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Mathematics of crossover with selection in genetic algorithms: haploid and diploid populations

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The purpose of the thesis is to describe the theoretical evolution of populations under the action of crossover and selection, with finite population size where possible. The study is extended to cover the theoretical effectiveness of different types of crossover and the generalized outcome of selection and crossover on two specific problems, including comparisons between haploid and diploid representations. Though crossover would appear to offer a route whereby different good properties of parents can be recombined to provide an offspring that is better than either parent, it has by no means been established that this is the case. Two simple problems are chosen to test rigorously this conjecture and the associated conjecture that the diploid representation confers an advantage.

The critical problem which arises in the theoretical study of populations of finite size is that the evolution cannot, in general, be described by the evolution of the expected population from generation to generation. In Chapter 2 we introduce a general framework to describe the evolution of a finite population and prove a theorem which establishes the sufficient conditions under which some property of a finite population evolves precisely as that of the sequence of expected populations. The importance of this theorem is that it describes those conditions under which the evolution of a finite population may be tractable to mathematical treatment.

A sequence of theorems, corollaries and lemmas in Chapter 4, making use of the general theorem, lead to the main result that a population of finite size converges, under the action of crossover only, to a population of maximum diversity. The link between the mask or template which defines a crossover operation, the parent strings and a schema is developed and is used to simplify the proofs.

The theoretical effectiveness of different types of crossover on the disruption and recombination of schemata is described in Chapter 5. The types of crossover include a new variation of multi-point crossover, an analogue of the recombination method that

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occurs in nature. The conclusion is that the optimum crossover parameters depend critically on the problem and the length of the building block schemata it is required to recombine but that, in any case, uniform crossover gives clearly inferior results.

We show that the operation of selection does not satisfy the conditions of the general theorem so, except for trivial cases, finite populations do not allow a tractable theoretical treatment of the evolution. Therefore, a study of crossover with selection is restricted to infinite populations, and, conversely, consideration of finite populations is restricted to crossover in isolation.

The two test problems (the one with a unimodal fitness landscape, the other bimodal) showed that crossover almost always improves the haploid populations but the conjecture that diploidy confers an advantage is proved invalid for both fitness landscapes with uniform crossover (Chapter 6). In the course of this part of the study we introduce a more flexible dominance regime than has been applied in previous studies of diploidy and prove general results for selection in isolation (Chapter 3).

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