Black Holes from Stars to Galaxies – Across the Range of Masses
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Radiation in models with cosmological constant

Hedvika Kadlecová and Jiří Podolský

Institute of Theoretical Physics, Faculty of Mathematics and Physics, Charles University, Prague, Czech Republic

email: Hedvika.Kadlecova@centrum.cz, podolsky@mbox.troja.mff.cuni.cz

Abstract. We analyze the asymptotic directional structure and properties of gravitational field of specific exact solutions belonging to the large family of black hole spacetimes of type D found by Plebański and Demiański. We discuss the structure in the case of de Sitter ($\Lambda > 0$) and anti–de Sitter ($\Lambda < 0$) conformal infinity. With the aim of further interpretation, we have found the relation between the structure of the sources (the mass m, the electric e and magnetic g charges, the NUT parameter l, the rotational parameter a, the acceleration α) and the properties of radiation generated by them. In particular, we studied the amplitude of radiation and illustrated it in several figures.

Keywords. Plebański–Demiański family of solutions – asymptotic directional structure of radiation – de Sitter, anti-de Sitter

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1. Introduction

The investigation of properties of gravitational radiation is an important problem of general relativity. In last few years, the theory was further developed for asymptotically de Sitter ($\Lambda > 0$) and anti-de Sitter ($\Lambda < 0$) spacetimes in Krtouš & Podolský (2004). The work assumed, in analogy with the asymptotically flat situation, that the radiative field components are decaying as η^{-1}, where η is an affine parameter along the null geodesics. We apply the new approach on particular exact model spacetimes of type D from the Plebański and Demiański family of solutions, concretely on the Kerr–Newman–de Sitter metric and then on the general form of the metric which contains not only the mass, the charge and the rotation, but also the NUT parameter, non-zero acceleration and the cosmological constant, for details see Plebański & Demiański (1976), Griffiths & Podolský (2006). In both cases, we found the explicit dependence on the parameters of the metric in the final formulae of the directional structure of radiation.

2. Results

The general form of the metric has the form (Plebański & Demiański 1976)

$$\mathbf{g}_{ab} = \frac{1}{\Omega^2}\left[-\frac{Q}{\rho^2}(\mathrm{d}\tilde{t} - (a\sin^2\vartheta + 4l\sin^2\tfrac{\vartheta}{2})\mathrm{d}\tilde{\phi})^2 + \frac{\rho^2}{\hat{P}}\mathrm{d}\vartheta^2 + \frac{\rho^2}{Q}\,\mathrm{d}r^2 \right.$$
$$\left. + \frac{\hat{P}\sin^2\vartheta}{\rho^2}(\mathrm{d}\tilde{t} - (r^2 + (a+l)^2)\mathrm{d}\tilde{\phi})^2\right], \tag{2.1}$$

where $\Omega = 1 - \frac{\alpha}{\omega}(l + a\cos\vartheta)$, $\rho^2 = r^2 + (l + a\cos\vartheta)^2$, $\hat{P} = 1 - a_3\cos\vartheta - a_4\cos^2\vartheta$, $Q = (\omega^2 k + e^2 + g^2) - 2mr + \epsilon r^2 - 2\alpha\omega^{-1}nr^3 - (\alpha^2 k + \Lambda/3)r^4$.

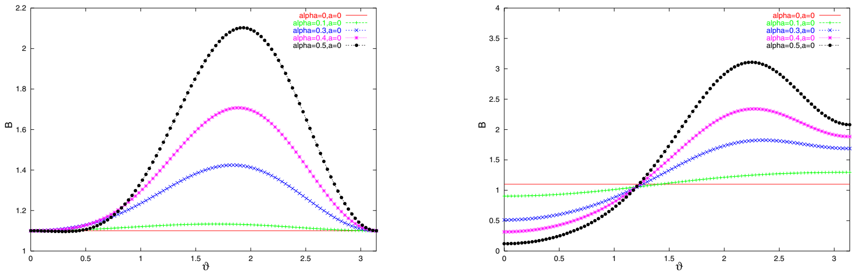

Figure 1. Left panel: the magnitude of radiation $B(\vartheta)$ on spacelike $\mathcal{I}$ for C–metric without charges, where $a = 0$ and α varies. Right panel: the magnitude of radiation $B(\vartheta)$ on spacelike $\mathcal{I}$ for charged C–metric, where $a = 0$ and α varies.

The directional pattern for $\Lambda > 0$ is determined by two principal null directions (PNDs) oriented outwards with respect to the future spacelike conformal infinity $\mathcal{I}^+$, explicitly

$$|\Psi_4^{\mathrm{i}}| \approx \frac{3}{2}\frac{1}{|\eta|}B(\vartheta)\mathcal{A}(\theta, \phi, \theta_{\mathrm{s}}), \tag{2.2}$$

where $B(\vartheta)$ is the amplitude of radiation, which contain parameters of the metric. $\mathcal{A}(\theta, \phi, \theta_{\mathrm{s}})$ is the directional structure, where θ and ϕ parametrize the direction along geodesic which approaches the conformal infinity, θ_{s} is a parameter between two tetrads. In particular, for the C–metric the amplitude $B(\vartheta)$ has the form

$$B(\vartheta) = (1 + \tfrac{3}{\Lambda}\alpha^2\mathcal{P}\sin^2\vartheta)\sqrt{m^2 - 4m\alpha(e^2 + g^2)\cos\vartheta + 4\alpha^2(e^2 + g^2)^2\cos^2\vartheta}$$

where $\mathcal{P} = 1 - 2\alpha m\cos\vartheta + \alpha^2(e^2 + g^2)\cos^2\vartheta$. The dependence on parameters is illustrated in two graphs in Fig. 1. We verified results for C-metric obtained in Krtouš & Podolský (2003) and in Podolský, Ortaggio & Krtouš (2003). For Kerr–Newman–de Sitter, only the mass parameter occurs in the amplitude $B(\vartheta) = m$. We also investigated the structure near timelike $\mathcal{I}$, for details see Kadlecová (2006).

3. Conclusions

We investigated influence of the physical parameters of the sources, namely α, a, m, e, g and Λ on the amplitude of the radiation $B(\vartheta)$ which appears at spacelike and timelike conformal infinity in the subcase $l = 0$ of the general metric (2.1) and on its special subcases: the Kerr–Newman–de Sitter solution and the C-metric. Generally, we observe from Fig. 1 and other figures that the acceleration parameter α and the cosmological constant Λ have *dominant* influence on the amplitude. The acceleration parameter α has stronger influence than the rotational parameter a. The charges e and g have a smaller influence than acceleration but their influence is quite stronger when the rotational parameter a is small or vanishes. The asymptotic structure depends substantially on *rotation* when the acceleration is non-vanishing. We hope that these results will provide a deeper insight into the general theory of radiation in general relativity.

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