# SOLAR MODELS WITH LOW OPACITY*

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**Abstract.** Evolutionary models of the present Sun with standard and artifically low opacity of stellar matter are obtained and adiabatic nonradial oscillations of the models are computed. The opacity $\varkappa$ in nonstandard model (which is the first in a series of future models with low $\varkappa$) was taken half as much as the Cox–Stewart opacity. The central temperature in such model is approximately 10% below the 'standard' values, which can explain the neutrino experiment results. Unlike solar models with very low heavy element abundance, the low $\varkappa$ model has approximately standard mass concentration and distribution of the matter in the outer layers – specifically, the standard characteristics of the convection zone. Hence, the spectrum of adiabatic oscillations is similar to that of the standard models and has the same capabilities for the explanation of the observed pulsations.

## 1. Introduction

The solar models with very low heavy element abundances in interiors (say, $Z = 0.002$) have low central temperature and can explain the experimental results on detecting solar neutrinos (see Bahcall *et al.*, 1973, and other references in Davis and Evans, 1978). However, these models have some defects, too. There is a very shallow convection zone in these models: it is only 30–100 thousand kilometers deep and has temperature at the bottom less than $10^6$ K (Christensen-Dalsgaard *et al.*, 1979; see also Iben and Mahaffy, 1976). This does not agree with the interpretation of observed 5-min oscillations of small horizontal scale as acoustic modes of non-radial oscillations at high values of spherical harmonics (Rhodes *et al.*, 1977) and contradicts the hypothesis of thermonuclear destruction of lithium (Herbig, 1965; simple evaluations show that to explain the observed deficiency of lithium at normal abundance of berillium, the temperature at the bottom of the convection zone should approximately range from $2 \times 10^6$ to $3 \times 10^6$ K). The interpretation of 5-min oscillations of low degree (Grec *et al.*, 1980) also favours the standard solar models with a deep convection zone of about $200 \times 10^3$ km (Christensen-Dalsgaard and Gough, 1980; Shibahashi and Osaki, 1983). Moreover, the origin of models with low heavy element abundances in interiors and normal abundances on the surface requires a detailed investigation of interaction between accretion of interstellar matter and solar wind throughout the evolution of the Sun.

The main 'useful' effect of very low $Z$ – low central temperature – could be reached at normal $Z$, if it turned out that solar interiors are more transparent than it follows from the radiative transfer law. Newman and Fowler (1976) considered solar models with postulated additional energy transport mechanisms. From the formal point of view this phenomenon corresponds to the reduction of effective opacity of the solar matter. Quite recently a very important physical argument for that reduction has appeared. Opher (1981) showed that the proper treatment of the plasmon – electron interactions leads

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to an increase in the number of the high energy electrons compared with the Maxwell–Boltzmann distribution and, hence, to the opacity reduction approximately by half. The author emphasizes that his results contain no free parameters. Note that other mechanisms of the effective $\varkappa$ decrease are possible as well; for example, under

TABLE I

Solar model characteristics

| Model<br>Effective opacity | 1<br>$\varkappa_{\text{table}}$ | 2<br>$\varkappa_{\text{table}}$ | 3<br>$\varkappa_{\text{table}}$ | 4<br>$\frac{1}{2}\varkappa_{\text{table}}$ |
|---|---|---|---|---|
| Initial heavy element abundance, $Z_0$ | 0.01 | 0.02 | 0.03 | 0.02 |
| Initial hydrogen abundance, $X_0$ | 0.77 | 0.73 | 0.693 | 0.86 |
| Mixing length: pressure scale height, $\alpha = l/H_p$ | 1.24 | 1.41 | 1.52 | 1.285 |
| Luminosity, $L/L_\odot$ | 1.0049 | 0.9997 | 1.0016 | 1.0090 |
| Radius, $R/R_\odot$ | 1.0014 | 0.9985 | 0.9990 | 1.0021 |
| Age ($10^9$ yr) | 4.50 | 4.55 | 4.58 | 4.58 |
| Central hydrogen abundance, $X_c$ | 0.39 | 0.34 | 0.30 | 0.46 |
| Central temperature, $T_c$ ($10^6$ K) | 14.2 | 14.6 | 14.9 | 13.2 |
| Central density (g cm$^{-3}$) | 127 | 135 | 142 | 133 |
| Density at the bottom of convection zone (g cm$^{-3}$) | 0.115 | 0.185 | 0.241 | 0.178 |
| Depth of convection zone ($R_\odot$) | 0.244 | 0.279 | 0.297 | 0.274 |
| Mass of convection zone ($M_\odot$) | 0.0141 | 0.0245 | 0.0315 | 0.0230 |
| Temperature at the bottom of convection zone, $T_b$ ($10^6$ K) | 1.72 | 2.11 | 2.36 | 1.89 |

the influence of propagation to the solar center of high frequency internal gravity waves generated at the bottom of the convection zone (Press, 1980)*. So, Opher's result is not confirmed, it is reasonable to investigate these models in more detail. Also, the decrease of effective opacity in stellar interiors can be useful in solution of some other astronomical problems, for example, of the problem of Cepheid masses, as it was investigated by Fricke *et al.* (1971).

* More detailed analysis by Press (1981, *Astrophys. J.* **245**, 286) and by Press and Rybicki (1981, *Astrophys. J.* **248**, 751) shows that internal waves may cause the increase of the effective opacity in radiatively stable regions.

At present we are investigating the main characteristics of models with artificially low opacity of solar matter. Some preliminary results are described below.

## 2. The Models

All models have solar luminosity, radius, and age of about 4.5–4.6 × $10^9$ yr. Evolution from initial main sequence was calculated according to Schwarzschild's fitting method with the Cox and Stewart opacity tables and with treatment of convection according to the mixing – length theory. The rates of nuclear reactions of proton – proton and CN cycles were taken from Fowler *et al.* (1975). The main characteristics of models are listed in Table I and illustrated in Figure 1.

Models 1, 2, 3 are probably a reasonable set of standard models of the Sun. They have been calculated with opacity values from tables and the results may be compared with the results of similar calculations (Iben and Mahaffy, 1976; Dziembowski and Pamyatnykh, 1978; model A by Christensen-Dalsgaard *et al.*, 1979). Model 4 has been calculated with $Z$ = 0.02 and with opacity coefficients which are half as much as the

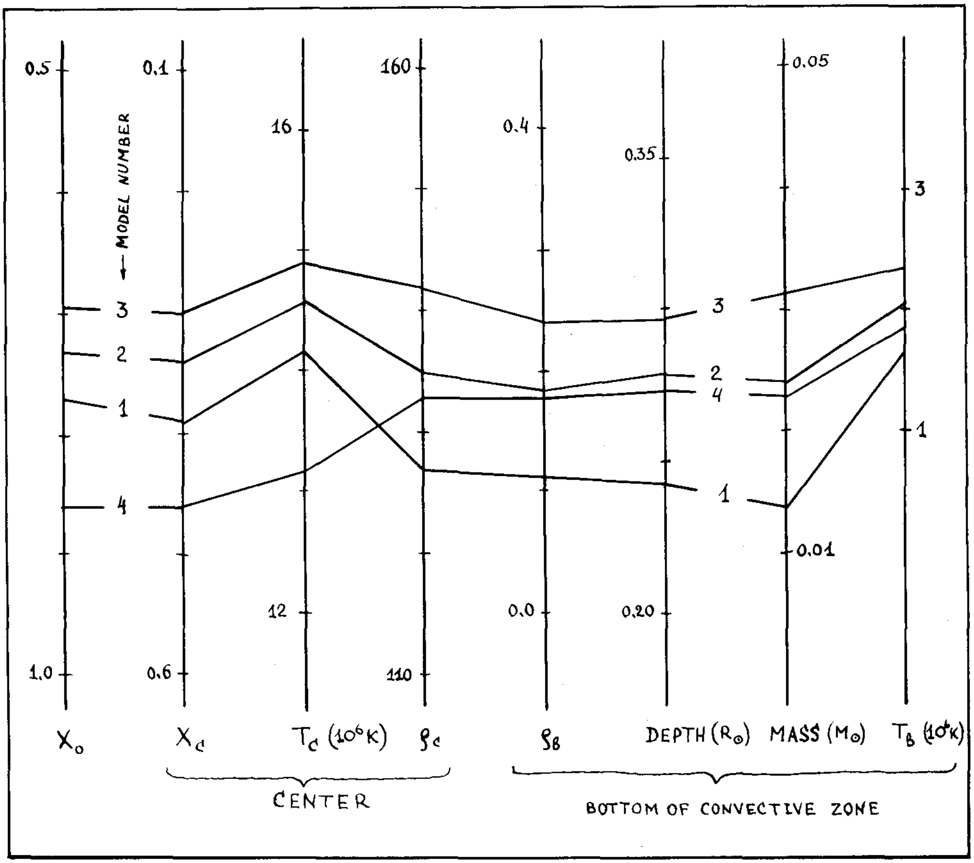


Fig. 1. Basic characteristics of different solar models. All scales are linear.

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corresponding table values. This is the first in a series of future models with low $\varkappa$. Some parameters of this model may be compared with the results reported by Newman and Fowler (1976). In calculations of thermonuclear reactions we, for simplicity, did not take into account the branch *pp* III of $p-p$ cycle, whose role is negligible in the total energy generation, but which produces the main contribution (from decay of $B^8$) to the high energy neutrino flux. Therefore, in Table I there are no theoretical evaluations of the rate of neutrino capture by $Cl^{37}$. In models without mixing in interiors the high energy neutrino flux is approximately proportional to $T_c^{13}$ (see, for example, Iben, 1969), and for agreement of the experimental results with theoretical predictions it is enough to decrease $T_c$ by about $\sim 10\%$, which will reduce the flux $\sim 3.5$ times (for new data see, for example, short review by Peak, 1980). Model 4 has just this change of $T_c$ in comparison with model 2, and besides both models have the same heavy element abundance: $Z = 0.02$.

The important difference between our model with low $\varkappa$ and models with low $Z$ (say, $Z = 0.002$) is the standard distribution of matter and standard characteristics of the convective zone. These results are clearly displayed in Figure 1. For example, the central densities in models 2 and 4 do not differ by more than 2% and there is a similar difference in depths of the convection zones. So, the mechanical properties (specifically, the periods of the oscillations) of our model with low $\varkappa$ may be expected to be similar to those of the standard models. The lithium-berillium problem can also be solved in the models with low $\varkappa$ according to the hypothesis of the lithium thermonuclear depletion at the bottom of the convection zone*.

### 3. Oscillations of the Models

We have computed periods of several modes of adiabatic oscillations of low degree (both nonradial and radial) using the programs which were kindly placed at our disposal by W. Dziembowski (the programs are described by Dziembowski, 1977). The frequences of oscillations were computed as eigenvalues of the equations. The Eulerian perturbation of gravitational potential was taken into account. The outer boundary conditions were applied at optical depth $\tau = 0.01$ (the atmosphere was computed in the diffusion approximation). The results of computations are listed in Table II, where the conventional classification of gravity and acoustical modes is used. Frequences (in mHz), and not periods, for acoustical modes of high order are listed to simplify the comparison with other investigations.

Comparison of the periods of low order modes for different models shows that the periods of model 4 with low $\varkappa$ lie between those of standard models 1 and 2 as it should be expected from the above mentioned general considerations. Oscillations with periods

* In the course of Sun evolution from ZAMS to the present state the depth of the convection zone is continuously decreased. Lithium could be depleted at early phases of Sun evolution on the main sequence. Hence, the models of the Sun with $T_b \sim 2.0 \times 10^6$ K may be sufficient for solving the problem, since at early phases these models had $T_b \sim 2.5 \times 10^6$ K. The problem of lithium depletion and the effect of uncertainty of the mixing-length theory parameters was discussed, for example, by Ergma and Pamyatnykh (1971).

TABLE II

Solar models oscillations

(a) Oscillation periods of several gravity modes (min)

| Model | $l = 1$ | | | $l = 2$ | | | |
|---|---|---|---|---|---|---|---|
| | $g_1$ | $g_2$ | nearest to $160^m$ | $f$ | $g_1$ | $g_2$ | nearest to $160^m$ |
| 1 | 69.44 | 96.36 | $g_5$: 168.26 | 51.73 | 60.65 | 69.83 | $g_9$: 160.20 |
| 2 | 66.53 | 91.86 | $g_5$: 160.25 | 49.38 | 59.33 | 68.02 | $g_{10}$: 165.28 |
| 3 | 64.29 | 88.34 | $g_5$: 154.26 | 47.61 | 57.87 | 66.53 | $g_{10}$: 159.06 |
| 4 | 67.59 | 93.44 | $g_5$: 163.19 | 50.00 | 59.77 | 68.75 | $g_9$: 155.49<br>$g_{10}$: 168.40 |

(b) Oscillation periods of the low order acoustic modes (min)

| Model | $l = 0$ | | $l = 1$ | | $l = 2$ | | $l = 3$ | |
|---|---|---|---|---|---|---|---|---|
| | $p_1$ | $p_2$ | $p_1$ | $p_2$ | $p_1$ | $p_2$ | $p_1$ | $p_2$ |
| 1 | 63.33 | 41.93 | 58.55 | 37.28 | 45.25 | 32.69 | 40.66 | 29.78 |
| 2 | 64.84 | 41.69 | 58.94 | 37.14 | 44.86 | 32.49 | 40.44 | 29.57 |
| 3 | 65.74 | 41.69 | 58.95 | 37.19 | 44.55 | 32.48 | 40.44 | 29.54 |
| 4 | 64.95 | 41.76 | 59.24 | 37.31 | 44.93 | 32.65 | 40.57 | 29.70 |

(c) Frequences of the high order acoustic modes of low degree (mHz)

| Model | $l = 0$ | | | $l = 1$ | | |
|---|---|---|---|---|---|---|
| | $p_{20}$ | $p_{21}$ | $p_{22}$ | $p_{20}$ | $p_{21}$ | $p_{22}$ |
| 1 | 2.8999 | 3.0387 | 3.1785 | 2.9641 | 3.1032 | 3.2428 |
| 2 | 2.9253 | 3.0650 | 3.2056 | 2.9900 | 3.1298 | 3.2704 |
| 3 | 2.9297 | 3.0690 | 3.2096 | 2.9942 | 3.1340 | 3.2745 |
| 4 | 2.9093 | 3.0492 | 3.1892 | 2.9742 | 3.1139 | 3.2540 |

| Model | $l = 2$ | | | $l = 3$ | | |
|---|---|---|---|---|---|---|
| | $p_{20}$ | $p_{21}$ | $p_{22}$ | $p_{20}$ | $p_{21}$ | $p_{22}$ |
| 1 | 3.0270 | 3.1669 | 3.3070 | 3.0859 | 3.2260 | 3.3671 |
| 2 | 3.0537 | 3.1944 | 3.3351 | 3.1133 | 3.2545 | 3.3959 |
| 3 | 3.0582 | 3.1989 | 3.3395 | 3.1182 | 3.2593 | 3.4007 |
| 4 | 3.0378 | 3.1779 | 3.3188 | 3.0970 | 3.2377 | 3.3795 |

of about 160 min are similar in all models: this period is corresponding to the dipole mode $g_5$ and the quadrupole mode $g_9$ or $g_{10}$. Hence, the problem of 160-min oscillations in models with low $\varkappa$ may be the same as in the 'standard' models.

The frequences of acoustic modes of high order are corresponding to the observed 5-min pulsations of large horizontal scale (Grec *et al.*, 1980). The results for models with low $\varkappa$ lie between those for standard models 1 and 2, too. However, as it can be seen from Table IIc, the frequency separation between the adjacent eigenfrequences for a given $l$ is greater than the double difference (136.0 μHz) between peaks in power spectrum of the observed pulsations. For computed eigenmodes $p_{15} - p_{23}$ (in the Table IIc frequences only for $p_{20} - p_{22}$ are listed) with $l = 1$ we have the average frequency separation $\Delta_{nl}\nu$ = 138.2 μHz for model 1, 139.0 μHz for model 2, 138.9 μHz for model 3, and 138.8 μHz for model 4. The differences between the calculated eigenfrequences and the observed values result partly from the neglecting of role of the atmosphere as it was shown by Christensen-Dalsgaard and Gough (1980). (We have computed the lower atmosphere, but very roughly.) Besides, the frequency separations $\Delta_{nl}\nu$ in our models are approximately 1 μHz greater than the corresponding values for model A by Christensen-Dalsgaard *et al.* (1979) also computed without the account of the atmosphere. Note that the depth of the convection zone in their model A with $Z = 0.02$ is lower than in our 'shallowest' model with $Z = 0.01$. It may be connected with differences in age: the age of the model A is $4.75 \times 10^9$ yr. In future we intend to investigate both the influence of the atmosphere and the effect of age (say, between 4.5 and $5.0 \times 10^9$ yr) in models with solar luminosity and radius. It is important to us here that the low $\varkappa$ models have, in fact, the 'standard' frequences and the frequency separations of acoustic modes of high order: e.g., the frequences listed in Table IIc for model 4 lie between those for models 1 and 2.*

## 4. Conclusions

Thus, our preliminary results show that the solar models with low $\varkappa$ and with normal heavy element abundance are potentially very useful for the explanation of such observed facts as the solar neutrino flux, the lithium and berillium abundances in solar atmosphere and 5-min oscillations of low and high degree. The defect of these models (as well as the models with low $Z$) is a fairly low initial helium abundance, $Y = 0.12$ in model 4. We hope, as it may be expected from the results by Newman and Fowler (1976), that the models with low $\varkappa$ in interiors and with standard $\varkappa$ in the envelope will have more reasonable values of the helium abundance, e.g., $Y = 0.2$.

In conclusion we have a comment about the frequency separation $\Delta_{nl}\nu$ between the frequences of the acoustic modes of high order for the standard models. In our models the frequency separation increases by about 0.8 μHz as $Z$ increases from 0.01 to 0.02, i.e. as depth of the convection zone increases from $170 \times 10^3$ to $195 \times 10^3$ km (see Tables I and IIc). This result is similar qualitatively to the one reported by Christensen-

* See note added in proof.

Dalsgaard *et al.* (1979), Christensen-Dalsgaard and Gough (1980) and does not agree with the results by Scuflaire *et al.* (1981) (in any case, with the difference $\Delta_{nl}\nu$ between their models 1 and 2 whose convection zone parameters are similar roughly to those of our models 1 and 3). The decrease of $\Delta_{nl}\nu$ with the increase of the depth of the convection zone in the envelope models computed by Scuflaire *et al.* (1981) may be easily understood since these models do not fit the corresponding interior models. In such envelope models the average temperature decreases with the increase of the depth of the convection zone. It would lead to the decrease of the average sound speed and, thus, to the decrease of the value $\Delta_{nl}\nu$ (see the discussion of the $\Delta_{nl}\nu$ in Christensen-Dalsgaard *et al.*, 1979). However, for the fitting evolutionary models the situation will be more complicated: the larger is the depth of the convection zone the larger is, on the average, temperature in interiors (which leads to the increase of the sound speed and $\Delta_{nl}\nu$) and the higher is helium abundance (which leads to the decrease of the sound speed due to the effect of the molecular weight). It is possible, that for the very deep convection zones the frequency separation $\Delta_{nl}\nu$ varies in the direction reported by Scuflaire *et al.* (1981) due to the decrease of the average temperature in the very extensive convection zone. However, for moderately deep convection zones (as is the case for our models 1 and 2) we have the opposite effect. This remark explains the difference in $\Delta_{nl}\nu$ behaviour between our models and envelope models by Scuflaire *et al.* (1981).

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**Note added in proof:** Our new computations show that the difference between calculated and observed $\Delta_{nl}\nu$ (see Section 3) results mostly from too small number of discrete mass zones ($\sim 450$) in our models: when we have $\sim 4 \times 450$ mass zones, $\Delta_{nl}\nu$ is in much better agreement with the observations and, for example, with the results by Shibahashi and Osaki (1983) where the same effect has been noted and discussed.

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