MIXING IN COLLISIONFREE SYSTEMS

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ABSTRACT. Results of dynamical, numerical simulations of self-gravitating systems are used to show the existence of non-uniform mixing in phase-space.

The systems under consideration (3-D dynamics , vanishing total angular momentum) are collisionfree, so only phase space conserving states are accessible within the exact dynamics. This class of states Φ_0 can be described to be the set of all one particle distribution- functions $f(\vec{r},\vec{v})$ leaving $g(\varphi)$ (the memory function) invariant.

$$\int \Theta(f(\vec{r},\vec{v})-g)d^3r\,d^3v = \varphi(g) \Rightarrow g(\varphi) \qquad \forall f \in \Phi_0. \qquad (1)$$

Θ denoting the unit step function. A globally stable equilibrium state within collisionless dynamics is the minimum energy state of all $f \in \Phi_0$. For given Φ_0 non-equilibrium states are separated from globally stable states by an energy difference ΔE (Wiechen et al., 1988).
Due to filamentation on microscopic scales equilibrium states may be reachable on macroscopic scales. Microscopic filamentation and coarse-graining of f on macroscopic scales effects the "memory"-function $g(\varphi)$ in the following way (Wehrl, 1978)

$$\overline{M}(\varphi) = \int_0^{\varphi} \overline{g}(\varphi')d\varphi' \leq \int_0^{\varphi} g(\varphi')d\varphi' \quad \forall\varphi \qquad (2)$$

with $\overline{g}(\varphi)$ being calculated from the averaged version of the distribution function $\overline{\varphi}(g) = \int \Theta(\overline{f}(\vec{r},\vec{v})-g)d^3r d^3v$. Because this "mixing" transformation of $g(\varphi)$ increases the energy of the corresponding lowest energy state (Ziegler and Wiechen, 1989) a globally stable state on macroscopic scales may be attainable and one can try to predict it by transforming the memory function $g(\varphi)$ obeying the mixing condition (2) to that extend, that the energy of the corresponding lowest energy state equals E_0. In that way the overall effect of mixing is calculated

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self-consistently, but the special form of transformation (2) generally is unknown and additional assumptions are needed.
In the case of a strong and uniform filamentation of $f(\vec{r},\vec{v},t)$ with peaks of small width δ with typical distances $\tilde{\delta}$ and a macroscopic scale Δ obeying $\delta \ll \Delta < \tilde{\delta}$ the transformation of g corresponding to the coarse-graining of f reads

$$\overline{g}(\varphi) = \frac{1}{\alpha} g(\varphi/\alpha) \quad \alpha = \Delta/\delta \quad \delta, \Delta \to 0 \tag{3}$$

For systems initially being not to far away from an equilibrium state ($\Delta E/|E| < O(1)$) the theoretical predictions using assumption (3) are in very good agreement with the simulational results (see e.g. Ziegler, Wiechen 1989). This situation changes significantly if we go over to initial data very far from a stable equilibrium state (see also Nozakura, 1992). To show this and to clarify the reason we have performed several simulational runs using the TREECODE (Hernquist 1990). $\overline{g}(\varphi)$ and $\overline{M}(\varphi)$ reconstructed from the particle data show that the final states are lowest energy states, being more mixed than the initial (exact) states according to relation (2). But the mixing turns out to be non-uniform. Especially for systems being initially very far away from an equilibrium state mixing is less efficient for small φ and more efficient for big φ compared to the assumption of strong and uniform mixing (equation (3)). For a lowest energy state (globally stable state) small φ correspond to high values of f and low values of h. So we can state, that the low energy part of the distribution function is less mixed and the high energy part is more mixed compared to the uniform case. Note that low energies typically correspond to particles in the core of the system while high energy particles are likely to be in the halo. Certainly the prediction for the final states can be improved significantly replacing (3) by a more appropriate operator. For the calculated examples a convolution of $g(\varphi)$ with e.g. a gaussian curve would help, but a sound motivation going beyond a good fitting reason is still missing.

References

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