On the theory of row-column designs

Kenneth Graham Russell

If an experiment is to be conducted in a heterogeneous medium, the
experimenter may endeavour to reduce the experiment's inherent variability
by dividing the medium into segments which are relatively homogeneous. If
this requires it to be divided in two different directions, the medium may
be considered to be partitioned into a number of rows and columns.
Treatment is then applied to the experimental units in each homogeneous
segment, and the resulting layout is known as a row-column design, or a
design for the two-way elimination of heterogeneity. The theory of such
designs is applicable to any design where the systematic variability is due
to three distinct factors.

This thesis examines the theory of row-column designs. In two
preliminary chapters, known results on estimability and orthogonality, and
on the testing of hypotheses, are presented. All work is done under the
assumption of the linear additive fixed-effects model.

In Chapter 4 the orthogonality of two factors after each has been
adjusted for a third factor is defined, and a necessary and sufficient
condition for its existence is obtained. The presence of adjusted
orthogonality is shown to simplify the requirements for a factor to be
connected.

Chapter 5 treats the estimability and connectedness of a factor of a
row-column design. A method of determining the estimable contrasts of a
factor's levels, and estimators of them, is presented. It is shown that,
in certain cases, these estimators are the least squares estimators. The
method of estimation is similar to that of Birkes, Dodge, and Seely [1].

Chapter 6 considers the introduction of a second blocking system into

balanced incomplete block designs to convert them to row-column designs.
As such, they are a sub-type, $0 : XB$, of $0 : XT$ designs (using Pearce's
notation [2], where X represents orthogonality or the degree of balance).
$0 : XB$ designs are shown to be pairwise connected, and a necessary and
sufficient condition is obtained for the overall connectedness of $0 : XT$
designs. The (M, S)-optimality criterion is used to optimise $0 : XB$
designs. M-optimal designs, which are obtained by use of systems of
distinct representatives, are shown to be connected, and are categorised
into three classes. Procedures are given for obtaining the (M, S)-optimal
design in two of these classes. The third class requires trial-and-error
methods, and the most efficient designs obtained are displayed.

The final chapter extends the results of Chapter 4 to the multi-factor
case, and the concept of adjusted orthogonality is considered in complete
generality. Classical orthogonality between pairs of factors is shown to
be a particular case of adjusted orthogonality.

The appendix contains several APL computer programs for use with row-
column designs. Additionally, a proof is given that, for many parameter
sets, all $0 : XB$ designs are connected.

References

[1] David Birkes, Yadolah Dodge and Justus Seely, "Spanning sets for
         estimable contrasts in classification models", *Ann. Statist.* **4**
         (1976), 86-107.

[2] S. C. Pearce, "The use and classification of non-orthogonal
         designs", *J. Roy. Statist. Soc. Ser. A* **126** (1963), 353-369.