from there develops the standard results of the subject as well as delving more deeply in some topics to which Professor Kasch has made important contributions. The original book in German was very well received, and so, no doubt, will be this very good translation by Professor Wallace.

The author believes "that the concepts of projective and injective modules are among the most important fundamental concepts of the theory of rings and modules" and accordingly starts with a chapter on the fundamentals of category theory. But this is kept within bounds and only the material needed later is developed. Categories are used as a tool here and the introduction is compact and informative without getting bogged down in "abstract nonsense". The next chapter introduces modules, submodules and factor modules; then there is a treatment of homorphisms of modules and rings. Chapter 4 is on direct products, sums and free modules. The next topics treated are: injective and projective modules, artinian and noetherian modules, local rings and the Krull–Remak–Schmidt theorem, semisimple modules and rings and the radical and socle. This covers what might be called the standard theory. Use is made of categorical ideas, in particular the idea of generators and cogenerators are used in many places.

The last third of the book develops the material of a more specialized nature. This consists of four chapters: the first on the tensor product, flat modules and regular rings, the next on semiperfect modules and perfect rings, then rings with perfect duality and finally quasi-frobenius rings. Many of the ideas studied here arise from the imposition of various finiteness conditions on the generators or cogenerators or both. This leads to a number of very satisfying characterisation results. Each chapter in the book has a collection of exercises at the end, of varying difficulty, which help to understand and apply further the ideas from the preceding pages.

This is a useful addition to the literature. It is quite pleasant to read and presents its material well. The first part of the book could be used as an optional final year undergraduate course on ring theory, and the last part as a series of seminars for postgraduate students. The specialised material presented is not too esoteric and presents a coherent and well-developed theory. So in terms of content this is a very satisfactory book. The standard of printing and production is high, but the number of misprints is rather on the high side. Fortunately none should cause real trouble, though some will cause the reader a few puzzled moments. The price, at nearly 10p/page, seems high or is that wishful thinking for a book of this nature?

<div align="right">J. D. P. MELDRUM</div>

ARROWSMITH, D. K. and PLACE, C. M., *Ordinary differential equations: a qualitative approach with applications* (Chapman and Hall, 1982), 250 pp., cloth £18, paper £7.95.

This undergraduate textbook develops the qualitative theory of differential equations and applications in mathematical modelling. It is intended for approximately the third year level (Scottish pattern) and assumes a knowledge of basic calculus with several variables and linear algebra. Naturally, great emphasis is placed on geometrical aspects and there is a very effective use of diagrams (well over half the text pages contain a diagram). The special orientation leads to a particular selection of material and the omission of certain important topics in differential equations. Consequently, the book may be more appropriate for a second course, following a more traditional one.

In the introduction, first order equations and two-dimensional systems are employed to introduce the fundamental geometrical notions of phase portrait, qualitative equivalence, flow and evolution. Existence and uniqueness theorems are stated. The geometrical emphasis means that an example using isoclines is worked out in the text, whereas techniques of separation, homogeneous equations, exact equations and integrating factors are reserved for exercises.

The second chapter, on linear systems, contains a classification of autonomous, homogeneous systems by the real, canonical form of their matrices ("algebraic type") and by the idea of qualitative equivalence ("qualitative type"). The evolution operator is defined and used in the solution of inhomogeneous (affine) systems.

The third chapter concerns non-linear systems in the plane and discusses local and global phase portraits. Local theory is represented by theorems on linearisation and stability; global theory by

study of first integrals and Poincaré–Bendixson theory. There is no attempt even to sketch a proof of the main theorems.

Applications appear mainly in chapters four and five, separated from the theory. A reader would miss a very great deal if he omitted these chapters. Specific applications are both "classical and modern". As well as the usual mechanical and electrical oscillations, there is an application of the linear theory in economics for which "negative damping" has a meaningful interpretation. Applications of the non-linear theory include competing species, prey–predator eco-systems, relaxation, regularisation, Liénard equation, further economic models, the Zeeman heartbeat and nerve impulse models, Liapunov functions, bifurcation, and a model for tumour growth. Care is taken to explain the thinking, both mathematical and non-mathematical, involved in each modelling process. This most difficult and important aspect of the applied mathematician's work is so often neglected because it is difficult to teach. There is a satisfying thoroughness about the treatment of these examples. For instance, the authors are not content to describe the Volterra–Lotka prey–predator model (as appears in many textbooks), but take the opportunity to teach valuable lessons by criticising it, and putting forward the more sophisticated Holling–Tanner model.

JOHN MARTIN

BOOKS RECEIVED

M. Kline, *Mathematics: the loss of certainty* (Oxford University Press), paper £4.95.
C. Bandelow, *Inside Rubik's cube and beyond* (Birkhäuser Verlag), paper £5.35.
Collected papers of J. E. Littlewood (ed. by a Committee appointed by the London Mathematical Society) (Oxford University Press), 2 vols. each £60.
R. D. Mauldin (ed.), *The Scottish book: mathematics from the Scottish Café* (Birkhäuser Verlag), £18.
P. Hall, *Rates of convergence in the central limit theorem* (Pitman), £10.50.
J. M. Hill, *Solution of differential equations by means of one-parameter groups* (Pitman), £8.50.
M. S. P. Eastham, *Schrödinger-type operators with continuous spectra* (Pitman), £12.50.
S. C. Power, *Hankel operators on Hilbert space* (Pitman), £7.
S. Campbell (ed.), *Recent applications of generalized inverses* (Pitman), £11.
P.-L. Lions, *Generalized solutions of Hamilton-Jacobi equations* (Pitman), £12.50.