A RELATIVISTIC THEORY OF MOTION OF THE INNER PLANETS

E.L.Akim, V.A.Brumberg, M.D.Kislik,
Ju.F.Koljuka, G.A.Krasinsky, E.V.Pitjeva,
V.A.Shishov, V.A.Stepanianz, M.L.Sveshnikov
and V.F.Tikhonov

ABSTRACT. The theory of the motion of Mercury, Venus, Earth and Mars is constructed by numerical integration. The theory takes into account relativistic corrections in the frame of Schwarzschild's space-time metrics. The constants of the theory are determined by discussion of the Soviet and American radar observations of Mercury, Venus and Mars, position astrometric observations of these planets (and the Sun) and observations of the Soviet artificial satellites of Venus. Apart from the planet elements the value AU and corrections to the adopted radii of Mercury, Venus and Mars are determined. Statistics of residuals is given and accuracy of the theory is estimated. The theory presented is an ephemeris base for deep space experiments.

This work was jointly prepared by the specialists of the Mission Control Center, the Institute of Applied Mathematics named after Keldysh and the Institute of Theoretical Astronomy.

The creation of the theories of the Solar System bodies motion is the major and the ancient problem of Theoretical Astronomy. In late nineteenth and in the first half of the twentieth century Newcomb, Clemence and others constructed sufficiently advanced theories for the inner planets (Mercury, Venus, Earth and Mars) that became the basis of National astronomical almanacs including the U.S.S.R. Astronomical Year-Book. These theories are atributed to classical ones. To describe the planets motion they make use of the approximate analytical methods of motion equations integration. The classical theories accuracy evaluation based on the treatment of the results of long-time optical observations of the planets and the Sun has shown that they enable us to calculate the planets coordinates with

J. Kovalevsky and V. A. Brumberg (eds.), Relativity in Celestial Mechanics and Astrometry, 63–68.

errors of several hundreds of kilometers.

The development of space technology gave rise to the direct planets investigation with the aid of automatic interplanetary stations in which Venus and Mars were among the first. To make these investigations possible the accuracy of planets ephemerides had to be considerably increased. The real possibility of developing the new highly-accurate theories of inner planets motion appeared due to availability in I96I and further rapid development of radar planetary astronomy. The distances from the Earth (the signal time delay) to the neighbouring planets started to undergo measurements with errors of several kilometers and even with higher precision by two or three orders exceeding the equivalent precision of angular measurements typical for optical observations. The U.S.S.R. made radar measurements with the aid of planetary radar designed by the Institute of Radio Technics and Electronics of the U.S.S.R. Academy of Sciences under assistance of several institutions on the technological basis of the Deep Space Communication Center located in the Crimea. These observations made during I962-I982 constitute the basis of the astrometric data bank. While developing the theory together with radar observations use was also made of the optical observations of the planets and the Sun despite their relatively low accuracy. This is explained by the fact that by means of optical observations only one can determine three angular parameters characterising the orientation of a planets configuration with respect to stars at each given epoch. The Optical measurement data used for the new theories development were acquired in the Soviet Union primarily by the Nikolaev Astronomical Observatory of the U.S.S.R. Academy of Sciences. During I964-I977 there had been made more than twenty two hundred observations of the Sun, Venus, Mars and Mercury.

To form the astrometrical data bank use was also made of the high accuracy observation results of the soviet automatic interplanetary Space stations "Venera-9" through "Venera-I6" motions. Furthermore, we included the radar observations of planets which had been made in the United States till I97I together with optical observations made in Gt.Britain and in the U.S.A. The data bank features are given in Table I.

The methods of constructing the relativistic theory of inner planets motion is connected with combined determination of the osculating orbital elements of Mercury, Earth, Venus, Mars, as well as of radii of Mercury and Venus, the equatorial radius of Mars, the astronomical unit and the dynamic oblateness of the sun. The data were processed by the least squares iteration Method. To increase the accuracy of agreement of calculated and experimental data the information on the Venúsian and Martian to-

https://doi.org/10.1017/S0074180900147990 Published online by Cambridge University Press

pography has been introduced while processing the radar observations.

To describe the heliocentric motion of the inner planets use is made of relativistic differential equations containing in their right hand sides along with the Newtonian terms the relativistic ones conditioned by the Schwarzschild field for the sun.

Equations of Motion .
(heliocentric standard relativistic coordinate system is used)
a) for Earth - Moon barycenter (3)

$$\ddot{\bar{r}}_3 = -k^2(1+m_3)(1+m_{☾})\left\{\frac{\bar{r}_3 + m_{☾}(\bar{r}_3 - \bar{\rho}_{☾})}{|\bar{r}_3 + m_{☾}(\bar{r}_3 - \bar{\rho}_{☾})|^3} + m_{☾}\frac{(1+m_{☾})\bar{r}_3 + \bar{\rho}_{☾}}{|(1+m_{☾})\bar{r}_3 + \bar{\rho}_{☾}|^3}\right\} + \frac{\partial R^{(3)}}{\partial \bar{r}_3} + \bar{F}^{(3)}_{REL} + \bar{G}^{(3)}_{\odot}$$

$$(\bar{\rho}_{☾} = \bar{r}_{earth} - \bar{r}_{moon})$$

TABLE I. OBSERVATIONS

Place of observations	The measurement interval			
	Sun	Mercury	Venus	Mars
		Radar data		
Crimea		1980,1982	1962-1982	1971,1980,1982
Millstone Hill		1964	1964-1966	
Goldstone			1965-1967	1969
Arecibo		1964-1965	1964-1965	1965
Haystack				1967-1971
		Optical data		
Washington	1960-1971	1960-1971	1960-1971	1960-1971
Greenwich		1960-1973		
Nikolaev	1964-1976	1970-1976	1964-1976	1964-1976
Pulkovo			1964-1976	1964-1976
		Tracking data of "Venus 9-16"		
Crimea		1975 - 1984		

Number of observations

Radar	Optical	Tracking	Total
4625	8131	1080	13836

https://doi.org/10.1017/S0074180900147990 Published online by Cambridge University Press

b) for Mercury (I), Venus (2), Mars (4)

$$\ddot{\bar{r}}_i = -k^2(1+m_i)\frac{\bar{r}_i}{r_i^3} + \frac{\partial R^{(i)}}{\partial \bar{r}_i} + \bar{F}^{(i)}_{REL} + \bar{G}^{(i)}_{\odot} \qquad (i=1,2,4)$$

$$\left.\begin{aligned}
\frac{\partial R^{(i)}}{\partial \bar{r}_i} &= k^2\sum_{\substack{j=1\\ j\neq i}}^{8} m_j\left(\frac{\bar{r}_j-\bar{r}_i}{r_{ij}^3} - \frac{\bar{r}_j}{r_j^3}\right)\\
\bar{F}^{(i)}_{REL} &= \frac{k^2}{c^2 r_i^3}\left[2k^2\frac{\bar{r}_i}{r_i} - 2(\dot{\bar{r}}_i)^2\bar{r}_i + 3\frac{(\bar{r}_i\dot{\bar{r}}_i)^2}{r_i^2}\bar{r}_i + 2(\bar{r}_i\dot{\bar{r}}_i)\dot{\bar{r}}_i\right]\\
\bar{G}^{(i)}_{\odot} &= \frac{3}{2}\,\frac{k^2 J_{2\odot} a_{\odot}^2}{r_i^4}\left[\left(\left(5\frac{\bar{r}_i}{r_i}\bar{P}_{\odot}\right)^2-1\right)\frac{\bar{r}_i}{r_i} - 2\left(\frac{\bar{r}_i}{r_i}\bar{P}_{\odot}\right)\bar{P}_{\odot}\right]
\end{aligned}\right\} \quad (i=1,2,3,4)$$

$$|\bar{P}_{\odot}| = 1$$

The simultaneous integration of equations of motion of planets, the Moon and the spaceprobes has been done by the numerical method. In this case the calculation errors of the planets coordinates at the time interval of several decades don't exceed IO meters. The coordinate time t^K related to the Earth observer proper time t by the relation

$$\left(\frac{dt}{dt^K}\right)^2 = 1 - \frac{2k^2}{r_{st}} - \frac{(\dot{\bar{r}}_{st})^2}{2c^2}$$

is employed as an independent variable. During the observations results treatment at a calculated time of a back signal delay τ the corrections taking into account the relativistic radar effect are being introduced.

Formulas are the following:

$$\tau = \frac{|\bar{r}_i(t_2)-\bar{r}_{st}(t_1)| + |\bar{r}_i(t_2)-\bar{r}_{st}(t_3)| - 2a_i}{c} + \Delta\tau_{REL}$$

$$\Delta\tau_{REL} = \frac{2k^2}{c}\left[2\ln\frac{r_3+r_i+r_{3i}}{r_3+r_i-r_{3i}} + \frac{1}{2}\,\frac{(r_3+r_i)(r_i-r_3+r_{3i})(r_i-r_3-r_{3i})}{r_i r_3 r_{3i}}\right]$$

The values of osculating elements of planetary orbits and the up-dated constants resulting from the problem solution are shown in Table II.

The detailed analysis of the character and deviations value of the measured distances and angles from their calculated values corresponding to the developed theory gave the following results (See Table III). At all stages of measurement span of 25 years (I960-I984) there are practically no systematic deviations. Mean square root deviations of radar distances starting from I967 are equal

https://doi.org/10.1017/S0074180900147990 Published online by Cambridge University Press

TABLE II. RESULTS OF SOLUTION

Osculating orbital elements (q_i) at JED 2437000.5. Mean equator and equinox of I960.0

q_i	Mercury	Venus
a (a.u.)	0.3870983633	0.723330I2II
e	0.20563I2560	0.006773924I
ω (rad.)	I.I6992I037I	2.I62I956I87
i (rad.)	0.4990326326	0.42687I42I4
Ω (rad.)	0.I900688I63	0.I393904426
τ (JED)	2436987.344405II	2436908.09402323
q_i	Earth-Moon	Mars
a (a.u.)	0.9999904567	I.5237219705
e	0.0I67483794	0.0932936483
ω (rad.)	I.7830424234	5.7985896472
i (rad.)	0.409I8I6208	0.4309I30058
Ω (rad.)	6.283I832723	0.0585769555
τ (JED)	2436937.03239746	2436394.05690967

Improved astronomical constants

A(km)	$R_{☿}$ (km)	$R_{♀}$ (km)	$R^{e}_{♂}$ (km)
I49597867.3±0.3	2434.9±I.I	6050.I±0.I	3394.6±0.3

$J_{2\odot}$
$0.2\cdot10^{-6}\pm 0.2\cdot10^{-5}$

to 0,5 km for Venus, to 2 km for Mercury and to I km for Mars. Mean square root deviations of optical measurements are within the range of 0,6-I,2 arc seconds. Mean square root deviations of distances during the observation of the automatic interplanetary station motion are equal to 2 km.

By developing the inner planets motion theory aimed at evaluating the possibilities of Newtonian mechanics the processing of all measuring data has been conducted in Newton's version. As previously anticipated, the agreement of measured and calculated distances considerably

TABLE III. POST-FIT RESIDUALS

Final standard deviation for relativistic theory

Body	Radar and tracking data		Optical data	
	interval	σ_D (km)	interval	$\sigma_{\alpha,\delta}$
Sun	-	-	I960-I976	0.82"
Mercury	I964-I965 I980-I982	9 2	I960-I976	I.00"
Venus	I962-I969 I970-I982	4 0.5	I960-I976	I.I7"
Mars	I965 I967-I982	3.5 I	I960-I976	0.60"
Spacecrafts "Venus 9-I6"	I975-I984	2	-	-

Maximum residuals (D_O-D_C) for Newtonian solution

Mercury	Venus	Mars
390 km	8 km	I2 km

worsened. The systematic deviations reaching 390 km for Mercury, 8 km for Venus and I2 km for Mars took place at some parts of measurement interval.

Thus, the application of Newtonian mechanics for these purposes is unreasonable.

Good agreement of experimental and calculated data derived during the relativistic theory development represents the experimental verification of the general relativity by astronomical methods being of global nature and covering all possible relativistic effects of planetary motion and light propagation within the frame of the Schwarzschild problem solution for the Sun.