COLLISIONAL STELLAR DYNAMICS IN GENERAL RELATIVITY: AN OVERVIEW

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ABSTRACT. Recently, Israel and Kandrup (1984; Kandrup 1984 a,b,c,d) have formulated a new, manifestly covariant approach to non-equilibrium statistical mechanics in classical general relativity. The object here is to indicate how that formalism may be used to construct a theory of 'collisional' stellar dynamics, valid for a collection of point mass stars in the limit that incoherent radiative effects may be neglected.

The idealisation of point masses is of course a gross oversimplification which, however, at least in principle, should not be that difficult to overcome. The neglect of radiative effects is more important, since it implies ultimately that deviations from some 'average' mean field conditions may be modeled by a comparatively simple direct interaction. This means that one need not consider explicitly the degrees of freedom of the gravitational 'field'. It should, therefore, be stressed that this neglect really is legitimate: the time scale for energy loss via radiation reaction will exceed the evaporation time scale and all other relevant scales provided that the central redshift $z \lesssim 1$, a condition believed necessary to ensure dynamical stability.

Whether there exist realistic situations in which relativistic effects actually are important is at present an unanswered question. The object here is simply to understand how one might describe such relativistic systems if they do in fact exist. In this sense, the problem may, as suggested by Zel'dovich and Podurets (1966), be termed one of 'methodological interest.'

In order to see what is required, it is useful to recall the Newtonian theory. In its most rigorous form, the starting point for the description of a collection of N identical, point mass stars is an N-particle distribution function μ, the probability density for finding each star i at a given point $\vec{x}_i$ with momentum $\vec{p}_i$ at time t. The one-particle f, which is of more direct physical relevance, is constructed as a reduced distribution. Given this f, one may generate an average mass density ρ and other quantities of interest. The evolution of μ is of course governed by the N-particle Liouville equation. What one really wants, however, is an equation for the evolution of f.

Physically, one anticipates that any such equation will allow for two sorts of effects, namely (i) the 'average' gravitational force $-\vec{\nabla}\Phi$ associated with ρ, and (ii) 'fluctuating' forces that arise because the 'true' and 'average' potentials do not coincide. The important point is that such

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a decomposition of the total force can be implemented rigorously, so that one can derive an exact, closed, non-linear, non-local equation for the evolution of f (Kandrup 1981). In the limit that the fluctuating components are neglected, one recovers the standard 'collisionless' theory. In the limit that their effects may be evaluated as if the stars were following linear trajectories through the center of an infinite, homogeneous system, one recovers the Fokker-Planck equation of 'collisional' stellar dynamics.

The question is of course: Could one formulate analogous relativistic theories of 'collisionless' and 'collisional' stellar dynamics? An heuristic 'collisionless' theory is easy enough to obtain (see, e.g., Ipser and Thorne 1968). One again considers a one-particle f, now defined covariantly, and, given this f, one may construct an average stress-energy $T_{\mu\nu}$ which in turn serves as a source for an average spacetime metric $g_{\mu\nu}$. To the extent that 'fluctuations' may be neglected, one might suppose that the stars may be approximated as following geodesics in this average spacetime, and, in this limit, one recovers a simple relativistic generalisation of the 'collisionless' Vlasov equation.

Transcending a 'collisionless' description is more difficult. One obvious complication is the fact that one really ought to incorporate explicitly the degrees of freedom of the gravitational field. Thus, in analogy with relativistic plasma physics, one ought to start from a more complicated distribution function involving both particle and field variables (Kandrup 1984d). This complication can, however, be circumvented if one is concerned only with formulating an approximate kinetic theory. The important point is that, to the extent that incoherent radiative effects may be neglected, one can model the gravitational 'forces' by a direct interaction involving only the coordinates and momenta of the stars (Israel and Kandrup 1984, Kandrup 1984a,b).

Another complication arises when one tries to specify what is really meant by a gravitational 'force'. After all, the guiding principle of general relativity is that, in the absence of non-gravitational effects, the stars follow geodesics in the spacetime which they generate. To isolate upon a sensible gravitational 'force' one must re-express the 'true' physics — stars following geodesics in the irregular, rapidly fluctuating spacetime — in terms of another, smoothed background spacetime chosen to satisfy some appropriate field equations, e.g., the equations of the collisionless theory. The tensor $\delta\Gamma^{\lambda}_{\mu\nu}$, the difference between the Christoffel symbols for the true and background spacetimes, serves to generate a 'fluctuating force', the existence of which is manifest by the fact that the stars no longer appear to follow geodesics (Israel and Kandrup 1984). The form of this $\delta\Gamma^{\lambda}_{\mu\nu}$ must in general be determined by solving non-linear field equations which are virtually intractable analytically. To the extent, however, that the difference between the two spacetimes is small, so that the 'fluctuating forces' are weak, it will suffice to linearise the field equations about the background solution. A similar argument then suggests that, when solving these equations, one may proceed in a first approximation as if the stars were following geodesics in the background spacetime: this is the relativistic analogue of the standard 'impulse' approximation. What this means is that the gravitational forces will be generated from simple retarded potentials involving only the coordinates and momenta of the stars: one has achieved the desired direct interaction approximation.

Given these sorts of expressions for the inter-stellar forces, it is straightforward to proceed in comparatively close analogy with the Newtonian description to introduce an N-particle distribution function μ and a reduced one-particle f; and, starting from this 'complete' description, it is possible to derive an exact, closed equation for the evolution of f. What remains then is to approximate that exact equation by a tractable kinetic equation. This entails four general assumptions, the justification of which has been discussed elsewhere (Israel and Kandrup 1984, Kandrup 1984a,b,f). (i) It is assumed that any initial correlations have a negligible influence upon the evolution of f. (ii) It is assumed that there exists a 'collision' time scale short compared with the dynamical and relaxation time scales (in the context of an impulse approximation, this only makes sense if one introduces a cut off in the potential at large impact parameter to avoid the standard logarithmic divergences). (iii) It is assumed that the stars may be approximated as following geodesics in the average spacetime. (iv) It is assumed that, in evaluating the effects of the fluctuating forces at some point x^{μ}, one may introduce a local Lorentz frame and then proceed as if the spacetime were really homogeneous and flat ('localised fluctuations'). This final assumption enables one to express the forces in terms of their Fourier transforms, and one obtains thereby the kinetic equation (7) of Kandrup (1984c).

Various properties of this kinetic equation have been discussed elsewhere (Israel and Kandrup 1984, Kandrup 1984a,c,f) but certain implications should be reiterated here. (i) In the obvious Newtonian limit, one recovers the well-known Landau equation for a 1/r potential, and, if one performs the $\vec{k}$-space integrations by introducing suitable cut offs, one obtains the standard Fokker-Planck equation. (ii) Provided that one introduces a cut off appropriate for large impact parameter, one can conclude that the relativistic isothermal distribution constitutes an exact stationary solution. If, however, one does not introduce a cut off, this is no longer true. Just as in the Newtonian limit (Kandrup 1984e), one anticipates deviations from an isothermal! (iii) The kinetic equation guarantees energy-momentum conservation, i.e., it implies that the mean field $T^{\mu\nu}$ is intrinsically divergence-free: $\nabla_{\mu} T^{\mu\nu} = 0$. (iv) The kinetic equation implies an H-theorem inequality, i.e., the divergence of the Boltzmann entropy flow S^{μ} associated with f is non-negative: $\nabla_{\mu} S^{\mu} \geq 0$, with equality holding only for a locally Maxwellian distribution of velocities.

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