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Stability and efficiency properties of implicit Runge-Kutta methods

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This thesis is divided into two sections. The first section examines certain stability properties of implicit Runge-Kutta methods. In particular, two new stability properties are defined. The first is a modification to non-autonomous problems of the property of A-stability (introduced by Dahlquist [3]), and the second is a similar modification of B-stability (introduced by Butcher [1]). A comparison is made of these two stability criteria and it is shown that under certain mild conditions (namely that the abscissae of the Runge-Kutta method be distinct) these two concepts are equivalent, thus relating a stability property for nonlinear problems directly to a stability property for linear problems. A family of high order methods is constructed and examined in the light of these new stability criteria, and comparisons are made with the A-stability properties of this family. Finally in this section, it is shown that the growth of errors can be estimated by an extension of this new stability theory and a number of examples are given.

In the second section the efficient implementation of a certain family of Runge-Kutta methods, based on a technique due to Butcher [2], is considered. Butcher has shown that when using this technique the most efficient methods are those whose Runge-Kutta matrix has a single real eigenvalue. Based on this criterion a family of methods, called singly-implicit methods, is constructed and results concerning their maximum attainable order and stability properties are given. In particular, it is shown that an s-stage singly-implicit method can always attain order $s + 1$, but that there are no algebraically stable singly-implicit methods

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with more than three stages. Some consideration is also given to showing how local error estimates can be obtained, by the use of embedding techniques, for both singly-implicit methods and the more general family of implicit Runge-Kutta methods. Finally an algorithm called STRIDE, based on these singly-implicit methods, is presented and tested on a number of stiff differential equations. Since Gear's algorithm is recommended for the solution of all types of stiff problems, this has been included in our tests for the purpose of comparison. Although more testing needs to be done, it appears that STRIDE does have a future as an efficient solver of stiff problems, and that furthermore, it is now possible to implement a Runge-Kutta package with the same efficiency as other existing programs, such as Gear's.

References

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