

of mathematics. Andrews is well known as a world authority on the subject, having written many papers and books in that area, and Eriksson is a professor of applied mathematics who has recently written several papers on lecture-hall partitions. Together they have produced a book that is very suitable for adoption as a textbook for a course of lectures or for a reading course for honours degree undergraduates; it is very carefully structured to support self-study. I recommend it as an excellent introduction to a fascinating topic.

I. ANDERSON

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RUSS, S. *The mathematical works of Bernard Bolzano* (Oxford University Press, 2004), xxx + 698 pp., 0 19 853930 4 (hardcover), £125.

Bernard Bolzano (1781–1848) was born in Prague, his father being an Italian immigrant. In 1804 he took holy orders and in 1805 he was appointed to the newly established Chair in Philosophy of Religion at the University of Prague. However, the enlightened views that he put forward did not find favour with the authorities and he was dismissed from this post in 1819 with an interdict on publishing; persecution continued until the mid 1720s. Bolzano is of course familiar to us through his association with the Bolzano–Weierstrass theorem, but probably not a lot more is generally known about him. His mathematical work covered logic, foundations and rigorous analysis; in some aspects he anticipated, and even had priority over, other better-known mathematicians whom we associate with the development of key concepts.

This volume grew out of translations which appeared in Dr Russ's PhD thesis. The works presented are the following (the titles are given as in the translations); each item is provided with a detailed introduction and footnotes are provided to clarify textual and mathematical points.

- (i) *Considerations on some objects of elementary geometry (1804)*. Here Bolzano is concerned to develop a theory of triangles without assuming the existence of a plane; arguments involving motion are also disqualified.
- (ii) *Contributions to a better-grounded presentation of mathematics (1810)*. As the title suggests this is concerned with logic and proofs.
- (iii) *The binomial theorem and as a consequence from it the polynomial theorem and the series which serve for the calculation of logarithmic and exponential quantities proved more strictly than before (1816)*. Ideas of convergence are discussed, the Cauchy criterion is applied, there is an extensive discussion of the binomial series, and series for exponentials and logarithms are also discussed.
- (iv) *Purely analytic proof of the theorem that between any two values, which give results of opposite sign, there lies at least one real root of the equation (1817)*. The 'intermediate value theorem' is the main topic of this item; again the Cauchy criterion is assumed. The discussion of sups and infs incorporates Bolzano's version of the Bolzano–Weierstrass theorem.
- (v) *The three problems of rectification, complanation and cubature, solved without consideration of the infinitely small, without the hypotheses of Archimedes and without any assumption which is not strictly provable (1817)*. Taylor's theorem for functions of several variables is applied in a theory of length, area and volume.
- (vi) *Pure theory of numbers, seventh section: infinite quantity concepts (posthumous)*. Here Bolzano sets up the real numbers (measurable numbers) using a process of bracketing by related rational fractions; the Cauchy criterion is established for his system.

- (vii) *Theory of functions with improvements and additions to the theory of functions (post-humous)*. This is a course on rigorous analysis as far as continuity and differentiability, including Taylor's theorem with remainder term; functions of several variables are also considered. An example is given of a continuous function which neither increases nor decreases on any subinterval of its domain and in consequence is nowhere differentiable; this is different from the famous Weierstrass example. The 'Additions' are in note form and take up some ideas of uniform continuity and the convergence, continuity and differentiability of series of functions.
- (viii) *Dr Bernard Bolzano's paradoxes of the infinite, edited from the writings of the author by Dr F. Příhonský (1851)*. Infinite subsets of infinite sets are discussed and the idea of comparing infinite sets by pairing off their elements is introduced. However, much of this item is devoted to a philosophical discussion of the nature of space and similar ideas.

Bolzano intersperses his mathematics with many interesting comments and he is not slow to point out perceived deficiencies in the proofs or ideas of other mathematicians: Cauchy, Laplace, Lagrange and Euler, among others, come in for some criticism. However, he himself is not infallible: for example, in (vii) (§ 155) he appears to assert that the rule for differentiating a sum of two functions extends trivially to an infinite sum of functions; in the 'Additions' (§ 2) he has realized that this is not so straightforward but I do not find his revised version convincing.

Apart from dealing with the mathematics, the translator is faced with two main problems which are not always reconcilable: producing a text which is acceptable in the new language and which preserves the style and underlying philosophy of the author. Dr Russ explains his approach in his introductory 'Note on the translations' and I am sure that he has done an excellent job in making Bolzano's work available in English. However, I feel that a little paraphrasing could have eased the reading: for example, in my experience, the 'one' construction has never been common in mathematical English, whereas phrases such as *man kann zeigen* abound in German mathematics; I wonder if any harm would be done if this were to be translated as *it can be shown* or even as *we can show* rather than *one can show*; for my taste, the 'one' construction is used excessively in the translations. Another personal quibble is that the words *mere* and *merely*, again not common choices for me at least, seem to be much overworked.

The book will of course be of interest to mathematical historians but anyone involved in teaching rigorous analysis could benefit from studying it, especially item (vii), and Bolzano's construction of the real numbers in (vi) might provide an interesting alternative to the approaches of Dedekind and Cantor.

I. TWEDDLE

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SCHECHTER, M. *An introduction to nonlinear analysis* (Cambridge University Press, 2005), 376 pp., 0 521 84397 9 (hardback), £40.

This book is no ordinary introductory text. It is based on lectures where the author's aim is to quickly introduce some techniques of nonlinear analysis that can be used in a variety of situations. In order not to restrict the course to advanced students only, background material from functional analysis is called on as and when needed but is not presented in the main body of the text. However, for reference purposes this material is listed in some appendices.

The style is not 'theorem, proof, example', but begins by posing a problem to be solved and gradually develops the tools to solve it. The emphasis is on developing methods and showing how they are useful in many nonlinear problems. Problems from differential equations have been used to do this because of their familiarity, and accordingly they do not demand too much preparation.

The first part of the book concentrates on critical point theory. This is motivated by the study of periodic solutions of a second-order ordinary differential equation in the first chapter. The question posed is as follows.

Can the problem be thought of as some equation $G'(u) = 0$ for some differentiable functional in a suitable Hilbert space?