COSMIC STRUCTURES' FORMATION IN THE MULTICOMPONENT UNIVERSE: NEW TESTS

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Abstract

Scaling of the transfer function of primordial perturbations in the multicomponent Universe is found. New tests for detecting light weakly interacting particles and dark barions are proposed.

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Both fundamental physics and astronomical observations evidence the existence of non-barionic dark matter in the Universe. This may be in the form of non-relativistic weakly interacting particles (heavy cosmions) which$\sim 10 \div 30$ times the barion density of the Universe and, thus, add the total cosmic mass nowdays to about the critical value. There are also possible relativistic weakly interacting particles (light cosmions) which strongly influenced cosmological expansion in the past. Together with the ordinary matter (barions, photons) cosmions make the Universe multicomponent which crucially changes the density perturbation growth in the Early Universe.

Two questions arise on this way:

1) Which (and how) the parameters of the dark matter determine in the first hand the development of the primordial perturbations in the dominating components?

2) Does any additional possibility appear in the density (and velocity) contrast distributions to be set initially for the non-dominating components, say for barions?

These can be reformulated in more astrophysical terms:

1) How many light particles are in the Universe? and

2) How many barions are in cosmic voids and islands?

Obviously, neither non-dominating cosmions no dark barions can be disclosed dynamically now. The answer requires the per-

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turbation considerations in the Early Universe when the influence of the relativistic particles and barions on the dynamics was important. Then one can make some observational predictions about the large-scale structure properties of the Universe and of the relic microwave background. And the comparison of these consequences of the past dynamics with the reality serves as a test of the parameters we are looking for.

Further on we briefly present some new results concerning the above two questions.

I. Light particles and the large-scale structure of the Universe.

We know already one test of the number of species of light particles in the Universe. It is the primordial nucleasyntesis [1,2]. Here we show that the supercluster scales, the fluctuations of the relic radiation and other large-scale structure parameters are also very sensitive to the presence of relativistic particles in the Early Universe.

Let us consider a simple 3-component model of the Early Universe: two callisionless components

(α) light relativistic particles described by the Boltzman-Vlasov equation , and

(β) heavy 'dust' *) particles which will maintain the contemporary critical density

and

(γ) ideal fluid with the equation of state $p=\varepsilon/3$

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*) Here we neglect the pressure connected with barionic part of the non-relativistic component.

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This model is fully specified by a single parameter

$$\alpha = \frac{\varepsilon^{(\alpha)}}{\varepsilon^{(\alpha)} + \varepsilon^{(\gamma)}} = const \quad \in (0, 1) \tag{1}$$

that fixes the total number of the light cosmions as compared to the total number of all relativistic particles. Hence the component energy densities can be wright down as *)

$$\varepsilon^{(\alpha)} = \frac{3\alpha}{R^2 a^4} \quad , \quad \varepsilon^{(\beta)} = \frac{12}{R^2 a^3} \quad , \quad \varepsilon^{(\gamma)} = \frac{3\gamma}{R^2 a^4} \tag{2}$$

where $\gamma = 1 - \alpha$; $a = \tau(1+\tau)$ is the normalized scale factor and R is a scale constant. Time parameter τ is related to the cosmic time as follows:

$$t = R\int a\, d\tau = \frac{1}{2} R \tau^2 (1 + \frac{2}{3}\tau) \tag{3}$$

In our normalization the values $a_{eq} = 1/4$ and $\tau_{eq} \simeq 0.2$ correspond to the moment of equality of the 'cold' and 'hot' component densities ($\varepsilon^{(\alpha)} + \varepsilon^{(\gamma)} = \varepsilon^{(\beta)}$) ; and the up-to-date values are given as

$$a_o = 1.05\ 10^4 \gamma h^2 \quad , \quad \tau_o = 1.02\ 10^2 h \sqrt{\gamma} \gg 1 \tag{4}$$

where $h \in (0,5;1)$ is the Hubble parameter in the units 100 km/s/Mpc and T = 2.7 K is the background temperature. E.g., for three sorts of massless neutrinos (α) and photons (γ) we have

$$\alpha = 0.405, \quad \gamma = 0.595, \quad \tau_c = 79\, h \quad . \tag{5}$$

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*) We use units $c = \hbar = 8\pi G = 1$

Finaly, the model $\mathcal{R}$ - constant is calculated

$$\mathcal{R} = 5.6\ 10^{11} \gamma^{-3/2} h^{-4}\ s \simeq 45\ t_{eq} \qquad (6)$$

The perturbations were analysed in gauge - invariant form (for general formalism see ref. 3). Here we present them in the comoving with the β - component sinchronous reference system

$$ds^2 = (a\mathcal{R})^2 (d\tau^2 - (\delta_{ij} + h_{ij}) dx^i dx^j) \qquad (7)$$

where $h_{ij} = A\delta_{ij} + B_{,ij}$; $u_i^{(\beta)} = 0$. The γ- and α - perturbations are described by the velocity potential $v \equiv v(\tau, \vec{x})$ (the non-zero 4-velocity components: $u_i^{(\gamma)} = a v_{;i}/2$) and by the distribution function of α - particles in the phase space with the particle momenta $\vec{q} = \{q_i\}$ ($= const$: the integrals of motions). The equation system for the Fourrier-harmonics ($\sim e^{i\vec{k}\vec{x}}$) is the following one

$$\begin{aligned}
&\dot{F} - i\kappa\mu F + \dot{A} - (\kappa\mu)^2 \dot{B} = 0 \\
&\ddot{v} + \frac{\kappa^2}{3}(v - \dot{B}) + \dot{A} = 0 \\
&\dot{A} + a^{-2}(2\gamma v - 3i\alpha\kappa^{-1}\int_{-1}^{+1} F\mu\, d\mu) = 0 \\
&\ddot{B} + 2\frac{\dot{a}}{a}\dot{B} - A + 3\alpha(\kappa a)^{-2}\int_{-1}^{+1} F(3\mu^2 - 1)\, d\mu = 0
\end{aligned} \qquad (8)$$

where dot denotes the derivative over τ - parameter and $\mu = \vec{\kappa}\vec{q}/\kappa q$ with $\kappa = |\vec{\kappa}|$, $q = |\vec{q}|$. Here and further on $v = v_\kappa(\tau)$, $A = A_\kappa(\tau)$, $B = B_\kappa(\tau)$. The function $F = F_\kappa(\tau, \mu)$ is related to the integral of the distribution function over the momentum modulus q [4)].

An important quantity is the density perturbation of the β - component which can be derived by the solution of eqs. (8):

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$$\delta^{(\beta)} \equiv \frac{\delta\varepsilon^{(\beta)}}{\varepsilon^{(\beta)}} = \frac{1}{2}(\kappa^2 B + 3(1-A)) + const \tag{9a}$$

Eqs. (8,9) have the first integral for the total perturbation energy:

$$4\delta^{(\beta)} + a^{-1}(\alpha\delta^{(\alpha)} + \gamma\delta^{(\gamma)}) = \dot{a}(\dot{A} - \frac{\kappa^2}{3}\dot{B}) + \frac{\kappa^2}{3} a A \tag{9b}$$

where

$$\delta^{(\alpha)} \equiv \frac{\delta\varepsilon^{(\alpha)}}{\varepsilon^{(\alpha)}} = \int_{-1}^{+1} F d\mu \quad , \quad \delta^{(\gamma)} \equiv \frac{\delta\varepsilon^{(\gamma)}}{\varepsilon^{(\gamma)}} = 2\dot{v}$$

Primordial cosmological perturbations, produced by any mechanizm near the cosmological singularity, form the initial condition set for eqs. (8). Below we consider adiabatic 'growing' mode normalized at $\tau \to 0$:

$$A = 1 \quad , \quad B = \dot{B} = F = v = \dot{v} = \delta^{(\beta)} = 0, \; (\dot{A} = 0) \tag{10}$$

where the condition in brackets is needed for the equation terms to be formally defined at $\tau = 0$. Thus, to find the perturbation amplitude at any moment of time, one should take just the solution of eqs. (8), (10) times the fundamental primordial spectrum $S(\kappa)$, the latter being defined at $\tau \to 0$,

$$< A^2 > = (2\pi)^{-3} \int \frac{S^2(\kappa)}{\kappa^3} d^3\vec{\kappa} \tag{11}$$

For the flat spectrum $S(\kappa) = S \equiv$ const $(\sim 10^{-4})$.

It is convenient to express the result in terms of the transfer function $C(\kappa)$ which describes the solution at large $\tau \gg 1$ (just before the nonlinear perturbation dynamics develop):

$$A = C(1 + \frac{1}{5a}) \; , \; B = \frac{a}{10} C \; , \; v = \frac{\dot{a}}{10} C \; , \tag{12}$$
$$\delta^{(\alpha)} = \delta^{(\gamma)} = \frac{2}{5} C \; , \; \delta^{(\beta)} = \frac{\kappa^2 a}{20} C \; ,$$

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$$F = \frac{1}{5} C + ik\mu v + F_k \exp(ik\mu\tau)$$

where $C = C(k)$ and $F_k = F_k(\mu)$ are the gauge-invariant functionals of the evolution history. So the transfer function $C(k)$ is, indeed, the ratio of the two spectra - initial spectrum for the nonlinear simulations at small redshifts and the primordial perturbation spectrum at the expansion beginning:

$$C(k) = \frac{A_k(\tau \gg 1)}{A_k(\tau \to 0)} = \lim_{\tau \gg 1} \frac{\delta_k^{(\beta)}}{k^2 a S_k / 20} \tag{13}$$

The function $C(k)$ was analytically and numerically calculated for different parameters α [5,6] (see fig. 1) and the result turned out rather puzzling. Before we present it, let's consider an asymptotic behaviour of $C(k)$ at large $k \gg 100$ in two limit cases for the α - parameter :

$$C(k) = \frac{120}{k^2} \cdot \begin{cases} 3 \ln(k/32), & \text{for } \alpha = 0 \ ^{7)} \\ 4 \ln(k/55), & \text{for } \alpha = 1 \ ^{6)} \end{cases} \tag{14}$$

For very small scales $(k \to \infty)$ the difference is seen to reach $\sim 30\,\%$, but the physically interesting are the larger scales ($k < 500$, $\lambda > 1$ Mpc, see eq. (16)).

Now we can formulate the main result: $C(k)$ is practically independent of α for $k < 500$. The limit curves (for $\alpha = 0$ and $\alpha = 1$) are maximally deviated at $k \sim 10$ where the discrepancy consists in about 8 %, then they converge with growing and overlap at $k \sim 100$; further on they diverge, reaching again descrepancy $\sim 8\,\%$ at $k \sim 500$, and proceed according to eq. (14). All the other curves $C(k)$ with the intermediate values of α $(0 < \alpha < 1)$ are placed in between the two limits considered.

Such a weak dependence of $C(k)$ on α in no way means

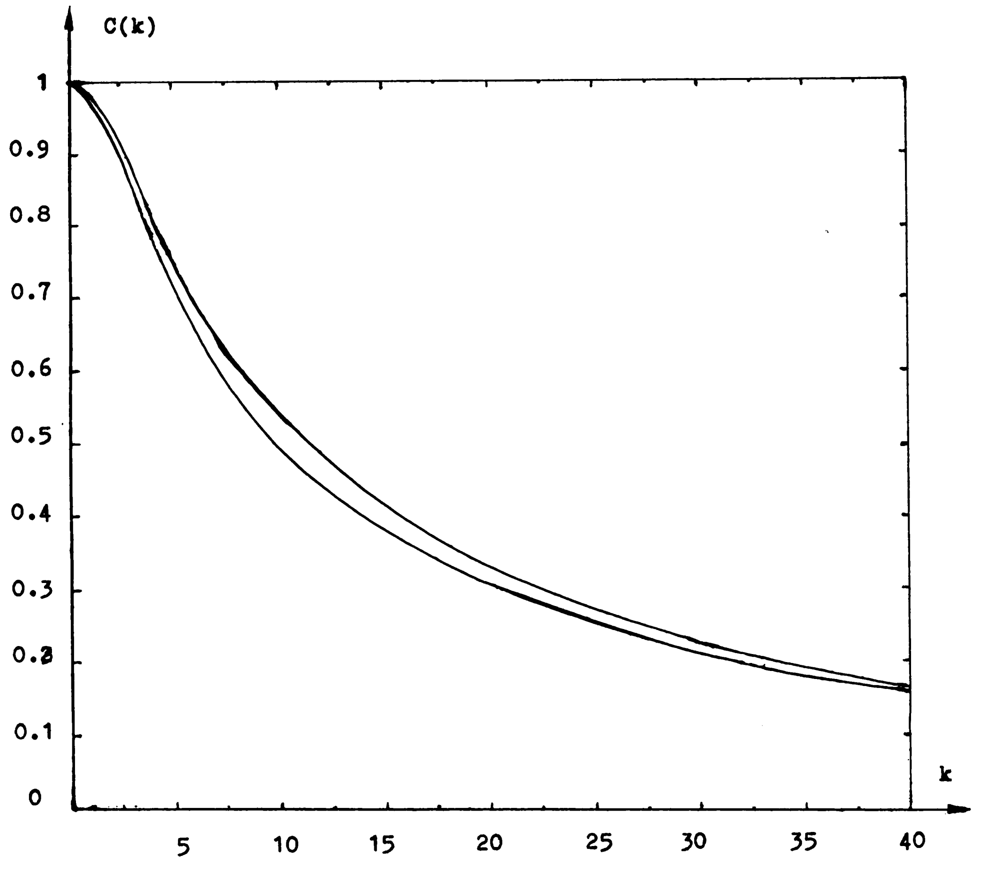

Fig. 1

Transfer functions C(k) for $\alpha = 0$ (upper curve) and $\alpha = 1$ (bottom curve).

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that the light cosmions do not impact the structure formation process. This effect is connected with the lucky choice of the normalization of the K - parameter. In the physical space all the structural scales crucially depend on the α - parameter.

So, we can reformulate our result as a discovery of the scaling of the transfer function. The function shape is practically invariant while it uniformly stretches or shrinks as a whole when the relativistic weak-interacting particle density increase or descrease, respectively.

Indeed, let us find a characteristic scale of the transfer function when the spectrum shape changes. As it is seen from the figure, with a good accuracy we can put it as $K = K_{eq}$, where

$$C(K_{eq}) \simeq 0.5 \quad , \quad K_{eq} = 10 \tag{15}$$

Then, the physical scale is easily related to the K - parameter:

$$K = 10\,\lambda_{eq}/\lambda \quad , \quad \lambda_{eq} = 36.8\, h^{-2}(1-\alpha)^{-1/2}\, Mpc \tag{16}$$

where $\lambda = 2\pi a R/K$ is the perturbation wave-length in Mpc.

So, the scale of the spectrum cut in large wavelengths grows when the relativistic particles (at the early epoch near t_{eq}) increase in number. And it goes up to infinity ($\lambda_{eq} \to \infty$) with $\alpha \to 1$. At the same time $C(K \equiv 10\lambda_{eq}/\lambda)$ as a function of K , is in fact insensitive to the α - parameter in the region $K < 500$.

Let us now turn to the background temperature fluctuations $\Delta T/T(\Theta)$ due to the Sachs-Wolfe effect as to be dependent on the light cosmions' amount. First , a qualitative look. If one makes a standard normalization of the primordial spectrum with the help of the density correlation function now (see eq. (12))

$$\xi(r) = \langle \delta^{(\beta)}(0)\,\delta^{(\beta)}(r) \rangle^{1/2} , \quad r = aR|\vec{X} - \vec{X}'| \qquad (17)$$

with $\xi(r_c \equiv 5h^{-1}Mpc) = 1$ at $t = t_0$ 8), then we infer simple relations (cf. eq. (11))

$$S \backsim C^{-1}(K_c) \quad , K_c \backsim h^{-1}\gamma^{-1/2} \qquad (18)$$

which imply that the cosmic background fluctuations grow with α growing:

$$\Delta T/T \backsim S \rightarrow \frac{1}{1-\alpha} \quad , (\alpha \rightarrow 1) \qquad (19)$$

A thorough analysis shows that the cosmic microwave background anisotropies are a very sensitive tool for measuring the α - coefficient. For instance, with the quadrupole upper limit $\Delta T/T(\ell=2) < 2\ 10^{-5}$, we get a reasonable upper bound for α - parameter within the framework of the model mentioned above:

$$\alpha < 0.8 \qquad (20)$$

with $h \simeq 0.5$, the Gaussian antenna beamwidth $\Theta_a = 6^o$, and assumption that galaxies trace the mass distribution. Similar estimates of α are calculating when different $\Delta T/T$ observational data are taken into account.

Note in conclusion, that this test of light cosmions resulting in the α -detection gives us the information not only about the number of the families of relativistic particles which initially were in the thermal equilibrium, but it also counts gravitons and other nonequilibrium weak interacting massless particle species whose revealing by any other means seem improbable now.

II. Barion deficits and enhancements in space

Amongst different schemes for biased galaxy formation proposed recently there are few that speculate on the idea of large special perturbations in the barion component: from hypothesis about the deficit of barions in large cosmic voids embracing scales $\sim 30 \div 50$ Mpc (cosmic bubbles) [9], and up to the assumption than all the visible matter in the Universe was born in a barionic island extanding from here to the redshifts $z \sim 4$ and barion density vanishes beyond the cosmic island $(z > 4)$ [10,11].

These and similar suggestions originate from a more general assertion that nondominating mediums do not manifest themselves dynamically now. It means that even if they were highly perturbed now (but the dominating dark matter remains homogeneous) they would not contribute to the gravitational potential, i.e. to the 'growing' adiabatic mode of perturbations.

However, the works mentioned above miss one point: since these large perturbations were created very early (e.g., barions were produced at GUT's times, $t \sim 10^{-35}$ s), they did influence the expansion dynamics during the equality epoch $t \sim t_{eq} \sim 10^{3-5}$ yrs and , as a result, they induced the 'growing' adiabatic mode with a high amplitude which, in turn, caused later large background $\Delta T/T$ - fluctuations due to the Sachs-Wolfe effect and, as a consequence, - contradictions with the observations *).

*) In principle, a special geometry (e.g., high degree spherical symmetry of the cosmic void or island, a certain position of the observer e.t.c.) could conceal these temperature fluctuations from the observer, but here we consider arbitrary density configurations (for discussion see ref. [11]).

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In other words one can say that the large perturbations in non-dominating components cannot be isocurvature now; they were purely isocurvature ($\equiv$ isothermal) at the very beginning when relativistic particles predominated, but now they are accompanied by the gravitational adiabatic perturbations of approximately the same amplitude.

Below, we briefly outline the proof.

Let us consider an early multicomponent Universe consisting of the relativistic (τ) components (which include (α) and (γ) particles, see Sect. I), and of the non-relativistic 'dust' β - particles. The latter include 'cold dark matter' (CDM) and 'barions' (b) whose portion is fixed by the parameter

$$\Omega_b \equiv \Omega_b(\vec{x}) = \varepsilon_b / \varepsilon^{(\beta)} \in (0,1) \qquad (21)$$

where $\varepsilon^{(\beta)} = \varepsilon_{CDM} + \varepsilon_b$. For simplicity, we neglect the barionic pressure and therefore barions and cold particles move together *) . Thus, Ω_b is the integral of motion, i.e. it depends only on space coordinates in the synchronous comoving frame.

Now, we consider early evolution, when the cosmological horizon was much less than a characteristic scale of variation of the Ω_b function:

$$t \ll \ell \equiv |\Omega_b / \nabla_i \Omega_b| \qquad (22)$$

Let all the components be initially at rest ($t \to 0$). Then

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*) This simplification allows one to generalize the problem : instead of 'barions' one may consider any heavy particles, e.g. the CDM-particles which inverse the problem. (Note that Ω_b takes any values from zero up to unity, see (21)).

under condition (22) the metric is locally isotropic

$$ds^2 = dt^2 - (aR)^2(dx^2 + dy^2 + dz^2) \qquad (23)$$

where $a \equiv a(t, \vec{x})$, $R \equiv const \sim t_{eq}$ *) ; and the only non-trivial equations are

$$\varepsilon^{(\tau)} = \frac{3C_1^2}{R^2 a^4} \quad , \quad \varepsilon^{(\beta)} = \frac{12 C_1 C}{R^2 a^3} \quad , \qquad (24)$$

$$\dot{a}^2 = C_1^2 + 4aC_1 C \quad , \quad (\dot{\ }) \equiv aR\partial/\partial t$$

where $C_{(1)} \equiv C_{(1)}(\vec{x})$ - space functions fixed by initial conditions. The solution for the scale factor is the following (cf. eq. (3))

$$a = C_1 \tau (1 + C\tau) \quad ,$$
$$t = R\int a\,d\tau = \tfrac{1}{2} R C_1 \tau^2 (1 + \tfrac{2}{3} C\tau) \qquad (25)$$

Let now set the barion excess to be produced extremely perturbed initially while the CDM and the dominating τ - component were spacially homogeneous (for physical mechanisms see ref. [12]). This means the following choice of $C_{(1)}$ - functions (see (21)):

$$C_1 = 1 \quad , \quad C = \frac{1}{1 - \Omega_\ell} \qquad (26)$$

So that the equality time $(\varepsilon^{(\tau)} = \varepsilon^{(\beta)})$

$$a_{eq} = \frac{1 - \Omega_\ell}{4} \quad , \quad \tau_{eq} \simeq 0.2(1 - \Omega_\ell) \qquad (27)$$

---

*) a-function and R - parameter are gauge - invariant since the reference system (23) is fixed unambiguously by the condition $u_i^{(\beta)} = 0$

For further estimates we shall substitute $\delta\Omega_b \simeq \Omega_b \simeq 0.1$ for the barion contrast on scales $\sim \ell$ (see (22))

At $\tau \ll \tau_{eq}$ when relativistic particles predominate, we have

$$
\begin{aligned}
&a = (2t/R)^{1/2} \quad , \quad \varepsilon^{(\tau)} = \frac{3}{4t^2} \quad , \\
&\varepsilon^{(\beta)} = \frac{6}{1-\Omega_b} \cdot (2R)^{-1/2} \cdot t^{-3/2} \ll \varepsilon^{(\tau)}
\end{aligned}
\tag{28}
$$

in accordance with the initial conditions. At $\tau \gg \tau_{eq}$ eqs. (24), (25) yield

$$
\begin{aligned}
&a = (1-\Omega_b)^{-1/3} \cdot (3t/R)^{2/3} \quad , \quad \varepsilon^{(\beta)} = \frac{4}{3t^2} \quad , \\
&\varepsilon_b = \frac{4\Omega_b}{3t^2} \quad , \quad \varepsilon^{(\tau)} \ll \varepsilon^{(\beta)}
\end{aligned}
\tag{29}
$$

which, in fact, is a sum of two perturbation modes. The upper line of eq. (29) presents the first expansion term (over the parameter $(t/\ell)^2 \ll 1$, see (22)) of the 'growing' adiabatic mode ($\equiv$ the quasiisotropic solution [13]). The second line in (29) describes the isocurvature perturbations in the barions and in the non-dominating τ - component.

Eqs. (28), (29) display an important conclusion: Gravitational field perturbations ($\equiv$ scale factor perturbations here), being initially vanished, arise at the non-relativistic matter dominating era $(t > t_{eq})$

$$\frac{\delta a}{a} \simeq \frac{1}{3}\Omega_b \simeq 0.03$$

These large perturbations of metric bring about cosmic microwave background fluctuations $\Delta T/T \sim 10^{-3}$-10^{-2} at angular scales $\sim 10^{\circ}$-90° which depend on the value of the variation scale of barion density contract ℓ (see (22)).

Conclusions

We claim to find scaling of the transfer function $C(\kappa) \equiv \equiv f(\lambda_{eq}/\lambda)$ of adiabatic perturbations in the multicomponent Universe. Up to an accuracy of 8 % (see ref. 6) it appears to be independent of α, the relative number of massless weakly interacting particles, for the wavelengths $\lambda > \lambda_{eq}/50 \sim h^{-2}(1-\alpha)^{-1/2}$ Mpc (see eq. (16)) while the λ_{eq}- scale is very sensitive to the α - parameter.

We found how large scale temperature fluctuations of the relic radiation grow with light particle number growing, which allows for an upper estimate $\alpha < 0.8$. It is marginally satisfied for four flavours of particles like neutrino. This is a test of the total number of particles which were relativistic at the equality epoch, $t \sim 10^{3-4}$ yrs (including gravitons and other non-equilibrium species).

We analysed observational consequences of a possible barions' deficit in large cosmic voids and of a possible existence of even larger barions' islands, and obtained that it would contradict the upper $\Delta T/T$ limits in scales $\Theta \sim 10^{\circ}$–90° .

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