

Relativistic Aspects of Reference Systems and Time Scales

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Mindful of the fact that any time and space coordinates of General Relativity Theory (GRT) are not, in general, physically meaningful and measurable quantities, one may choose between three possibilities in applying GRT to ephemeris astronomy:

- 1) avoid coordinates completely, i.e., construct coordinate-independent theories of light propagation and solar system body motion involving the removal of all coordinate-dependent quantities from the present system of astronomical constants;
- 2) use any coordinate system to describe observational procedures and to solve dynamics problems, provided that one and the same coordinates be used for both the kinematics and dynamics of a specific problem;
- 3) adopt IAU recommendations specifying reference systems and time scales to be used in ephemeris astronomy.

The third possibility seems to be the most adequate for astronomical practice.

A reference system (RS) is given in GRT by the four dimensional metric form,

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu, \quad \mu, \nu = 0, 1, 2, 3 \quad (1)$$

with time coordinate x^0 and three space coordinates, x^i ($i = 1, 2, 3$). (The Einstein summation convention is used throughout this paper.) The quantity $c^{-1}x^0$, c being the velocity of light, is called the coordinate time. In quasi-Galilean coordinates which are used in actual problems, the metric tensor components,

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \quad (2)$$

differ little from the Galilean metric of an inertial RS of special relativity, ($\eta_{00} = 1$, $\eta_{0i} = 0$, $\eta_{ij} = -\delta_{ij}$).

Expansions of $h_{\mu\nu}$ in powers of v/c , v being the characteristic velocity of the bodies, appear as:

$$h_{00} = h_{00}^{(2)} + h_{00}^{(4)} + \dots, \quad h_{ij} = h_{ij}^{(2)} + h_{ij}^{(4)} + \dots, \quad h_{0i} = h_{0i}^{(3)} + \dots, \quad (3)$$

with figures in parentheses indicating the order of smallness with respect to v/c . Retaining $h_{00}^{(2)}$ alone gives the Newtonian equations of motion of bodies. This is also sufficient for treating, at the present level of accuracy, the problems of time scales and clock synchronization. Adding $h_{ij}^{(2)}$ enables one to obtain post-Newtonian equations of light propagation in a static field. Inclusion of $h_{0i}^{(3)}$ permits the solution of the same problem for a non-static field taking into account the motion of the bodies.

This term is also crucial for distinguishing between dynamically or kinematically non-rotating systems. If a RS rotates in the Newtonian sense with angular velocity Ω^j , it would be reflected in the first order term in h_{0i} as follows:

$$h_{0i}^{(1)} = -c^{-1} \epsilon_{ijk} \Omega^j x^k \quad (4)$$

with ϵ_{ijk} being the fully anti-symmetric Levi-Civita symbol ($\epsilon_{123} = 1$). Such systems are excluded from the hierarchy of basic reference systems. If $h_{0i}^{(3)}$ does not contain a third order term similar to (4), then the corresponding RS is called a dynamically non-rotating RS (DRS). However, such a term is included in $h_{0i}^{(3)}$ if one wishes to have a RS with invariable directions of space axes with respect to distant celestial objects. Such a RS is called a kinematically non-rotating RS (KRS). Proceeding further, and adding $h_{00}^{(4)}$, one gets the post-Newtonian equations of motion of celestial bodies. Finally, inclusion of $h_{ij}^{(4)}$ leads to the post-post-Newtonian equations of light propagation.

The present (1991) recommendations specify only $h_{00}^{(2)}$ and $h_{ij}^{(2)}$ terms with the isotropic coordinate condition,

$$h_{ij}^{(2)} = h_{00}^{(2)} \delta_{ij} \quad (5)$$

This condition permits the use of harmonic or PPN coordinates, but excludes, for instance, the standard Schwarzschild coordinates. The recommendations contain an implicit restriction for $h_{0i}^{(3)}$ to ensure the absence of kinematical rotation, i.e. to have a KRS. It is of importance also that the recommendations prescribe the description of the influence of the external mass only in the form of tidal terms. As a result, the relativistic hierarchy of astronomical reference systems may be constructed as shown in Figure 1.

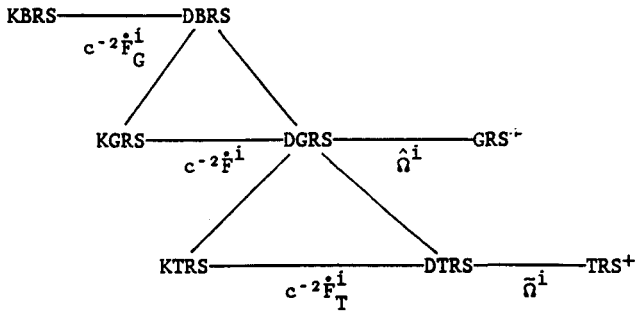


Figure 1

Relativistic Hierarchy of Astronomical Reference Systems

Each of the three systems: barycentric (BRS), geocentric (GRS) and topocentric (or satellite) observer's (TRS) is represented both in dynamical and kinematical versions, differing one from another by the amount of Galactic ($c^{-2}F_G^i$), geodesic ($c^{-2}F^i$) and topocentric ($c^{-2}F_T^i$) precession, respectively (the notion of precession is used here in the broad sense, including the periodic part, i.e., nutation as well). The GRS⁺ (or TRS⁺) result from the Newtonian rotation of the space axes of the DGRS (or DTRS) with angular velocity $\hat{\Omega}^i$ or $\bar{\Omega}^i$, respectively so as to take into account the rotation of the Earth.

The coordinate times t , u and τ of the BRS, GRS and TRS respectively, define unambiguously the relativistic time scales to be used in astronomy. In doing this it is to be noted that τ coincides with the physically measured observer's proper time at the origin of the TRS. The transformation between t (TCB) and u (TCG) at the point with BRS space coordinates x^k is as follows:

$$u = t - c^{-2} [S(t) + v_E^k (x^k - x_E^k)] , \tag{6}$$

with x_E^k and $v_E^k = \dot{x}_E^k$, being the BRS coordinates and velocity of the Earth, and $S(t)$ to be determined from the equation:

$$\dot{S} = v_E^2/2 + \bar{U}_E(\underline{x}_E) , \tag{7}$$

the external mass potential $\bar{U}_E(\underline{x})$ is to be evaluated at the geocenter,

$$\bar{U}_E(\underline{x}_E) = \sum_{A \neq E} GM_A / r_{EA} . \tag{8}$$

Under the limited accuracy of the Keplerian planetary motions, the solution of Eq. (7) may be split into secular and periodic parts,

$$S(t) = S^*t + S_p(t) \quad (9)$$

Taking into account the actual planetary motions this split loses its sense and cannot be unambiguously defined. The time scales TDB and TDT introduced by the IAU (1976) resolutions are in fact proportional to t and u , respectively:

$$TDB = k_B t, \quad TDT = k_G u \quad (10)$$

In putting,

$$k_B/k_G = 1 - L_C; \quad L_C = c^{-2}S^* (\approx 0.46 \text{ s/y}) \quad (11)$$

one has instead of (6),

$$TDB = TDT + c^{-2}[S_p(t) + v_E^k(x^k - x_E^k)] \quad (12)$$

In just the same manner, one has the transformation between u and r at the point with GRS coordinates w^k ,

$$r = u - c^{-2}[V(u) + v_T^k(w^k - w_T^k)] \quad (13)$$

with w_T^k , $v_T^k = dw_T^k/du$, being the GRS coordinates and velocity of the TRS origin. The function $V(u)$ satisfies the equation,

$$dV/du = v_T^2/2 + \hat{U}_E(w_T) + Q_k w_T^k + T(w_T) \quad (14)$$

where $\hat{U}(w)$ is the GRS geopotential taken at the topocenter, Q_k is a very small quantity describing the deviation of the BRS motion of the Earth from geodesic motion, and $T(w)$ is the external mass tidal potential evaluated at the topocenter. Presently, the time service includes only the clocks situated at rest on the Earth's surface, and the precision of these clocks permits one to neglect the tidal terms in (14). Under these two circumstances it is reasonable to present the solution of Eq. (14) in the form:

$$V(u) = V^*u + V_p(u) \quad (15)$$

with V^* being one and same constant for all possible TRS. Introducing the notion of the geoid, one may identify V^* with the value of the force of gravity on the geoid.

Then, choosing,

$$k_G = 1 - L_G, \quad L_G = c^{-2}V^* (\approx 0.022 \text{ s/y}) \quad (16)$$

one has instead of (13),

$$TDT = \tau + c^{-2}[V_p(u) + v_T^k(w^k - w_T^k)] + 32.184 \text{ s} \quad (17)$$

with the additive constant offset now adopted. TAI is determined presently from averaging the readings of the clocks synchronized with respect to TDT, i.e.,

$$TAI = \text{Mean}[\tau + c^{-2}V_p(u)] \quad (18)$$

in other words, TAI represents the physical realization of TDT (TT by modern terminology), and presently,

$$TDT = TAI + 32.184 \text{ s} \quad (19)$$

Only the correction for the observer's height above the geoid is now taken into account in $V_p(u)$. In fact, this function includes the tidal correction as well (with a factor, $1 + k_2 - h_2$, taking into account the Love numbers) and the correction due to geophysical factors (tectonic displacements). Considering these corrections, one will have to define more rigorously the notion of the geoid. In this case, the splitting (Eq. (15)) will lose its advantages. Moreover, for moving clocks participating in the time service (clocks on satellites) such a split will become absolutely inefficient. One may foresee that in the future, the split, the notion of the geoid and the auxiliary time scale TT will become useless. Instead one may introduce the time scale TAIM (TAI Modified) representing the physical realization of TCG,

$$TCG = TAIM + 32.184 \text{ s} \quad (20)$$

which results from the averaging,

$$TAIM = \text{Mean}[\tau + c^{-2}V(u)] \quad (21)$$

using the function $V(u)$ as determined directly from Eq. 13.

The possible tendency in the IAU standards of time scales may be illustrated in Table 1.

Year	1976	1991	?
IAU TIME SCALES	TDB TDT TAI	TCB TCG TAI	TCB TCG TAIM
A T T R I B U T E S	s* v* - GEOID	- v* TT GEOID	- - - -

Table 1.
Possible Tendency in IAU Time Scales

Finally, let us mention once again that the great advantage of the TCB and TCG time scales is the possibility of using SI units both in the BRS and GRS without any scaling factors. On the contrary, using TDB and TDT leads to the appearance of time and length scaling factors as illustrated in Table 2. Here the SI second and SI meter are denoted respectively by s and m , while t , u , x , w , GM_A , c and AU are supposed to be expressed in SI units.

	BRS(*)	GRS(')
Time Scale	$TDB = k_B t$	$TDT = k_G u$
Length Scale	$x^* = l_B x$	$w' = l_G w$
Time Unit	$s^* = s/k_B$	$s' = s/k_G$
Length Unit	$m^* = m/l_B$	$m' = m/l_G$
Mass Factor	$(GM_A)^* = GM_A l_B^3/k_B^2$	$(GM_A)' = GM_A l_G^3/k_G^2$
Light Velocity	$c^* = cl_B/k_B$	$c' = cl_G/k_G$
Astronomical Unit	$AU^* = l_B AU$	$AU' = l_G AU$

Table 2
BRS and GRS Scaling Factors When Using TDB and TDT

Three evident options for choosing the length scaling factors are:

- a) $l_B = 1$, $l_G = 1$, meter invariance,
- b) $l_B = k_B$, $l_G = k_G$, light velocity invariance, and
- c) $l_B = (k_B)^{3/2}$, $l_G = (k_G)^{3/2}$, mass factor invariance.

In any case, the appearance of the scaling factors in BRS planetary and GRS satellite equations of motion is unavoidable, and every quantity in the system of astronomical constants should be expressed in three versions with BRS, GRS and SI units. Introducing TCB and TCG instead of TDB and TDT removes these inconveniences.

References

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DISCUSSION

Fukushima:

I would like to make one comment. As Dr. Brumberg has said, the present time convention we are now using is the cause of the difference of the GM of the Earth from the satellite laser ranging measurements and from lunar laser ranging measurements, as was very clearly pointed out in the IERS standards, especially in the numerical parts, and I feel that we should avoid this type of confusion. GM should have a single value through all the coordinate systems, and it is very confusing that the constants have different values in different coordinate systems.