A MONTE CARLO SIMULATION OF THE MASS DISTRIBUTION IN AN ACCRETING SYSTEM OF DUST PARTICLES

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This paper investigates the way in which the mass distribution index, s, of the particles in a dust cloud varies as a function of time and particle mass, assuming that accretion, and only accretion, takes place after interparticle collisions. The approach is an extension of one first put forward by Napier and Dodd (1974). Two cases are considered. In the first the cloud starts with 500 particles of unit mass and as accretion takes place, more unit mass particles are added to keep that number constant at 500. This simulates a constant source feeding the low mass end of the distribution. In the second case the cloud starts with 5000 particles of unit mass and no additions are made. This corresponds to accretion in a closed system.

Let the number of mass m_i particles be C_i. The probability p_{ij} of a collision between a particle of mass m_i and one of mass$_j$ is proportional to C_i and C_j and also the total cross-sectional area of m_i and m_j particles.

$$p_{ij} = C_i.C_j.(m_i^{\frac{1}{3}} + m_j^{\frac{1}{3}})^2.P^{-1} \qquad j \neq i \qquad (1)$$

$$p_{ii} = 4.C_i.(C_i - 1).m_i^{\frac{2}{3}}.P^{-1} \qquad j = i \qquad (2)$$

Normalisation requires that the sum of the collision probabilities for all particles must be unity. P, the normalising function in equations 1 and 2 is given by

$$P = \tfrac{1}{2} \sum_i \sum_{j \neq i} C_i.C_j.(m_i^{\frac{1}{3}} + m_j^{\frac{1}{3}})^2 + \sum_i 4.C_i.(C_i - 1).m_i^{\frac{2}{3}}.$$

The first term accounts for collisions between particles of different masses and the second term for collisions between particles of the same mass. As i and j are just mass labels covering the same mass range they are interchangeable and it can be shown that

$$P = ((\sum_i C_i) - 4).(\sum_i C_i\ m_i^{\frac{2}{3}}) + (\sum_i C_i.m_i^{\frac{1}{3}})^2 + 2.(\sum_i C_i^{\,2}.m_i^{\frac{2}{3}}).$$

This can be computed more readily as each term involves only a single summation.

I. Halliday and B. A. McIntosh (eds.), Solid Particles in the Solar System, 325–328.

A Monte Carlo technique is applied to the collision problem. Masses m_i and m_j of the two colliding particles are picked independently by using a random number generator such that all masses are equally likely to be picked. Then p_{ij} is computed taking into account the number of particles in each mass class and their cross-sectional areas. p_{ij} is compared with a randomly generated number r which has a value between zero and unity. p_{ij} also has a value between zero and unity, and if $p_{ij} \geq r$ accretion occurs, a particle of mass $m_i + m_j$ is produced and the collision number increased by one. If, however, $p_{ij} < r$ the collision is not allowed. P is recalculated after each collision.

The results of the first run, which had $C_1 = 500$ for all collisions, are shown in Fig. 1. The massing of lines in the $1 < m < 100$ range shows that an equilibrium distribution is being approached after a large number of collisions.

In the second run the total mass of particles remained constant. At the beginning $C_1 = 5000$, $C_{i \neq 1} = 0$. Eventually, after many collisions, the distribution will be $C_{5000} = 1$, $C_{i \neq 5000} = 0$. The distribution obtained is shown in Fig. 2. Lines are again drawn for the distribution after 100, 200, 300 etc. collisions have taken place. In the second case it is obvious that N_i does not reach an equilibrium

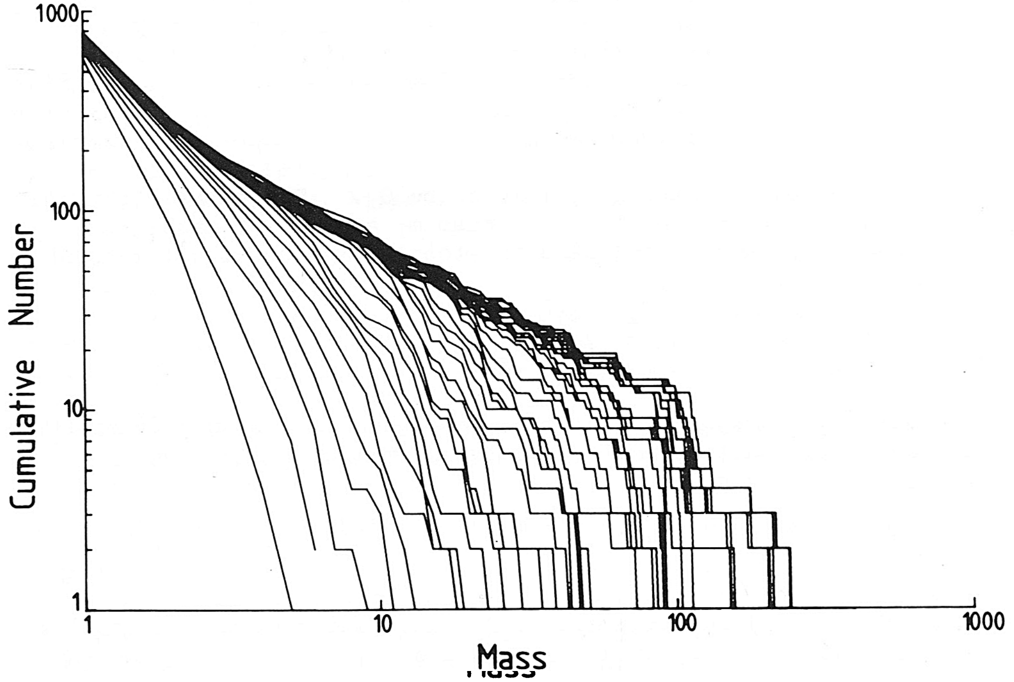

Fig. 1 A logarithmic plot of the cumulative number of particles N with masses greater than m as a function of m for the case where C_1 remains constant at 500. The left hand line is for the case after 100 collisions and lines are plotted for subsequent additions of 100 collisions, the last (right hand) line being the distribution after 3700 collisions.

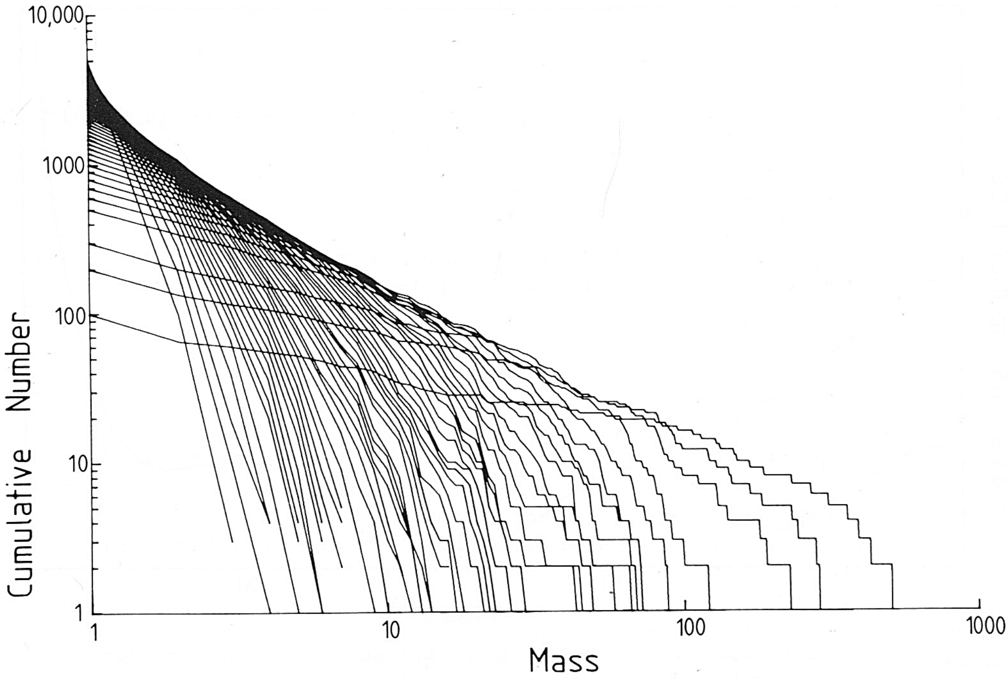

Fig. 2 A logarithmic plot of the cumulative number of particles N, with masses greater than m, as a function of m for the case where the total mass remains constant. Curves are plotted for the distribution after every 100 collisions.

value but again the gradient of log N against log m becomes reasonably constant over the mass range $1 < m < 100$. This gradient is (1-s), s being the mass distribution index defined by the formula

$$N = A.m^{1-s} \quad \text{where A is a constant.}$$

The values of s at specific masses were obtained for the highest collision number curves given in figures 1 and 2. This was done by fitting a 7th order polynomial to the 3700 collision number curve in Fig. 1 and a 7th order polynomial to the 4900 collision number curve in Fig. 2 and using these fits to calculate s as a function of mass. The results are shown in figure 3. It can be seen that in both cases the equilibrium s values are considerably less than 2.0. This indicates (see Hughes 1972) that the majority of the mass eventually resides in the few large particles. This situation is similar to that found in the comet, asteroid, micrometeorite, planet and planetary satellite populations. The mean s value for the constant mass equilibrium distribution over the range $m < 200$ is 1.43. Continual replenishment of the m_1 particles leads to an equilibrium s value of 1.72 ($5 < m < 70$). Any fragmentation of the colliding particles would invariably lead to a higher mass distribution index.

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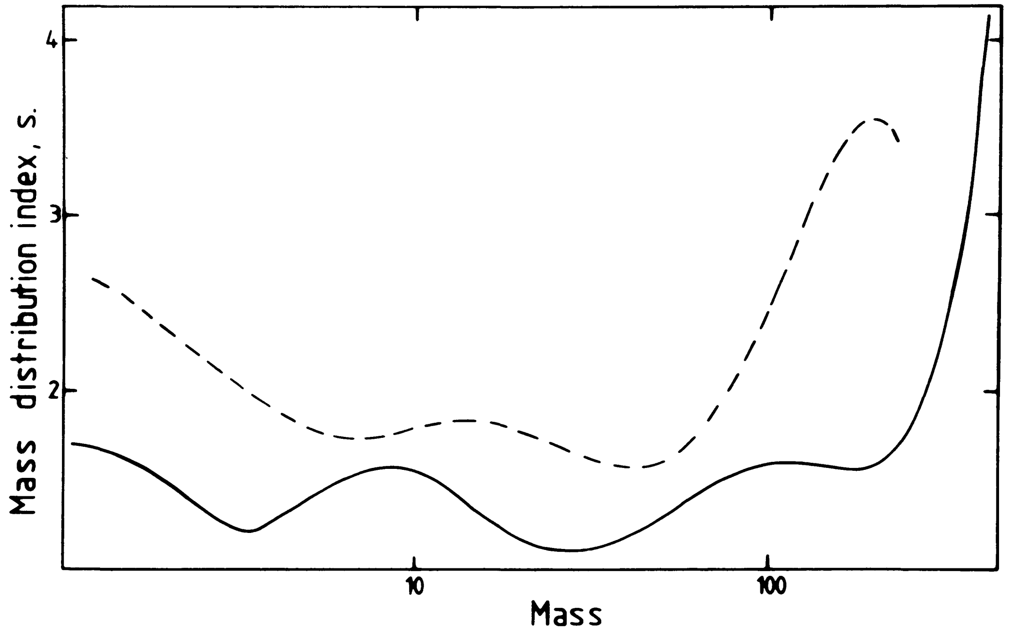

Fig. 3 The mass distribution index as a function of m for the equilibrium distributions. Dashed line, after 3700 collisions and a continual replenishment of the C_1 number to 500. Bold line, after 4900 collisions with a constant mass of material.

discussions and one of the authors (PAD) is grateful to the Science Research Council for a research studentship.

REFERENCES

Napier, W. McD., and Dodd, R. J. 1974, Mon. Not. R. astr. Soc. *166*, p.469.
Hughes, D. W. 1972, Planet. and Space Sci., *20*, p. 1949.

DISCUSSION

Hawkes: What is the minimum number of particles needed for consistent results?
Daniels and Hughes: Certainly the onset of equilibrium is achieved after a larger number of collisions if there are more particles but it is basically a trial-and-error process.

Cook: Am I correct that once the differential mass index drops below 5/3 even the total cross section is dominated by a few large particles?
Hughes: Yes; see Hughes 1978, in J.A.M. MacDonnell (ed.) "Cosmic Dust" p. 147.