UNIVERSITY TRAINING IN APPLIED MATHEMATICS

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The problem of encouraging students of marked mathematical ability to spend the rest of their lives working in fields where the correct application of advanced mathematics is of paramount interest and importance, and of training such students for such work is, to me, as an applied mathematician, naturally one of continuing interest and concern.

Today, it is more than ever important that an applied mathematician should receive a thorough grounding in the fundamental concepts underlying mathematical analysis. Only if he has a basic understanding of the nature of mathematics itself (as opposed to an acquired facility in manipulating formulae) can he have any hope of developing the rigorous approach needed in the mathematical treatment of applied problems. To aim at anything less in the training of an applied mathematician is to invite disaster. To illustrate this, integral equations are met with in the solution of certain applied problems but work published recently by some individuals directly involved with scientific problems where these have arisen has subsequently been shown to be erroneous. The explanation appears to be lack of specialist mathematical knowledge on the part of the authors concerning the theory of the pure mathematics field involved (in this case, that of integral equations). Again, two of my own papers find their raison-d'être in a similar way, namely, correction of previous mis-applied pure mathematics theory. Such misuse of mathematics is getting all too common and would clearly not occur if the applied mathematical analysis was carried out in the first place by an applied mathematician. Unfortunately, however, there is a very great shortage of people qualified in this way. (There are plenty of mathematicians with no experience and training working in applied fields and plenty of workers in scientific and other applied fields with no

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intensive training in pure mathematics - but few mathematicians whose interest lies mainly in the applications of mathematics to external situations.)

In my view the reason for the shortage of such people (applied mathematicians) lies in the inadequacy and unsuitability of the mathematics courses at present offered in most of our universities. I will now elaborate on this.

Lack of Suitable Courses and Unsuitability of Present Courses

1. Unnecessarily wide generality and abstraction

A pure mathematician is usually not satisfied with a theorem until it has been stated in the most general form possible. This explains why Pure Mathematics has become steadily more abstract over the years, and this trend to extreme abstraction has now affected even the elementary undergraduate mathematics courses.

An applied mathematician, on the other hand, (while equally insistent upon rigorous arguments and proofs) does not need all this generality of treatment because scientific applications are usually limited to (what are for the pure mathematician) a few very special types and functions.

The effect of this generality and abstract approach upon a student of marked mathematical ability hoping to become an applied mathematician is evident. The only way at present that he can obtain a mathematical grounding sufficiently strict and rigorous to enable him to meet successfully the exacting demands of modern applied mathematical research is to take most of the standard courses in Pure Mathematics provided by the mathematics department of his university. Since these are run by pure mathematicians, the prospective applied mathematician is forced to spend a lot of time in learning the subject to a high degree of generality and abstraction (to him - unnecessary; to the pure mathematician - vital) and consequently has no time left over for the task (to him - vital; to the pure mathematician - unnecessary) of learning the techniques of successful application of mathematics in external fields. Thus the applied mathematician is "strangled at birth", ergo, we produce no applied mathematicians (only import them).

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2. Omission of subjects important to the applied mathematician

Again, apart from the unnecessarily high degree of abstraction and generality above-noted, the standard pure mathematics courses leave out many topics essential to applied mathematicians [e. g., in analysis: methods of approximations (including asymptotic expansions), theory of special functions, solution of integral equations by series of special type functions, etc.]. Also, in such vital subjects as integral equations and partial differential equations too much stress is laid on proving existence theorems under unnecessarily general conditions and not enough upon examples (taken from practical situations) of equations which can be solved by special methods (e. g. by Fourier-Bessel series): - again a result of the above noted drive of pure mathematics towards the maximum possible generality.

3. Lack of courses giving the opportunity to learn application techniques

As noted under (1) above, there is at present no provision for training the student in the art of applying mathematics to practical situations. To do this successfully requires, in addition to a good knowledge of mathematics itself, a delicate judgement and insight and an ability to cooperate successfully with specialist workers in the fields in which the applied mathematician proposes to apply his mathematics.

All these qualifications usually take years of experience to acquire and so training to obtain them should start at the undergraduate level in special "application techniques" classes.

Proposed Remedial Action

In view of the above, the need is twofold; and can be met by the provision of two new types of courses (both of which are now working well at Carleton University).

1. Mathematical methods courses

These would be courses in Pure Mathematics especially adapted for the needs of the Applied Mathematician. Here all

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the central theorems of the branches of mathematics useful in applications would be developed - with no loss in rigor - but without the high degree of generality and abstraction of the corresponding courses for the pure mathematician.

For scientists and engineers, these courses might well be useful, as they often want to gain mathematical understanding (as opposed to facility in manipulative rules) but clearly have not the time to follow the central theorems through to the extent of generality required by the pure mathematician.

For the prospective applied mathematician, the time saved in these courses by omission of unnecessary general treatment would be available for:-

2. Applied mathematics ("application technique") courses

These would be courses at the undergraduate and graduate level where the mathematical analysis learnt under the preceeding Mathematical Methods courses is applied to practical situations in the various scientific (and other) fields of application. As noted previously such courses (although of course unnecessary to the pure mathematician) are vital for the training of an applied mathematician as he will later be required to apply his mathematical knowledge in collaboration with workers specializing in his field of application.

Summary

If we are to train applied mathematicians in our universities (instead of importing them) we shall (in view of the present and continuing trend towards the abstract in undergraduate mathematics) have to stream them into separate courses; namely, the "mathematical methods" and "applied mathematics" courses described above. Since, however, an applied mathematician should be able to reason to a high degree of generality on the rare occasions when this is necessary, he should also take a few of the pure mathematics courses (especially those in analysis and differential geometry). This is the procedure adopted at Carleton.

Carleton University

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