of the spectrum produced in an atmosphere where the only velocity fields are thermal.* – Our attention is directed at analysis of stellar atmospheric spectra to infer the character of any existing non-thermal velocity fields, as contrasted to the purely thermal velocity field. Consequently, our interest in this case **3**·2 centers around its use as a « control », to clarify some of the questions raised in the survey of astrophysical analytical methodology in sect **3**·1. Such a control has two aspects. First, there is the wholly theoretical one of answering the a priori methodological questions raised in **3**·1, which in essence comes down to an inquiry into the form of S_ν. Second, there is the question of the application of these results to situations where earlier analyses may have proceeded on the basis of assumptions not in harmony with the theoretical conclusions on the proper form of S_ν. We should ask whether the inferred velocity fields actually exist, or whether they are simply the consequence of a bad choice on S_ν. Since it is not our object in this paper to survey the actual results obtained in astrophysical analyses, this being left to the detailed summary-introductory papers at Varenna, we can only attempt to indicate the direction of an effect resulting from such a bad choice on S_ν, with a few simple examples after we have surveyed the general theoretical expectation for S_ν.

In Sect **3'1**, we have shown that concern with the form of S_ν centers on two points: the ν-dependence of S_ν; and the degree to which the local value of S_ν is fixed by the local value of T_e: Our approach in the present Section is: given the values of the local thermodynamic parameters characterizing the atmosphere, what is the theoretical expectation on S_ν and how do we analyse the line-profile using this theoretical form for S_ν in order to obtain empirical values for S_ν and possibly of T_e. We have in the introduction referred to indirect evidence on the existence of aerodynamic phenomena. Were we to investigate such indirect evidence, our interest would center on the relation between T_e and the local radiation field, with respect to the existence of cyclic processes associated with a non-radiative energy supply such as might come from local dissipation of energy from a macroscopic velocity field. Here, however, we simply take the local values of T_e and other thermodynamic parameters as given, not asking how these values were fixed, then ask what values of S_ν are consistent with them, in order to formulate the methodology to invert this procedure.

3'2.1 The form of S_ν. For the discussion of physical expectation, it is essential to break up S_ν into contributions from the continuum and from the line, since the contributions originate from different processes. Thus, we write

$$S_\nu = \frac{S_s + r_\nu S_c}{1 + r_\nu} = \frac{\eta_\nu S_s + S_c}{1 + \eta_\nu}. \tag{12}$$

We see that S_ν can be treated as ν-independent in only three cases: (i) when $S_s = S_c \neq f(\nu)$; (ii) when $r_\nu \ll 1$ and $S_s \neq f(\nu)$; (iii) when $r_\nu \gg 1$, and $S_c \neq f(\nu)$. There is the fourth case that S_s and S_c each depend upon ν, but in such a way that their variation combined with that of r_ν leaves S_ν independent of ν. As a general possibility, this last seems too fortuitous to consider, when combined with the following remarks on the ν-dependence of S_s and S_c.

Case (i) corresponds to LTE in both S_s and S_c, for S_s and S_c refer to different processes and can only coincide in the degenerate case. The results have already been discussed in Section **3'1**.

Case (ii) corresponds to the core of a strong line, and the condition that S_s is ν-independent over such a core. Since r_ν increases monotonically outward

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from the line-center, there must come some region on the line-profile where $r_\nu S_c$ is not negligible compared with S_s. In this region, S_ν varies with ν because of r_ν, even though S_s and S_c may be independent of ν. Thus, there is at most a limited region of the profile which may be treated by a ν-independent S_ν. Whether even this limited region exists, must be shown by asking the form of S_ν.

Case (iii) corresponds to a very weak line and the wings of stronger lines, and would appear to be the only case outside strict LTE where S_ν remains ν-independent over the whole profile. The variation of S_c with ν over the very small width of the line can almost certainly be ignored. Actually, the situation is not so straightforward; for we must retain some quantity referring to the line in either S_ν or τ_ν in order to produce a line. We return to this case under the designation of « weak-line approximation » below.

Consider the expectation on S_c and S_s.

In general, it appears sufficient to set $S_c = B_\nu(T_e)$—*i.e.* to assume LTE for the continuum—for discussions of most lines observed in the solar Fraunhofer spectrum. The point is the following. In the lower solar atmospheric regions, where r_ν is not negligible for the lines formed in such regions, our present knowledge suggests that S_c does not depart appreciably from $B_\nu(T_e)$ (cf. PAGEL, 1959; THOMAS and ATHAY, 1961). In the upper atmospheric regions, we must expect S_c to depart from $B_\nu(T_e)$; for example, the Lyman continuum of hydrogen shows an S_c very different from $B_\nu(T_e)$ (cf. THOMAS, 1952 and THOMAS and ATHAY, 1961, relative to the often-expressed, but incorrect, viewpoint contained *e.g.* in WOOLLEY and ALLEN, 1950). However, in these regions r_ν can be shown to be so small that the value of S_c is not very important in the Doppler core, where the velocity field is important. (Note that the non-LTE effect drops S_c below $B_\nu(T_e)$, further reducing the relative importance of $r_\nu S_c$.)

In the case of hot stars, and the outer solar atmosphere, S_c may depart from $B_\nu(T_e)$ for two reasons. On the one hand, electron scattering plays the major role in the continuous opacity for hot stars. On the other hand, whenever the bound-free opacity arises in a region where occupation numbers differ from a Boltzmann distribution, S_c departs from $B_\nu(T_e)$. (For example, in the hydrogen Lyman continuum, as already mentioned.) These conclusions on S_c have two immediate consequences for our discussions of velocity fields. First, in the deeper atmospheric regions, one can use observations made in the continuum to fix T_e, hence the thermal velocity field. Then we may use the analysis of those lines whose « effective » depth of formation lies in the range covered by the data from the continuum to provide two kinds of wholly empirical check. On the one hand, one may infer a value of S_ν, and compare it with $B_\nu(T_e)$, to check the applicability of the LTE assumption. Such an analysis has been initiated by PECKER and associates (1959) and leads them to the conclusion that $S_\nu \neq B_\nu(T_e)$ for a large number of weak lines of Ti, Ti^+, Fe, V, Cr and A^+. On the other hand, one may assume a ν-independent S_ν, and analyse the profile as outlined in Sect 3·1 to infer a velocity field. If the field agrees with the thermal value, it is temping to infer both that the ν-independence assumption is correct, and that the only velocity fields existing are thermal. In the event that the velocity fields derived do not agree with the thermal value, one can either question the assumption on S_ν or ascribe the discrepancy to a non-thermal velocity field. There exist a variety of analyses and results on this last procedure, which will be reported in detail in the various Parts of the program at Varenna.

Second, in hot stars and in the upper atmospheric regions where data from the continuum do not exist, the analysis of the lines must be used to fix both thermal and non-thermal velocity fields. Thus, considerable attention must be paid to the form of S_s. We mention the single exception, a limited region of the lower solar chromosphere, where it appears that eclipse observations made in the continuum provide independent data on T_e (cf. THOMAS and ATHAY, 1961). Throughout the outer stellar atmospheres, however, and over most of the outer solar atmosphere, both thermal and non-thermal velocity fields must be determined from analysis of the line-profiles alone. While a consistency requirement can be placed on thermal fields inferred from different lines of different ions—provided the relative regions of origin of the lines can be identified—the same consistency from ion to ion cannot be an *a priori* requirement on non-thermal velocity fields (for example, a superposition of gyromagnetic and turbulent motions).

When one turns to theoretical expectations on S_s, he must distinguish two kinds of treatment existing in the astrophysical literature. One is a kind of « working » approach to the analysis of spectral lines, based on formal rather than detailed physical analysis of the process of line-formation, which was developed mainly for discussion of total absorption in a line rather than of details in the line-profile. The other is a very specific attempt to treat in great detail the problems of line-formation from the single requirement that the observed spectrum does not change in time, thus, that the occupation numbers of internal energy levels of the various ions be constant in time.

The « working » approach characterizes by far the greatest bulk of existing astrophysical analyses of stellar spectra. The adopted expression for S_s follows from the process of simply writing down several possible mechanisms of producing radiation—which are taken to be « coherent scattering », « non-coherent scattering », and « pure absorption »—then assuming S_s is a linear combination of these alternatives, with coefficients whose numerical values are not specified a priori in terms of either atomic constants or thermodynamic

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parameters of the atmosphere, but are to be fixed by the analysis. (Coherent and non-coherent scattering refer to frequency, not phase; S_s for pure absorption is $B_\nu(T_e)$, cf. UNSOLD, 1955, for a detailed summary.) With the exception of very strong Fraunhofer lines, the « working approach » is again simplified, in the great majority of analyses, by assuming that only the pure absorption term is significant—or that non-coherent scattering prevails in some strong lines and gives $S_s = kB_\nu(T_e)$ (k being a frequency-independent constant, usually smaller than 1 for absorption lines). So long as either of the latter alternatives is valid, we have S_s independent of ν, case (i) above, or case (ii), and the analysis is straightforward as discussed in Section **3·1**. If the more general alternative including all terms mentioned were valid, the presence of the coherent-scattering term makes S_s, hence S_ν, ν-dependent. For reasons discussed below, we reject this general alternative in the central parts of the line, where the absorption coefficient is mainly fixed by the velocity field. Thus, returning to the two points developed in Section **3·1** and summarized in Section **3·2**, our concern with the form of S_ν reduces to just one point, that of the degree to which the local value of S_ν depends upon the local value of T_e. That is, the question of how literally the results of most of the existing astrophysical analyses can be taken, in discussing stellar atmospheric velocity fields, rests on how satisfactory is the assumption of LTE.

Investigations of the general form of S_s to be expected on the basis of the treatment of a gas in a statistically-steady, but not necessarily LTE, state have been motivated primarily by just this question of how significant are departures from LTE. General results from such investigations are presently few in number. There have been a number of detailed, numerical « brute-force » calculations, aimed mainly at producing results which can be compared with solar observations to see if details which are anomalous under the LTE approach become resolved under the non-LTE approach (cf. THOMAS and ATHAY, 1961, for a summary). Mainly, these calculations have been limited to hydrogen helium, and calcium. A sequence of algebraic investigations of simulated atoms, having a limited number of energy levels, has been initiated by JEFFERIES and THOMAS (1957, 1958, *et seq.*) in order to make more clearly explicit the thermodynamic parameters upon which S_s depends, and to link this more modern work with similar attempts in the early 1930's at more detailed investigation. We emphasize the significance of this older work; it was carried out under the conceptual limitations of no local energy dissipation other than radiative in the atmosphere. Thus, two points were missed: the possibility of an underestimate of the importance of collisional terms, because of the possible existence of regions where T_e exceeds the value inferred from the continuous spectrum alone; an underestimate of the height of formation of a spectral line relative to that of the continuum, again resulting from the greater value of T_e. On the other hand, much of the contemporary feeling that

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these more modern non-LTE effects are confined to the stellar chromosphere overlooks the kind of non-LTE effects implicit in the older work.

The approximate results obtained from this sequence of algebraic investigations are most likely to be applicable to actual atoms where one treats strong resonance lines, or strong subordinate lines in atmospheric regions having very high opacity in the resonance lines. Work on a general methodology to extend the treatment to weaker lines is promising and suggestive, but only so, at this stage of development. The empirical work already cited, by PECKER and associates, suggests that the *general* physical results on the direction of departure from LTE may remain valid for weaker lines. We will now summarize briefly the results from these somewhat-idealized algebraic investigations, which indicate the extent to which S_s is ν-independent, and to which the local values of S_s may be considered to depend only upon local parameters, particularly T_e.

One can write, quite generally,

$$S_s = B_\nu(T_{ex}) j_\nu / \varphi_\nu \,, \tag{13}$$

where j_ν and φ_ν represent the profiles of emission and absorption coefficients, normalized such that their integrals over ν (and solid angle, for j_ν) are unity. T_{ex} is the « excitation-temperature » defined as a Boltzmann temperature-parameter giving the actual ratio of occupation numbers in upper and lower levels of the transition. A solution of the equations of statistically-steady state for the occupation numbers, ignoring mass diffusion terms, gives (cf. JEFFERIES and THOMAS, 1958 *et seq.*; THOMAS and ATHAY, 1961)

$$B_\nu(T_{ex}) = \frac{\int \bar{I}_\nu \varphi_\nu \, d\nu + \varepsilon B_\nu(T_e) + \eta B^*}{1 + \varepsilon + \eta} \,, \tag{14}$$

ε is the ratio of rates of collisional to radiative de-excitation in the line, evaluated at the local value of T_e and n_e; the term ηB^* represents a ratio of upward excitations by radiative ionizations to spontaneous transitions downward in the line. Generally, in the stellar atmosphere, and for strong lines to which this two-level approximation has some degree of applicability, the first term on the right of eq. (14) is very much larger than either of the second two terms. In this event, the solution of the radiative transfer equation, using eq. (12), (13) and (14), is a diffusion problem, with the second two terms on the right of eq. (14) serving as « source » terms for the diffusion. With these expressions (13) and (14), consider the two questions we have raised: (i) the relation between the local value of S_s and the local values of the thermodynamic parameters characterizing the atmosphere; (ii) the relation between the ν-dependence of S_ν, that of the absorption coefficient, and that of I_ν (emergent).

First, from a purely formal standpoint it is clear that only in two cases will $B_\nu(T_{ex}) = B_\nu(T_e)$, and thus will the local value of $B_\nu(T_{ex})$ be fixed wholly by the local value of T_e.

a) If everywhere the first term on the right of eq. (14) is completely dominant, and satisfies $\int \bar{I}_\nu \varphi_\nu \, d\nu = B_\nu(T_e)$. Such a condition could at most hold under highly exceptional circumstances, and certainly not throughout the atmosphere and for all lines.

b) If the second term on the right of eq. (14) dominates completely. For resonance and strong subordinate lines, $\varepsilon \ll 1$ in stellar atmospheric situation; so this possibility is excluded. Uncertainty on cross-sections for higher-lying subordinate lines couple with the uncertain applicability of the expression (14) to leave the situation unresolved. It would seem plausible that there exist pairs of energy levels close enough to the continuum that this second term dominates; the problem is to specify them, and for this we require cross-sections and more detailed treatments of the statistically-steady-state.

If neither of these two cases hold—and it is clear that neither will for the stronger lines in the stellar spectrum—then the local value of T_{ex} is not fixed by the local values of the thermodynamic parameters characterizing the atmosphere, but by their depth distribution.

Second, since opacity of the atmosphere to the continuous radiation is generally several orders of magnitude smaller than to line radiation involving the same lower level, there is a strong difference in result according to which is the larger of the second or third terms on the right of eq. (14). If the second term predominates over the third, then indeed the *distribution* of T_e fixes the values of S_s; so that from an analysis of $S_s(\tau_\nu)$, we have a possibility of inferring $T_e(\tau_\nu)$. We must, however, treat the atmosphere as a whole. Examples of lines for which the second term in eq. (13) predominates over the first — which category we have called collision-dominated — are the *H* and *K* lines of ionized calcium, the Mg^+ lines near $\lambda\, 2\,700$—and generally, the ionized metallic resonance lines—and the Lyman lines of hydrogen in the chromosphere. If, on the other hand, the third term predominates, then the source-term in the diffusion problem is simply the radiation field in the ionization continuum associated with the lower level of the line. This continuum originates in a much deeper atmospheric region than that where the line originates. Thus, if this continuum can be represented in terms of some temperature value, the value must be expected to differ very considerably from the local values of T_e in the region of line-formation. Examples of lines falling in this last category — which we have called photoionization-dominated — are the early Balmer lines of hydrogen, the sodium *D* lines and generally, the neutral metals.

Third, the value of S_s is independent of ν at a given atmospheric position *only* if j_ν/φ_ν is ν-independent. It has been shown (THOMAS, 1957) that this latter condition is satisfied over that part of the line in which the profile of φ_ν is fixed by thermal motion. (If there exists a non-thermal motion, random over a scale much less than a photon free path, with mean velocity exceeding the thermal velocity for the atom in question, this same conclusion should apply to the larger line-core specified by this non-thermal velocity.) The behavior of S_s outside this central core—which is generally between 2 and 3 Doppler widths in size—has not yet been explored with conclusive results.

Fourth, since the source-terms in eq. (14) contain atomic parameters characteristic of the particular energy levels involved in producing the line studied, one must generally expect T_{ex} to differ from line to line, even in those cases where the lines may have one level in common. Exceptions may occur; these must be extablished by detailed investigation in each case.

Summarizing these results on S_ν as they bear on the problem of the analysis of line-profiles for velocity fields, as outlined in Section **3'1**, we can say the following:

α) For strong resonance lines, or strong subordinate lines where lower-lying lines satisfy detailed balance; $T_{ex} \neq T_e$; S_s is ν-independent over the core of the line where the profile of φ_ν is essentially determined by the random velocity fields present; S_ν is ν-independent over the same core *if* over the same region $r_\nu \ll 1$; in general, S_ν differs from one line to another.

β) For weaker lines, and higher-lying subordinate lines, we have at present essentially no sound theoretical guide. On the one hand, we expect that resonance lines approximately described by eq. (14) will have $T_{ex} \neq T_e$; but if they do not have $r_\nu \gg 1$, S_ν will depend upon ν. On the other hand, for transitions between sufficiently-high-lying levels, we may expect $S_s \to B_\nu(T_e)$. No work has yet estabilished the transition region. The empirical results by PECKER and associates suggest that this last regime of LTE has *not* been reached in the case of many lines for which it is usually assumed.

3'2.2. Application. In the following, we restrict our attention to absorption lines, from which most astrophysical information on velocity fields has come. There are exceptions, such as the discussion of very broad emission features in Wolf-Rayet stars and in novae, which ultimately must lead to very important information on velocity fields in atmospheres thought to be unstable in one way or another. Cf. the pioneering work by BEALS (1941), the extensive discussion by SOBOLEV (1947), such recent summaries as that by PAGEL (1959), and a short critique from the non-LTE viewpoint by THOMAS (1949). But by far the greatest body of information, upon which most astrophysical thinking is based, deals with absorption lines.

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It is usually customary, in discussing absorption lines, to work with the depth of the line, $R_\nu = I_c - I_\nu$, rather than the residual intensity, I_ν, in the line. From eq. (1) and (2) we have

$$(15)\qquad R_\nu = \int_0^\infty \{S_c(1 - \exp[-\tau_s/\mu]) - \eta_\nu S_s \exp[-\tau_s/\mu]\}\exp[-\tau_c/\mu]\,\mathrm{d}\tau_c/\mu\,.$$

It is also customary to express the above in terms of the « weighting-function », $g(\tau_c)$, introduced for weak lines by UNSÖLD (1932), MINNAERT (1948), and extended to stronger lines by PECKER (1951):

$$(16)\qquad \mu g(\tau_c) I_c = \int_{\tau_c}^\infty S_c \exp[-\tau_c/\mu]\,\mathrm{d}\tau_c/\mu - S_s \exp[-\tau_c/\mu]\,,$$

in terms of which, eq. (15) becomes

$$(17)\qquad R_\nu = \int_0^\infty \eta_\nu\,(\mu I_c g(\tau_c))\,\exp[-\tau_s/\mu]\,\mathrm{d}\tau_c/\mu\,.$$

The equivalent width of the line is then given—converting I and R to wave-length rather than frequency units—by

$$(18)\qquad W_\lambda = \int_0^\infty R_\lambda I_c^{-1}\,\mathrm{d}\lambda\,.$$

The weighting-function approach is *mainly* used under the LTE assumption on S_s. In this case, the integrand in eq. (17) is a product of two factors, one—$\eta_\nu \exp[-\tau_s/\mu]$—involving the line and depending upon ν, and the other—the weighting-function $g(\tau_c)$—independent of the line and ν. The latter, in the LTE case, is essentially the gradient of $B_\nu(T_e)$, and can be computed once and for all for a given atmospheric model. PECKER (1957) has emphasized that in the non-LTE case, the weighting-function, $g(\tau)$, can be written

$$(19)\qquad \mu I_c g(\tau) = \left\{\int_{\tau_c}^\infty S_c \exp[-\tau_c/\mu]\,\mathrm{d}\tau_c/\mu - S_c \exp[-\tau_c] + (S_c - S_s)\exp[-\tau_c/\mu]\right\},$$

which becomes, in the linear case of eq. (9) for S_c,

$$(20)\qquad \mu I_c g(\tau) = \{\mu\,\mathrm{d}S_c/\mathrm{d}\tau_c + (S_c - S_s)\}\exp[-\tau_c/\mu]\,.$$

Thus, for non-LTE effects to introduce significant change into the results of analysis of a line-profile, $(S_c - S_s)/S_c$ need not be of order unity, but only of order $\mu \, \mathrm{d}\ln S_c/\mathrm{d}\tau_c$. We also note that in the general non-LTE case, where the scattering term, $\int \bar{I}_\nu \varphi_\nu \, \mathrm{d}\tau_\nu$, entering eq. (14), is of major importance, it is quite misleading to retain the usual physical picture of the weighting-function as something characteristic of the model of the atmosphere and independent of the particular line considered. The scattering term often depends upon τ_s almost independently of τ_c, particularly for strong lines. Thus, the utility of $g(\tau)$ as a function that can be computed once and for all, independently of the line, largely disappears when non-LTE effects must be included. It is, however, a very useful concept to demonstrate, as in eq. (20), the quantities with which non-LTE effects must be compared to assess their importance.

If we consider the case where S_s is independent of ν, then we may regard $g(\tau)$ as that part of the integrand which is independent of ν, the ν-variation coming from the factors η_ν and $\exp[-\tau_s/\mu]$. The first factor alone would give something resembling the laboratory case of a thin atmosphere, since it is just the profile of the absorption coefficient. The second factor selects the atmospheric region contributing most to the particular point on the profile thus giving the contribution to the profile arising from the variation in g with τ—through the variation in the S with τ. Thus, in astrophysics it is customary to distinguish two kinds of lines: « weak » lines, for which $\tau_s \sim 0$ is a good approximation; and all other lines, for which non-zero τ_s must be considered. We consider those two kinds of lines, in turn.

a) The weak-line approximation. The basic assumption is $\tau_s/\mu \ll 1$, so that $\exp[-\tau_s/\mu]$ may be taken as unity. It is often assumed, in studying such weak lines, that the profile of R_ν mimics the profile of the absorption coefficient, through the ν-dependence of η_ν (Cf. Bell, 1951; Bell and Meltzer, 1958; Rogerson, 1957). If the assumption were valid, we should have the astrophysical analogue of the case of a uniform, thin gas in the laboratory, and the observed line-profile would give immediately the velocity field. We see that this assumption requires a constant profile of φ_ν over whatever region of the atmosphere contributes significantly to the line. Such a constant profile of φ_ν is often justified by the argument that the line is formed in a narrow region of the atmosphere (the so-called Schuster-Schwarzschild case), because of strong variation in excitation conditions. Thus, in applying this weak-line approximation to study the distribution of velocity fields in the atmosphere, it is important to distinguish the case just cited from that where the ion is distributed more-or-less uniformly over the region where the continuum originates (the so-called Milne-Eddington case), but the ion considered has a very low abundance.

We have already mentioned the second assumption, that S_s must be

ν-independent, if η_ν is to give the whole ν-dependence. This second assumption is invariably overlooked, because the analyses invariably assume LTE. However, its neglect is not serious, in view of our proof, mentioned earlier, that S_s is ν-independent over the Doppler core. The weak-line criterion of $\tau_s \ll 1$ ensures that the line will have significant opacity only over this Doppler core.

Thus, a departure of the profile of R_ν from a simple Doppler profile, that of φ_ν, comes only from a variation of φ_ν over the atmospheric region contributing to the line. Alternatively, such an observed departure may be taken as evidence that the line really does not satisfy the weak-line criterion. Finally, departure from agreement of profiles of several « weak-lines » originating from several ions either signifies a variation of conditions within the atmosphere, or varying velocity fields from ion to ion producing the lines.

The alternative among these effects must be considered carefully and seriously. An example lies in the suggested procedure to distinguish thermal from non-thermal velocity fields in the solar atmosphere by comparing profiles of R_ν for weak lines from several ions of differing mass (Bell, ibid). Houtgast (1953) has stressed the difficulties entering such an analysis from the standpoint of differing distributions within the atmosphere of the ions considered. For several of her lines, Miss Bell has found it necessary to interpretet the profiles with Doppler plus damping contributions to the absorption coefficient, which implies that the lines do not satisfy the weak-line approximation.

Finally, the criterion used to select lines satisfying the weak-line approximation is often not based on a computation of the validity of the condition $\tau_s \ll 1$, but only upon the observational criterion that $R_\nu <$ some small fraction. Such a procedure essentially neglects the effect of the difference $(S_c - S_s)$, assuming this quantity to be zero. In a similar way, we note that $\tau_s \ll 1$ does not imply that for all regions of the atmosphere, $\eta_\nu \ll 1$. A weak line may originate entirely within the chromosphere, where τ_c is essentially zero, for example. Both these points warrant investigation.

b) Lines not satisfying the « weak-line » approximation. Since η_ν is directly proportional to α_ν, whose integral over ν is $\pi\varepsilon^2 f_{12}/mc$, we see from eq. (17) and (18) that in the weak-line approximation, W_λ is independent of the velocity field. Thus, only the line-profile gives information on velocity fields. When observational conditions are such that only integrated intensities, or equivalent widths, not profiles, can be observed, the weak-line case gives no information on velocity fields. However, as τ_s increases to the point where the weak-line approximation becomes invalid, W_λ depends upon the velocity field and we may use both profile and integrated intensity to study the velocity fields.

For illustration, consider the LTE case, $S_s = S_c = B_\nu(T_e)$, and an extreme case of the (Schuster-Schwarzschild) kind of model already mentioned, where the emitting ion is confined to a narrow atmospheric layer, at τ_c, of negligible thickness in τ_c. Then eq. (17) integrates to

$$(21) \qquad R_\nu \sim \mu(1 - \exp[-\tau_s/\mu]) \exp[-\tau_c/\mu]\, \mathrm{d}S_c/\mathrm{d}\tau_c\,.$$

If we consider a sufficiently-strong line, we see that the line « saturates » in its central regions, maintaining practically a constant value of R_ν until φ_ν decreases sufficiently to drop the value of the bracket in eq. (21) below unity. Thus, the integrated profile of R_ν, or the equivalent width W_λ, depends strongly upon the parameters fixing the rate of drop of φ_ν. In a rough way, for wholly random velocity fields

$$(22) \qquad \varphi_\nu/\varphi_0 \sim \exp[-[\Delta\nu/\Delta\nu_D]^2] + F([\Delta\nu]^{-2})\,,$$

so that the greatest rate of decrease in φ_ν comes over the core of the line. F relates to the radiation and collisional broadening processes. For weak lines, W_λ depends only upon τ_{s_0} (subscript 0 referring to the line-center). As τ_s increases W_λ begins to depend upon $\Delta\lambda_D$ until, for strong enough τ_s, W_λ is proportional to $\Delta\lambda_D$, and varies only slightly with τ_{s_0}. When the line becomes so saturated that the second term on the right of eq. (22) becomes of major importance before the saturation begins to disappear, we enter the well-known « pressure-broadening » regime, and W_λ varies as $\tau_{s_0}^{\frac{1}{2}}$. A plot of this dependence of W_λ on τ_{s_0}, with parameter $\Delta\lambda_D$, is called the curve of growth. Fig. 3, in our discussion of a rough approximate treatment of the total absorption by a line, represents a rough approximation to a curve of growth. For a detailed discussion free from the special assumptions underlying eq. (21), cf. the classical discussion by MENZEL (1939), and the recent summary by VAN REGEMORTER (1958).

Let us now summarize the classical theoretical computation of a curve of growth from the weighting-function approach. It is based on the use of the weighting-saturation functions (see, for instance, in the limited case of LTE for $g(\tau_c)$, UNSÖLD, Physik der Sternatmosphären, p. 109). The expression of W can then be written

$$(23) \qquad W = \int_0^\infty \frac{R_\lambda}{I_c}\, \mathrm{d}\lambda = k \int_0^\infty g(\tau_c)\, \frac{\eta_{\nu_0}}{\overline{\eta}_{\nu_0}}\, \Phi(\overline{\eta}_{\nu_0}, \alpha) \sqrt{T_e}\, \mathrm{d}\tau_c\,,$$

$g(\tau_c)$ being the weighting-function, as defined in eq. (16), $\Phi(\overline{\eta}_{\nu_0}, \alpha)$ (Fig. 8) being the saturation function ($\Phi = 1$ if $\overline{\eta}_{\nu_0} \ll 1$ — weak lines —, and $\Phi < 1$

in the other cases—intermediate strength, and strong lines). The function Φ has been extensively tabulated. η_{ν_0} is the value of η_ν at the center of the line; and one has

$$(24)\qquad \bar{\eta}_{\nu_0} = \frac{1}{\tau_c}\int_0^{\tau_c} \eta_{\nu_0}\,\mathrm{d}\tau_c\,.$$

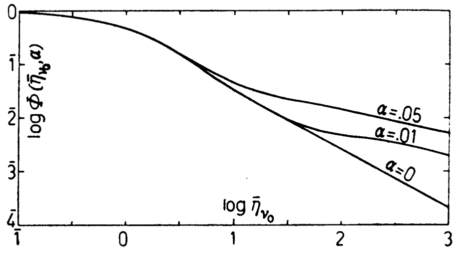

Fig. 8. – Saturation function.

One can write, in an approximate way, just introduced to show the respective importance of each of the physical factors involved,

$$(25)\qquad W \sim k[g(\tau_c)\sqrt{T_e}]\int_0^\infty \frac{\eta_{\nu_0}}{\bar{\eta}_{\nu_0}}\,\Phi(\bar{\eta}_{\nu_0}, \alpha)\,\mathrm{d}\tau_c\,.$$

The variation of $\eta_{\nu_0}/\bar{\eta}_{\nu_0}$ with τ_c fixes essentially (and only) the detailed shape of the curve of growth; the values of $g(\tau_c)$ and of T_e fix the value of W corresponding to the plateau of the curve of growth (Fig. 9). Through this

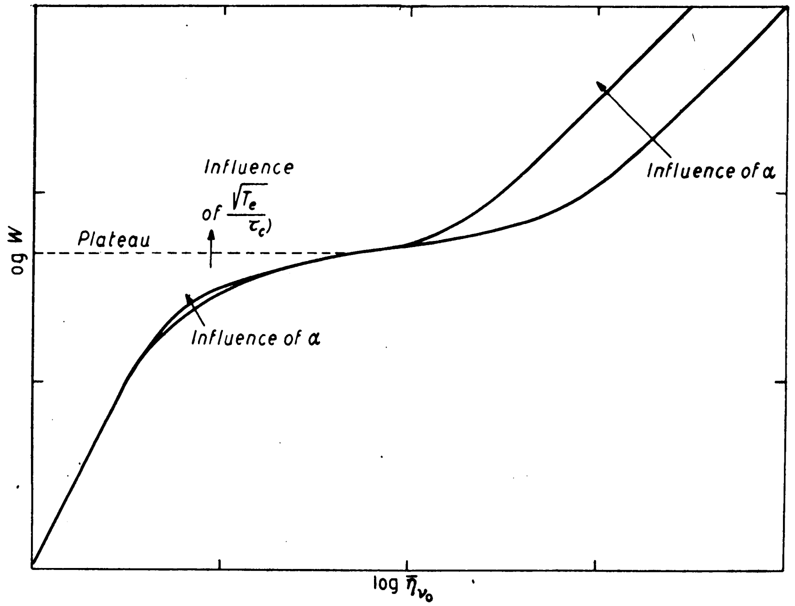

Fig. 9. – Influence of saturation function on plateau of curve of growth.

value, determined from measurements, and provided one knows the variation of $g(\tau_c)$, it is thus possible to derive a mean value of the thermal motions.

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It must be noted that the above analysis assumes S_c and S_s to be not ν-dependent $(i.e.\ g(\tau_c)$ non ν-dependent$)$; the case with a ν-dependent $g(\tau_c)$ has never been treated in this approach: this limitation thus puts a great concern with this weighting-saturation approach. It has been treated, however, rigorously in limited cases (pure coherent scattering and approximations on η_{ν_0} of the Milne-Eddington or Schuster-Schwarzschild type — see *e.g.* WRUBEL, 1949). But in those limited cases, the « pure-scattering » restriction on S_s and the overlooking of the actual stratification through the approximations on η_{ν_0} put again a great doubt on the results that can be derived, through such curve of growth, about velocity fields.

We denote lines not strong enough to enter the « pressure-broadening » regime as « lines of intermediate strength »; others, as « strong lines ». In the former, both profile and W_λ depend upon the velocity fields; in the latter, W_λ is most insensitive to the velocity field, and only the profile may be used.

Generally, in analysing a line for velocity fields, three effects must be considered: the distribution of φ_ν through the atmosphere; the effect of departure of S_s from S_c; the effect of ν-dependence of S_s outside the Doppler core. There exist no systematic investigations of the latter two effects. In discussing the problem of variable φ_ν, analyses reported thus far have simply made some assumption on variation of φ_ν, then compared results from lines thought to originate in differing atmospheric regions, to construct a better approximation (or, if center-limb observations exist, they may be used in place of several lines having differing heights of origin). So long as we restrict attention to atmospheres having only thermal velocity fields, as in this Section 3·2, and to those parts of the atmosphere where the *form* of the distribution in T_e is known, an a priori assumption on the kind of variation of φ_ν is feasible. Such a situation is possible for weak and intermediate strength lines. Because η_{ν_0} is so large for strong lines, those regions of the line where velocity fields are important are often formed outside the atmospheric regions where the general distribution of T_e is known. In the more general case of Section 3·3 and 3·4, any a priori estimate of general distribution of velocity field is unsound, because of our present complete lack of theoretical knowledge of the kinds of velocity fields to be expected in a stellar atmosphere. The most one can ask, in any iterative procedure, is numerical consistency (cf., *e.g.*, HUANG, 1951).

In summary, on the basis of presently-developed methods of analysis, the most important aspect in an a priori approach is that of assigning « effective » depths of formation for differing lines. The question of such effective depths has, until now at least, been investigated from the LTE basis (cf. VAN REGEMORTER, 1958). It is now well-established that non-LTE effects combine with the existence of a stellar chromosphere to introduce very serious change, over the classical LTE computations, in depth of formation of the Doppler cores of strong, and intermediate lines (cf. THOMAS and ATHAY, 1961,

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for a summary of work leading to this conclusion). It is a problem to be settled, how far such effects extend out from the core of the line, and their influence on W_λ. Another problem is to make a clear distinction between what could be called « effective depths of formation of Doppler widths » and « effective depths of formation of central intensity » etc. In an exact treatment, no problem arises; but the iterative methods used are hardly exact. Every method of measurement corresponds to a different « effective depth » and the interrelations between them have not been satisfactorily analysed.

We would emphasize that, since there is no *a priori* knowledge whether an atmosphere satisfies the condition of wholly thermal velocities, it is critical that an analysis of the atmosphere provide a check of inferred velocity field against the thermal value. For those weak lines formed in regions covered by analysis of the continuum, an immediate check exists. For intermediate-strength lines, a comparison of profile to total absorption gives a check. (Cf. the discussion of micro- and macro-turbulence in Sections **3**.3 and **3**.4 on this point.) For strong lines, such a check is more difficult. We must require either a determination of T_e from the magnitude of S_ν, then a determination of velocity field from absorption coefficient via the ν-dependence of I_ν; or a determination of velocity field *at a given point in* the atmosphere from several lines of different ions, and intercomparison to see whether a wholly thermal origin is consistent. Thus, the question of non-LTE effects becomes of primary importance, in attempting to analyse observations of such strong lines for atmospheric velocity fields.

We have already differentiated, in the last few paragraphs of Sect. **3**.2.1, between essentially two types of S_s—one of which depends upon collisional effects for the source term, thus upon the distribution of T_e through the region of line-formation; the other of which depends upon photo-excitation, thus is insensitive to T_e in the region of line formation. An analysis of a line of the first type may by itself provide a set of data with internal checks. An analysis of a line of the second type gives information on thermal velocity fields only from the ν-dependence of I_ν, and has no checks for consistency of the assumption that the velocity fields are wholly thermal. Thus, several lines of different ions must be analysed, whose Doppler cores are formed in the same atmospheric region. Locating the region of formation is a problem comparable to specifying the velocity field, and the two must be solved together.

3.3. *Analysis of a line formed in an atmosphere where non-thermal velocity fields exist, but are assumed to consist of random motions of groups of atoms of dimension smaller than a photon free-path.* – Clearly, the photon free-path in question (a length corresponding roughly to an optical depth unity) must be that corresponding to the largest value of the line absorption coefficient, that at the line center. Then, for a given ion, this motion is indistinguishable from the thermal motion in its effect upon absorption coefficient and ν-dependence of S_s. We simply write the resultant mean square velocity:

$$\overline{V^2_{\text{tot}}} = \overline{V^2_{\text{therm}}} + \overline{V^2_{\text{micro}}} = \frac{kT_e}{m_i} + \overline{V^2_{\text{micro}}} \,. \tag{26}$$

We use the subscript « micro » in conformity with astronomical usage of the term « microturbulence » to describe this small-scale, non-thermal velocity component. The « microturbulence » differs from the thermal motion in two respects: it need (*) not vary with the atomic mass, m_i; it need not be isotropic.

The same type of analysis may, consequently, be used under this condition 3·3 as under condition 3·2. The difference is, that what was a check between several measures of thermal motion becomes now a comparison of V_{therm} and V_{micro}.

This intercomparison may be made between T_e determined from S_s and from the ν-dependence of I_ν. Or, it may be made between values of T_e inferred from either of these methods applied to ions of different mass (*e.g.* the investigations like those of Bell already cited in Section 3·2. If there is only a random component in the non-thermal motion, eq. (26) may be used to obtain V_{therm} and V_{micro}. An inferred difference between these several quantities may either be taken literally, or used as a basis to question the validity of the analytical methodology, from the standpoint of the uncertainties raised in Sections 3·1 and 3·2.

Several analyses of astrophysical data have produced results implying $V_{\text{micro}} > V_{\text{sound}}$, where V_{sound} is essentially V_{therm} for hydrogen, evaluated at what is assumed, in these analyses, to be T_e in the atmospheric region analysed (cf. STRUVE and ELVEY, 1934; UNSÖLD, 1929, WILSON and BAPPU, 1957). It has been objected that such results are physically inconsistent from a gas-dynamical standpoint (THOMAS, 1948) if the atmosphere is to be in a time-steady thermodynamic state—they would lead to a rapid mechanical energy dissipation and a rise in T_e. Therefore, either the assumed values of V_{therm} in the atmosphere are too low, or the analytical methodology underlying the results is incorrect.

Probably the most controversial aspect of results on « microturbulence », aside from the above results concerning supersonic microturbulent velocities, lies in the question of anisotropy *vs.* depth-dependence. These results come from study of weak and intermediate strength lines in the sun, where

(*) Most authors assume that the microturbulence obviously does not depend upon atomic mass—but in the case of such motion as gyromagnetic, the velocity varies with m. Such possibilities must be clarified.

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centerlimb data may be obtained. (Cf. Allen, 1949; Richardson and Schwarzschild, 1950; Huang, 1951; Suemoto, 1957; Waddell, 1958; Rogerson, 1959.) In essence, as one observes along the line of sight progressing from center to limb, he both observes at lesser effective depth and along a non-radial direction. The problem is to distinguish, in an inferred change in « microturbulent » velocity, between the depth variation and a possible anisotropy. An argument (cf. Waddell) in favor of anisotropy comes from the fact that if no anisotropy is assumed, but all effects laid to a depth variation, the parameters describing such depth variation are found to depend heavily upon the line chosen. In our opinion, this discrepancy may also arise from differential non-LTE effects (cf. Pecker and Vogel, 1960). Much more work needs to be done on these questions before we can consider that we have a clear-cut kinematical picture of the velocity fields actually existing. Again, detailed discussion is best deferred to the presentation of results in the following papers. Here, we only emphasize the point as an important one from the standpoint of the analytical methodology.

3·4. *Analysis of a line formed in an atmosphere where quite general macroscopic velocity fields are admitted.* – In essence, we have four kinds of velocity fields to consider. In addition to those already treated: (i) thermal and (ii) non-thermal but random over all dimensions larger than some scale much smaller than a photon free-path — we have: (iii) mass motion of some type other than (ii) but having no gradient horizontally, and (iv) horizontal gradients in the mass motion. If we had arbitrarily-good geometrical resolution, we could restrict our attention to types (i)-(iii), or motion in a narrow cylinder of gas. Uniform systematic motion of the cylinder does not alter any of the approach already discussed, the line is simply displaced as a whole. What requires to be discussed, is a vertical velocity gradient in the motions of type (iii). Lacking good geometric resolution, the effects of type (iv) broaden the line profile. For example, note the simple case of a collection of columns, within each of which the gas moves up or down as a whole.

Generally, in astrophysics, motion of type (ii) is called « microturbulence », and the term « macroturbulence » is rather loosely applied to a compound of (iii) and (iv). In formulating an analytical methodology, for discussing the effect of velocity fields upon spectral line profiles, it is important to distinguish carefully between types (iii) and (iv); such a distinction is more often blurred in the astrophysical literature than not. For example, Huang and Struve developed their method of line-width *vs.* equivalent-width correlation in terms of a situation resting upon motions of type (i)-(iii), then applied it to situations which included the effect of type (iv). Huang has kindly answered our inquiry on this point by stating that it was their intent that the method should be applied only in situations where the « macroscopic » types (iii) and (iv) do

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not seriously alter the profiles obtained from « microscopic » motions of types (i) and (ii) alone. We could only emphasize that analyses and comparisons of results from different kinds of analyses (such as discussions of line-profiles *vs.* discussions of equivalent-widths) must be done with a very clear picture of the kind of motions assumed; since, for example, the differential effects of « microscopic » and « macroscopic » motions upon line-profiles and curve of growth are appreciable.

Little formal work has been done on this problem of interpreting line-profiles for generalized velocity fields, aside from that by HUANG and STRUVE just cited, mainly because of the observational difficulties cited, noting our earlier remarks that a good discrimination of non-LTE and velocity effects usually depends upon the analysis of intensity differences of several percent in the line-center. Modern photoelectric work with good gratings now begins to make such discrimination a possibility. So, we first summarize the Huang-Struve approach, then add a few comments from the standpoint of the developments summarized earlier in the present paper.

3·4.1. The Huang-Struve approach. They orient their discussion around a distinction between « physical Doppler broadening » and « geometrical Doppler broadening ». The former represents whatever line-broadening would result from the velocity distribution within a column of gas lying below some surface element. The effect of superposing several columns of gas distributed over the surface of the star, they call geometrical broadening. This last is the observed quantity, which they write as

$$R(\lambda) = \int_{-\infty}^{\infty} B(\Delta\lambda) R'(\lambda - \Delta\lambda)\, \mathrm{d}(\Delta\lambda) \,. \tag{27}$$

The quantity $R'(\lambda - \Delta\lambda)$ represents the « physical Doppler broadening ». thus, an integral over depth, and $B(\Delta\lambda)$ represents the geometric integration effect—limb-darkening, variation in systematic mass-motion of columns, stellar rotation, etc. Thus, the basic assumption is that the quantity R' can be determined for an atmospheric model having no « geometrical » broadening. One then introduces various broadening functions, $B(\Delta\lambda)$, and attempts to match the observed profiles, $R(\lambda)$. (Cf. HUANG and STRUVE, 1953, for a discussion of various broadening functions, and the difficulty of distinguishing between these functions for several types of mass motion.)

In the situations that either the effect of $B(\Delta\lambda)$ greatly predominates over that of R', or conversely, the resultant R is essentially the predominant one of the two quantities. When B and R' are comparable in their effect, HUANG and STRUVE have attempted to separate their effects by studying the relation between equivalent width, half-width, and central intensity of the line (1952*a*, 1952*b*, 1955). In the first two papers, the methodology developed rests upon the implicit—and somewhat paradoxical—assumption that the macroscopic motion does not seriously alter the profile obtained from considering only microscopic motions; the third paper attempts to remove this limitation. Huang and Struve recognize the uncertainty introduced by uncertainty on the theory for the central intensity of the line (which we would re-emphasize on the basis of our discussion of S_s). All we can really say, is that the problem remains to be investigated from the standpoint of a complete theory of S_s and its effect.

These discussions by Huang and Struve direct their attention to the very practical problem of analysing the observed stellar spectra. The solar case, with its resolution of the disk, provides an easier problem. Therefore, we conclude with a summary of the methodology in the case of two simple types of motion within « columns » that can be resolved. We deal, then, with the problem of specifying the R' function of Huang and Struve.

3.4.2. Vertical gradient in mass-motion; only thermal random motions. In essence, the presence of a gradient in systematic motions exhibits itself as an asymmetry in line-profile. For illustration, continue the assumption that S_ν is ν-independent, and restrict attention to the Doppler core of the line. Then we have

$$\varphi_\nu = \varphi_0 \exp\left[-\frac{[\nu - \nu_0(1 + V(\tau)/c)]^2}{(\Delta\nu_D)^2}\right]. \tag{28}$$

Again for illustration, adopt a caricature version of the result of the linear relation of eq. (7); viz., assume that rigorously

$$I_\nu(0, \mu) = S_\nu(\tau_\nu = \mu) . \tag{29}$$

Then if $V(\tau)$ has everywhere the same sign (*i.e.* always the motion is up or down), we see that the points to which we « see » at equal distance from the center of the line differ. That is, picking points of equal $I_\nu(0, \mu)$ on opposite sides of the line-center, we do not go equal distances ($\Delta\nu$) from the line-center. If we label these points of equal intensity by ν^+ and ν^-, and if V were constant at all heights above that corresponding to I_ν, we would determine V from points of equal φ_ν in eq. (28) as

$$V/c = [(\nu^+ - \nu_0) - (\nu_0 - \nu^-)]/2\nu_0 . \tag{30}$$

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Since $V(\tau)$ is not necessarily constant, one must proceed by successive approximation, but the principle is the same. Indeed, it is very similar to that discussed in Section 3·1 for determining $\Delta\nu_D$. Were these simplified assumptions satisfied, there would be no serious problem in determining $V(\tau)$ (cf. Fig. 10).

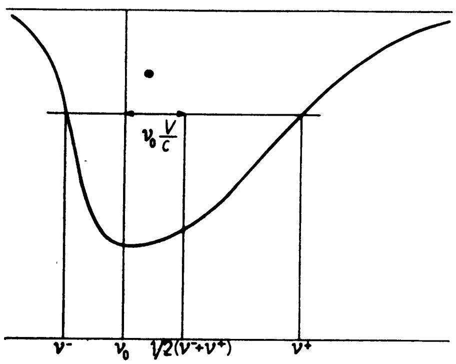

Fig. 10. – Vertical velocity gradient; eq. (30).

There are two kinds of difficulties associated with departure from the simplified assumptions of the last paragraph. First, there is the problem of inverting the integration of S_ν over τ_ν, to replace the simplified relation (29). The most direct approach is to investigate line-profiles via models of $S_\nu(\tau_\nu)$. For the LTE assumption, the procedure is fairly straightforward; for one uses the same function $T_e(\tau_c)$ for all lines. Then it is a question of investigating $V(\tau_c)$. The second problem is the more serious—if one does not assume LTE, what kind of function $S_\nu(\tau)$ is to be used in proceeding, even by the method of models?

One procedure is to attempt to duplicate the procedure sketched in Section 3·1 for an empirical determination of $S_\nu(\tau)$, based upon the assumption that several lines having the same S_ν can be found. It must first be shown that there are such lines. Second, since there are now two unknown functions to determine—$\Delta\nu_D(\tau)$, $V(\tau)$—more than two lines must be found satisfying the condition. Or, center-limb variations must be used, in the manner outlined in Section 3·1.

Thus, in any event, the problem comes down to discussing the question of S_s in an atmosphere with macroscopic velocity fields. We have already sketched the existing theoretical approach to S_s, in an atmosphere having only thermal motions, centered around eq. (13) and (14) as a first approximation. Consider quickly the modifications introduced by the presence of macroscopic velocity fields.

The formal procedure associated with the eq. (12) and (14) lies in investigating $S_s(\tau_0)$ in an atmosphere having some theoretically-prescribed distributions $T_e(\tau_0)$, $n_e(\tau_0)$. Then, one solves the equation of radiative transfer, using eq. (12) and (14) and these distributions. Thus, the changes in $S_s(\tau_0)$ which might be expected to occur in an atmosphere having macroscopic velocity fields as compared with one having thermal motions only are of two types.

First, there may occur a significant change in the $T_e(\tau_0)$, $n_e(\tau_0)$ which we would prescribe from wholly theoretical considerations, this change resulting from the added energy input from mechanical dissipation of the macroscopic velocity field. In addition to changing the details of the $S_s(\tau_0)$ distribution, this change in $T_e(\tau_0)$ may well change the *type* of S_s for a given line from the photoionization-dominated type (S_s largely independent of T_e) to the collision-dominated type (S_s dependent upon $T_e(\tau_0)$). Fixing n_e, T_e from a wholly empirical determination eliminates this problem.

Second, the presence of the macroscopic velocity field alters the opacity within the line, as given by eq. (30). To see the effect, we digress briefly on the method of solution of the transfer equation, using eq. (12) and (13). (Cf. JEFFERIES and THOMAS, ibid.) Since the resulting equation of transfer is an integro-differential equation, some method of algebric quadrature must be applied to the integral over ν in eq. (14). In the case of wholly-thermal motions, I_ν is symmetric about the line-center. Further, φ_ν falls off so rapidly with increasing $\Delta\nu$ that the investigations thus far completed have assumed it sufficient to treat only the Doppler core. The quadrature is then straightforward. The asymmetry introduced by the macroscopic velocity field, however, requires separate treatment of the two sides of the line, thus doubling the number of quadrature points, and introducing a more-elaborate depth dependence of φ_ν. The problem has not been touched to date.

3·4.3. All macroscopic motions random over a scale larger than some dimension smaller than a photon free-path. In thinking through the analytical approach to an analysis of the velocity field, we have essentially two alternative conceptual points: *a*) S_s has some given geometrical distribution, not dependent upon the particular line analysed—*e.g.*, LTE; *b*) the distribution of S_s is a strong function of τ_0, the opacity in the center of the line, and possibly the distribution of opacity about the line center.

a) S_s has a given geometrical distribution, not dependent upon properties of the line analysed.

Here we recognize that the essential quantity fixing I_ν is how geometrically-deep in the atmosphere do we see it at a given $\Delta\nu$ from the line-center. That is, how geometrically-deep must we go before encountering along the line of sight enough atoms having velocity V, where

(31) $$\Delta\nu = \nu_0 V/c\,,$$

to build up $\tau(\Delta\nu) \sim 1$. If we forget for the moment natural and collisional broadening of a line, we then see that in discussing line formation under this case *a*), the velocity distribution function of direct interest is not at a given point in the atmosphere. Rather, we want the function $\tau_c(V)$, which is the distance down into the atmosphere one must go before encountering N atoms,

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in the lower level of the line considered, having velocity V along the line of sight. Here, N is the same for all V, simply being given by

$$\frac{\pi\varepsilon^2}{mc} f_{12} N \sim 1 . \tag{32}$$

Clearly, $\tau_c(V)$ results from the integration of the distribution functions at a point, but these last are not the quantities of direct interest, nor are they obviously the easiest in which to formulate a description of random motions of varying scale. Given $\tau_c(V)$, we immediately have $\tau_\nu(V)$ from eq. (31) and the (assumed known) value of abundance of ion in question to the source of continuous opacity. Thus, we may integrate eq. (1) and obtain I_ν. Conversely, if we know $S_s(\tau_c)$, we may invert an observed I_ν to obtain $\tau_c(V)$.

The results from this inversion do not give the velocity distribution function at a point. To obtain this, one must analyse several lines, having different f_{12} values, then take the difference of the derived $\tau_c(V)$.

The actual presence of natural and collisional broadening must be included. To compute the relevant collision rates, we require an average over the local velocity distribution function. Since this last is *a priori* unknown, one must derive it as in the last paragraph, then iterate the procedure.

b) S_s depends upon τ_0 for the line considered, and possibly upon the distribution of opacity about the line-center. We return again to this question of the influence of velocity fields on the derived S_s. We have already commented on the two aspects changing the distribution derived for a quiescent atmosphere—a possible rise in T_e because of aerodynamic dissipation effects, and a change in opacity above a given geometrical point. There is no need to belabour the point, particularly since no work has been done on it.

Consider an extreme example, a column of gas consisting of two parts, one lying above the other, in relative motion at a speed large with respect to the internal thermal motion—we suppose there is no other motion. Now a photoionization-controlled S_s for a very strong line varies, over the central core, only with τ_0. Therefore, let each part of the column have thickness $\tau_0 \gg 10^4$, but not so great as to be opaque in the ionization continuum of the transitions considered. Then, if we consider the relative speed to correspond to, say, ten Doppler half-widths, the cores of the resulting two lines will be well-separated, and have the same profile, and intensity. As the relative speed decreases, the cores begin to merge, and the distributions $S_\nu(\tau_0)$ begin to be fixed by the conditions in the two parts of the column together, rather than there being two distinct parts of the column and two distinct line cores. The point which we would make here, is simply that a discussion of micro- and macroturbulence, in the usual sense, applied to a single column of gas requires a detailed discussion of the form and behavior of S_s.

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