THE ZZ CETI STARS AND THE RATE OF EVOLUTION OF WHITE DWARFS

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I. INTRODUCTION

The importance of the ZZ Ceti stars, and indeed the importance of all pulsating stars, derives from the fact that stellar pulsations probe the interiors of stars, and thus they test directly our models of stellar interiors and stellar evolution. The relative value of stellar pulsations as such a probe depends on, among other factors, the number of pulsation modes simultaneously excited in a star, as each additional mode depends on and constrains the properties of the star in a different way. Judged by this criterion, the pulsations of the ZZ Ceti stars should be unusually valuable because all ZZ Ceti stars are multi-mode variables. For example, among the ZZ Ceti stars with well studied light curves, the one with the fewest modes is R548 (= ZZ Ceti itself) with 4 pulsation modes simultaneously excited (Robinson et al. 1976), while some of the other ZZ Ceti stars can have dozens of pulsation modes simutaneously excited (cf. Robinson 1979).

A problem that has hindered the full use of these pulsations is the difficulty in identifying the modes. It does appear certain that the modes cannot be radial modes. Since the ZZ Ceti stars are otherwise normal white dwarfs, the periods of their radial pulsations would be less than perhaps 20 sec (Ostriker and Tassoul 1969), whereas the periods actually observed range from about 109 sec in G226-29 (McGraw, private communication) to 1186 sec in GD 154 (Robinson et al. 1978). In addition, it does appear certain that two groups of non-radial pulsations can have periods in the observed range: The g-mode pulsations and the r-mode pulsations (Cox 1976, Papaloizou and Pringle 1978). Unfortunately, within both of these groups there is such an enormous multiplicity of pulsation modes with periods in the correct range that the specific mode indices (the spherical harmonic indices, ℓ and m; and if the pulsations are g-modes, the radial overtone index, k) of the observed pulsations cannot yet be specified uniquely. Without these specific mode identifications the pulsations cannot be compared to theoretical predictions. At the same time, it also appears

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likely that the mode identifications cannot be made on purely observational grounds; some clues as to which modes are most likely to be excited must be provided by theoretical models. Thus, there is a knotted circularity in the mode identification process which will take some time to disentangle correctly.

Despite these difficulties, significant progress can be made if we look not at the pulsation periods themselves, but at the rate of change of the periods, $\dot{P}$. For white dwarfs, the $\dot{P}$ of g-modes can be related directly to the rate of cooling of the white dwarf, and is nearly independent of the specific pulsation mode. Thus, if the pulsations are g-modes the ZZ Ceti stars can be used to measure directly the cooling time scales of white dwarfs. These cooling time scales are critical numbers in the theories both of white dwarf structure and, by way of the white dwarf luminosity function, of the history of the stellar birth rate in the galaxy (c.f. Liebert 1980). Alternatively, if the pulsations of the ZZ Ceti stars are r-mode pulsations, $\dot{P}$ can be related directly to the rate of change of the rotation periods of white dwarfs. In this case, the ZZ Ceti stars will help to solve the problems of why white dwarfs rotate so slowly and when in their evolution they lose their angular momentum (Greenstein et al. 1977).

II. PERIOD CHANGES OF ZZ CETI STARS

i) Observed Period Changes

Accurate measurements of period changes have been made for only two ZZ Ceti stars, R548 and G117-B15A, and in both stars the period changes are less than the detection limit. The data are given in Table 1.

TABLE 1

The Pulsations of R548 and G117-B15A

Star	Pulsation Amp.(Mag)	Pulsation Period(Sec)	Upper Limit to $\dot{P}$ (Sec/sec)
R548	.0044	212.77	7×10^{-13}
	.0077	213.13	2×10^{-13}
	.0049	274.25	3×10^{-13}
	.0034	274.77	9×10^{-13}
G117-B15A	.0018	107.6	
	.0014	119.8	
	.0016	126.2	
	.025	215.20	6.5×10^{-14}
	.0077	271.0	
	.0090	304.4	

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The four pulsations of R548 are arranged in two close pairs, one close pair with periods near 213 sec and the other close pair with periods near 274 sec. According to Robinson et al. (1976) the periods demonstrate that R548 is pulsating in g-modes, but even for this well studied star the exact mode indices are uncertain. The most recent determination of the upper limits to the rates of change of the pulsation periods yielded $|\dot{P}| < 2 \times 10^{-13}$ for the 213.13 sec pulsation and $|\dot{P}| < 3 \times 10^{-13}$ for the 274.25 sec pulsation, these two having the largest amplitudes of the four pulsations (Stover et al. 1980). The time scale for period changes of both pulsations is $|P/\dot{P}| \gtrsim 10^{15}$ sec $\simeq 3 \times 10^{7}$ yr. In order to achieve this limit, 58 nights of photometry covering a time baseline 8 years long (9 seasons of data) were required.

The second star, G117-B15A, has six pulsation modes simultaneously excited in its light curve, but one of the six, with a period of 215.20 sec, has an amplitude significantly larger than any of the others and dominates the light curve. There are some striking relationships among the frequencies of the pulsations of G117-B15A. If the frequency of the single large amplitude pulsation is called ν_0 (=.004647 Hz), the remaining pulsation frequencies observed in G117-B15A can be generated by the following numerological steps.

1. Excite a second pulsation at precisely 3/4 ν_0.
2. Split the 3/4 ν_0 pulsation into two components at 3/4 $\nu_0 \pm \Delta\nu$, with $\Delta\nu$ = .000205 Hz. The splitting is small in the sense that $\Delta\nu/\nu_0 << 1$.
3. Add precisely ν_0 to ν_0 itself, and to the two frequencies generated in steps 1 and 2, and excite pulsations at these three new frequencies.

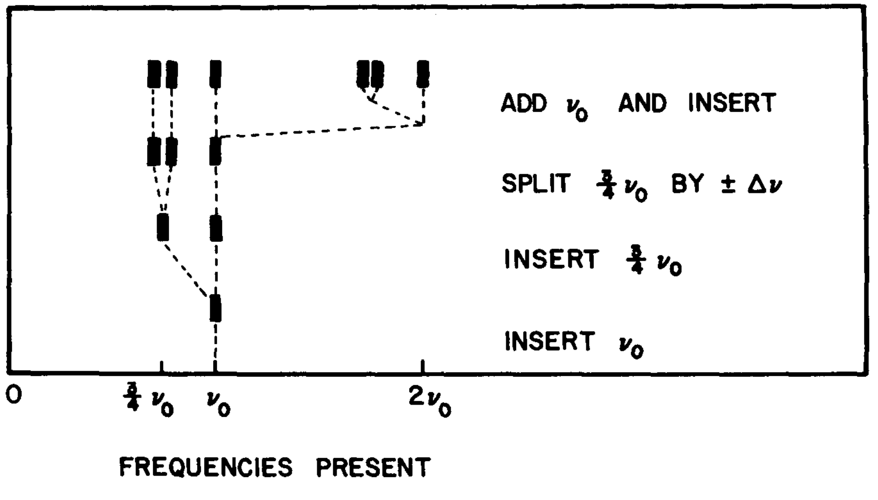

Fig. 1 - This figure displays in graphical form the sequence of numerological operations that generate the observed pulsation frequencies of G117-B15A. The generated and observed frequencies are compared in Table 2, and agree to ~1 in 1000.

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Figure 1 displays these surprisingly simple operations in graphical form; the numerical values of the generated frequencies are compared to the observed frequencies in Table 2. The agreement between the

TABLE 2

Relations Among the Pulsation Frequencies of G117-B15A

Relation*	Calculated (HZ)	Observed (HZ)
$3/4\ \nu_o - \Delta\nu$	.003280	.003285 ±5
$3/4\ \nu_o + \Delta\nu$	.003690	.003690 ±5
ν_o	.004647	.004647 ±0
$3/4\ \nu_o - \Delta\nu + \nu_o$	.007927	.007925 ±5
$3/4\ \nu_o + \Delta\nu + \nu_o$	.008337	.008345 ±5
$\nu_o + \nu_o$	.009294	.009295 ±5

* $\Delta\nu$ is .000205 Hz

generated and observed frequencies is accurate to about 1 part in 1000, and is within the observational error.

The steps used to generate the pulsations create considerable interpretive problems. If all of the pulsations are g-modes, it is difficult to understand the first step. To excite a g-mode pulsation at precisely 3/4 the frequency of the driving pulsation requires non-linear coupling of the two pulsation modes, but supporting evidence for non-linear coupling is lacking. There are, for example, no period or amplitude variations of the pulsations. If all of the pulsations are r-mode pulsations it is easy to excite pulsations with frequency ratios of 3/4, for example by choosing $m = 3$ and $m = 4$ with $\ell \geq 4$ (see equation 4), but the splitting of the pulsations into two components in step 2 becomes difficult to understand. Since we know that the pulsations of R548 are g-modes, and since properties of G117-B15A are similar to those of R548, at least some of the pulsations of G117-B15A are probably g-modes also; but it may finally be necessary to invoke some combination of both r-modes and g-modes to understand G117-B15A completely.

Kepler et al. (1980) have measured the upper limit to $\dot{P}$ for the strong 215.20 sec pulsation of G117-B15A and have found $|\dot{P}| < 6.5\times10^{-14}$ at the 68 percent confidence level, which yields $|P/\dot{P}| \geq 3.3\times10^{15}$ sec $\approx 1\times10^{8}$ yr. The data on G117-B15A included 36 nights of high-speed photometry covering a time baseline 5 years long.

ii) Expected Period Changes

The pulsation periods of the g-modes of a white dwarf depend primarily on the luminosity of the white dwarf if the star is slowly rotating. According to Osaki and Hansen (1973) the periods of the g-modes of white dwarfs with luminosities in the range $10^{-4} < L/L_\odot < 10^{-2}$ are given by

$$\log P = \alpha - \beta \log (L/L_\odot), \quad (1)$$

where α depends on the specific mode excited, but β is nearly independent of both the pulsation mode and the white dwarf mass (see Unno et al. [1979] for a discussion of this point). Values of β have been calculated by Osaki and Hansen (1973) and lie between .17 and .18 for $\ell = 2$ modes of white dwarfs with masses between .4 $M_\odot$ and 1.0 $M_\odot$. With these values of β, equation (1) yields

$$\frac{P}{\dot{P}} = \frac{-1}{\beta}\frac{L}{\dot{L}} \simeq -5.7\,\frac{L}{\dot{L}} . \quad (2)$$

The cooling time scale, $L/\dot{L}$, is itself a function of the luminosity of the white dwarf. The luminosities of the ZZ Ceti stars are all near $10^{-2.5}$ $L_\odot$ (McGraw 1979), and near this luminosity white dwarfs cool as $L \propto t^{-3/2}$ (Shaviv and Kovetz 1976). The cooling time scale is, then $L/\dot{L} = \gamma(L/L_\odot)^{-2/3}$, where γ is a proportionality constant equal to about $6\text{x}10^6$ yr for a 50% C^{12} and 50% O^{16} white dwarf, and is inversely proportional to the mean atomic weight of the core chemical composition. Equation (2) now becomes

$$\frac{P}{\dot{P}} \simeq 3.3\text{x}10^7 \left(\frac{L}{L_\odot}\right)^{2/3}, \quad (3)$$

and for the ZZ Ceti stars we expect the periods of the g-modes to vary on a time scale of $P/\dot{P} \sim 1.5\text{x}10^9$ yr.

According to Papaloizou and Pringle (1978) the pulsation frequencies, σ_r, of the r-mode pulsations are given by

$$\sigma_r = -m\Omega + \frac{2m\Omega}{\ell(\ell+1)}, \quad (4)$$

where Ω is the rotation frequency of the white dwarf. The rate of change of the pulsation period is given by

$$\frac{P}{\dot{P}} = -\frac{\sigma_r}{\dot{\sigma}_r} = -\frac{\Omega}{\dot{\Omega}}, \quad (5)$$

and therefore the pulsation period changes only if the rotation period changes.

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The rotation period of a white dwarf need not be constant. In particular, if the white dwarf has a magnetic field, several mechanisms may be envisioned for changing the rotation period on a short time scale. One mechanism is a stellar wind. If the white dwarf has a stellar wind and if it always rotates uniformly, its rotation frequency will change according to

$$\frac{\Omega}{\dot{\Omega}} = \left(\frac{gR}{R_D}\right)^2 \frac{M}{\dot{M}}$$

where M and R are the mass and radius of the white dwarf, $\dot{M}$ is the mass loss rate, R_D is the effective distance from the white dwarf at which the wind decouples from the white dwarf, and g is the radius of gyration and is typically about .5 for a white dwarf. This wind can be singularly effective at removing angular momentum from the white dwarf and slowing its rotation. Assume that the magnetic field of the white dwarf drops off as r^{-3}, and that R_D is approximately the distance at which the field becomes equal to the magnetic field of the interstellar medium, typically about 10^{-6} gauss. Then if the magnetic field has a strength of only 10^3 Gauss at the surface of the white dwarf, $\Omega/\dot{\Omega} \sim 10^{-5}\ M/\dot{M}$. Mass loss rates as low as $10^{-14}\ M_\odot\ yr^{-1}$ give $\Omega/\dot{\Omega} \sim 10^9$ yrs, so that even with these exceedingly small magnetic fields and mass loss rates the rotation period will change on time scales comparable to the cooling time scales of white dwarfs.

III. DISCUSSION

In summary, the results of the previous section are

1) The observed limits on $P/\dot{P}$ are $|P/\dot{P}| > 3\times10^7$ yr for R548, and $|P/\dot{P}| > 1\times10^8$ yr for G117-B15A.
2) The expected value of $P/\dot{P}$ is about 1.5×10^9 yr if the pulsations are g-modes. The expected value is indeterminant if the pulsations are r-modes, but could easily be as small as a few times 10^9 yr.

Thus, for the most favorable case, that of g-modes in G117-B15A, the observed lower limit to the rate of change of the period is a factor of 15 less than the expected rate of change. The observations do not yet challenge theory.

In the case of R548 there is little hope that the observational limits of $P/\dot{P}$ can be improved to the values expected from theory without a radical change in observing techniques, but in the case of G117-B15A there are grounds for considerable optimism that $P/\dot{P}$ can be improved rather easily to these interesting levels. Using available observational techniques, there are two basic ways to increase the lower limit to $P/\dot{P}$ in G117-B15A. The first way is simply to continue to observe the star for several more years. Since the accuracy with which $P/\dot{P}$ can be measured improves as the square of the time baseline over which a star is observed, and since the time baseline for G117-B15A is 5 years long, a total of 20 years would be needed to improve the limit on $P/\dot{P}$ by a factor of 15. Five of the 20

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years have been accumulated already, so that another 15 years of observations would be required. The second way to improve the limit on $P/\dot{P}$ is to improve the accuracy of measurement of the times of arrival of the pulsations of G117-B15A. In practice this amounts to increasing the amount of time spent observing G117-B15A each year. The accuracy with which the times of arrival can be measured, and therefore the accuracy with which $P/\dot{P}$ can be measured improves as the square root of the number of hours spent observing per year. This method would, then, require about 225 times the amount of observing time already invested to improve $P/\dot{P}$ by a factor of 15. G117-B15A has actually been observed an average of about 14 hours per year, so if the same telescope and instruments are used, more than 3000 hours of observing per year would be required.

Clearly neither method alone is sufficient: the one requires us to wait too long to measure $P/\dot{P}$, while the other requires us to invest too much telescope time. But by judiciously combining the two methods these restrictions can be avoided. It is reasonable, for example, to increase the number of hours spent observing G117-B15A by a factor of 4, as telescopes of only small and moderate size are needed for the observations. If this is done, observational limits on $P/\dot{P}$ as large as 1.5×10^9 yr can be achieved in less than 10 years, an eminently reasonable length of time. We conclude, therefore, that the prospects for improving the accuracy of measurement of $P/\dot{P}$ to interesting values are excellent.

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DISCUSSION

A. COX: In G117-B15A, you get the period ratio of 0.75, but one of the modes has been split. Any idea about that?

ROBINSON: No, it's a nasty star. If you say they are g-modes, then you have to invoke some sort of coupling to get 0.75. It doesn't just pop out. There is just no evidence that there is any kind of nonlinearities that would lead to coupling. On the other hand, if you say that all three of these modes are r-modes, then it is not clear how you can split an r-mode. As far as I know, you can't. I think that probably the thing to worry about here is some combination of g-modes and r-modes. In fact, I would like to make it clear that I believe quite strongly that we are dealing with g-modes in the vast majority of cases. I do not think it is wise to absolutely ignore the possibility that there is an admixture of r-modes in these stars.

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