SCHAEFER, H. H., *Banach Lattices and Positive Operators* (Springer-Verlag, 1974), xi+376 pp., DM 98.00, $42.70.

This book gives the first systematic account of positive linear operators acting on Banach lattices.

Chapter 1 is devoted to positive square matrices and provides valuable motivation to the remainder of the book. The ideal structure of finite-dimensional vector lattices is used extensively throughout this chapter, in which the Perron-Frobenius theory is presented. The examples of stochastic and doubly stochastic matrices and the applications of the theory to homogeneous Markov chains with finite state space illustrate the material very successfully.

Chapter 2 deals with the basic general theory of Banach lattices. The algebraic theory of vector lattices is developed in this chapter, with due emphasis being placed on duality. The special cases of AL-spaces and AM-spaces are discussed in detail; in particular their duality properties and their representation theorems are fully treated. Complexifications of vector lattices are also introduced here.

Chapter 3 begins with properties of closed ideals in Banach lattices and with valuations of vector lattices. This leads to representation theorems for a class of Banach lattices which contains all separable Banach lattices as well as all Banach lattices having an order-continuous norm. The second part of this chapter deals, amongst other things, with mean ergodic theory of semigroups of positive operators and with the representation of compact groups of positive operators on a Banach lattice.

Vector lattices of linear operators between Banach lattices are the subject of Chapter 4. Various types of tensor products of Banach lattices are introduced and relationships with the theory of integral maps and absolutely summing maps are shown. Special classes of operators are studied in detail, such as Hilbert-Schmidt operators, nuclear operators, compact operators, and kernel operators between Banach function lattices.

The final chapter of the book deals with applications of the earlier material to approximation theory, spectral theory and ergodic theory.

The book is written throughout in a clear and attractive style. It has many well-chosen examples, as well as an ample provision of exercises which should prove invaluable to the serious reader. The book is likely to become, and deserves to be, a standard reference work for both the research student and the experienced worker in this field.

A. J. ELLIS

STENSTRÖM, B., *Rings of Quotients* (Springer-Verlag, 1975), vii+309 pp., DM 92.00, $39.60.

The author states in the introduction that " the purpose of this book is not only to describe the theory of rings of quotients but also to give an introduction to the basic methods and results of ring theory at large ". He certainly succeeds in touching on an impressive number of topics in the theory of rings, some more fully than others, in the course of developing the theory of rings and modules of quotients. Rings of quotients are dealt with in a very comprehensive way and the abstract treatment is elucidated by numerous examples and exercises. The book begins with the definitions and elementary properties of modules and categories and this might seem to justify the author's claim that " the only prerequisites for a reading of this book are a knowledge of abstract algebra and the basic notions of set theory and general topology ". Although the theory is developed from the beginning I suspect that any with such flimsy apparatus would find themselves in some difficulty, and there are certainly much easier books giving an introduction to ring theory. This is a splendid book for the expert, however, and because of its comprehensiveness certainly one for the university library. The main criticism concerns the arrangement of some of the material. For example, Goldie's Theorem does not appear in the chapter on classical rings of quotients (Chapter 15) but in Chapter 2. Also it would be more natural to proceed more directly from the theory of Abelian and Grothendieck categories (Chapters 4 and 5) to the definition of quotient categories and their relationship with Grothendieck categories (Chapters 9 and 10). Instead the author interposes a discussion of torsion theories including hereditary torsion theories and their relationship to Gabriel topologies, fully bounded Noetherian rings, commutative Noetherian rings and Artinian rings (Chapters 6, 7 and 8). The first three chapters give a general introduction to modules, rings of fractions and modular lattices. Chapters 10-15 are concerned with particular rings of quotients—the maximal flat epimorphic ring of quotients, the maximal ring of quotients and the classical ring of quotients. A short preliminary version of this book was published in the Springer series " Lecture Notes in Mathematics " under the title " Rings and modules of quotients " (Volume 237, 1971).

P. F. SMITH

BLATTER, C., *Analysis* I, II and III (Heidelberger Taschenbücher, Springer-Verlag, Berlin-Heidelberg-New York, 1974), 204+180+184 pp., each volume $6.10.

This is a comprehensive first rigorous course on analysis of one and more real variables. In the first volume Dedekind sections are used to complete the rationals and form the real numbers $\boldsymbol{R}$, from which the complex numbers $\boldsymbol{C}$ are constructed in the usual way. Metric spaces are then introduced and the author achieves generality by working wherever possible in a metric space X, which can be either $\boldsymbol{R}$, $\boldsymbol{C}$ or $\boldsymbol{R}^n$. Thus continuity is defined for functions whose domain and range are subsets of the same space X. After a chapter on convergence the exponential, trigonometric and hyperbolic functions are defined by means of the series for exp z ($z \in \boldsymbol{C}$). The logarithmic function is defined on the positive reals as the inverse of the exponential function and the volume concludes with the usual theorems of differentiability, including Taylor's theorem.

The second volume is concerned with the Riemann integral including the usual techniques for evaluating definite and indefinite integrals. Curves and functions of bounded variation are discussed as well as uniform convergence and power series. The last two chapters are devoted to the study of functions from $\boldsymbol{R}^m$ to $\boldsymbol{R}^n$, including differentiability and the finding of extrema.

The third volume continues the discussion of functions of several variables with proofs of the inverse function, implicit function and immersion theorems, with applications to higher dimensional surfaces and Lagrange multipliers. Jordan content in $\boldsymbol{R}^m$ is introduced and multiple Riemann integrals. The final chapters deal with vector fields and the theorems of Green, Stokes and Gauss. The vector product $\boldsymbol{p} \times \boldsymbol{q}$ is introduced by noting that the determinantal function $\varepsilon(\boldsymbol{p}, \boldsymbol{q}, \boldsymbol{x}) = [\boldsymbol{p}, \boldsymbol{q}, \boldsymbol{x}]$ is a linear functional and so uniquely determines a vector such that $\varepsilon(\boldsymbol{p}, \boldsymbol{q}, \boldsymbol{x}) = \boldsymbol{a}.\boldsymbol{x}$ for all $\boldsymbol{x} \in \boldsymbol{R}^3$. The vector product $\boldsymbol{p} \times \boldsymbol{q}$ is then defined to be this vector $\boldsymbol{a}$. It is also of interest that the Heine-Borel Theorem (so called by the author) appears first on p. 126 of the third volume.

The book is beautifully produced and clearly written. It would be excellent as a textbook for a university course on analysis but would have to be supplemented by numerous examples and exercises as these are not included in the text.

R. A. RANKIN