CORRESPONDENCE

The Joint Editors   23 *July* 1953
The Journal of the Institute of
  Actuaries Students' Society

A Budget of Paradoxes

Sirs,

The letter from Mr H. L. Seal that you published in Vol. **12** (p. 55) accuses me (without naming me or giving the reference to enable readers readily to refer back) of five incorrect statements in one paragraph of my Birmingham address (*J.S.S.* **11**, 162). Before refuting four of them let me admit the fifth triviality (his point (4)). It was to say 'actuaries here and in Europe' instead of 'actuaries here and on the continent of Europe'.

With regard to the other four it is helpful to repeat my actual words as Mr Seal paraphrases them. They were 'the concept of the mortality table is essentially a distribution function—it is probably the oldest form of distribution function in the history of science, older than the normal distribution which originated with de Moivre'.

First, every student of actuarial science knows that the radix of a mortality table is arbitrary and without significance, that unity is a possible radix and that unity to represent certainty in probability theory is conventional and also operates purely as a radix. It is also obvious that if $F(x)$ is a distribution function with the conventional unit radix, so also is $1 - F(x)$ if we count from the other end. Thus there is no such inconsistency in referring to both l_x and $1 - l_x/l_0$ as distribution functions as is implied by Mr Seal. This disposes of his points (1) and (5).

As to the age of the concept of the mortality table, I know of no reason to suppose that this originated with Graunt. There is no need to confuse the issue by references to continuity (of l or of μl). The definition of l_x at integral ages is sufficient. Originally and also for a modern rigorous treatment the essential starting point is l and not μ or even μl. This and the fact that I said 'probably' and

was referring to 'the *concept* of the mortality table' disposes of Mr Seal's point (2). Nevertheless, it may be added that $_t p_x \mu_{x+t}$ was a corner-stone of life contingencies long before 1930, which Mr Seal gives as the date of 'the earliest explicit treatment of $l_x \mu_x / l_0$ as a probability distribution'.

There remains Mr Seal's point (3) regarding the originator of the normal distribution. There must be some subtlety here that escapes me because Mr Seal mentions the year 1733 which is the date of de Moivre's original work on the normal curve and this, according to Karl Pearson (*Biometrika*, **16**, 402, 1924), 'contains the first treatment known to me of the probability integral and essentially of the normal curve'. I might possibly agree with Mr Seal when he says that 'it was Laplace himself who first used the normal law as an independent entity' if I had some inkling of what he meant.

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Yours faithfully,

WILFRED PERKS

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P.S. Since writing the above letter Part II of *J.I.A.* **79** and the *Institute of Actuaries Year Book* 1953–54 have appeared. In the former (p. 196) Mr William Phillips refers to a mortality table in use in Rome in the 3rd century. In the latter (p. 13) there is a reference to de Moivre's work on the normal curve.

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W. P.

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