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RELATIVE RELATION MODULES OF FINITE GROUPS

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Let G be a fixed finite group and consider a short exact sequence

$$1 \to S \to E \xrightarrow{\psi} G \to 1$$

where E is a finitely generated group. The abelian group $\overline{S} = S/S'$ may be regarded as a $\mathbb{Z}_G$-module and, for a fixed prime p, the elementary abelian p-group $\hat{S} = S/S'S^p = \overline{S}/p\overline{S}$ may be regarded as an $\mathbb{F}_pG$-module. If E is a free group, $\overline{S}$ is called the relation module of G determined by ψ, and $\hat{S}$ the relation module modulo p. In general we call $\overline{S}$ the relative relation module, and $\hat{S}$ the relative relation module modulo p. When the minimal number of generators of G and E is the same, $\overline{S}$ and $\hat{S}$ will be called minimal.

Gaschütz, Gruenberg and others have studied relation modules and relation modules modulo p. The main aim of this thesis is to study relative relation modules modulo p when E is a free product of cyclic groups. To be more precise, let $X = \{g_i,\ 1 \le i \le d\}$ be a generating set of G, G_i the cyclic group generated by g_i, E the free product of the G_i, $1 \le i \le d$, ψ the epimorphism whose restriction to each G_i is the identity isomorphism, and S the kernel of ψ.

Some of the results may be summarised as follows. $\hat{S}$ is embedded in

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the direct sum of the augmentation ideals of the $\mathbb{F}_p G_i$, $1 \le i \le d$, induced to G , and the resulting factor module is isomorphic to the augmentation ideal of $\mathbb{F}_p G$. $\hat{S}$ may also be embedded in a free $\mathbb{F}_p G$-module of rank $d - 1$.

Two relative relation modules, isomorphic as $\mathbb{F}_p$-spaces, are rarely isomorphic as G-modules; that is, $\hat{S}$ not only depends on G, p and d but also on ψ . Some cases when $\hat{S}$ does not depend on ψ are established.

We say that p is semicoprime to the order of G if p divides the order of G and does not divide the orders of the G_i , $1 \le i \le d$. In the coprime and semicoprime cases a characterisation (including a criterion for counting projective summands) of $\hat{S}$ is given. Some relative relation modules (modulo p) of $SL(2, p)$ and $PSL(2, p)$ are described completely; the description may be useful in the study of the factor groups of $PSL(2, \mathbb{Z})$.

Given an unrefinable direct decomposition of a module, the direct sum of all the nonprojective summands is called the nonprojective part of the module. In the semicoprime case the nonprojective part of $\hat{S}$ is a uniquely determined, nonzero and indecomposable module (and is also the nonprojective part of $\hat{S}$ when E is a free group). The nonprojective part of $\hat{S}$ in the nonsemicoprime case may be zero or decomposable (and may not be a homomorphic image of the nonprojective part of $\hat{S}$ when E is free). When G is a p-group, we prove that $\hat{S}$ is nonprojective and indecomposable for $\hat{S}$ minimal.

Some of the above results can be generalised in the case when the cyclic factors of E are not restricted to be the generators of G , however it is not known whether in this case the minimal relative relation modules of p-groups are also indecomposable.

Some results may also be extended to $\bar{S}$.

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