Triangles AML and KDC are duplicates, since AK and LC are parallel chords, and LM, CD perpendicular to them. It follows that $AD = DK + CL$,

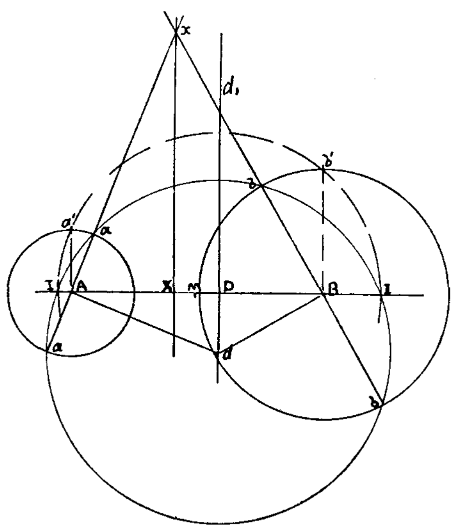
$$\therefore \tan A + \tan B + \tan C = \tan A \,.\, \tan B \,.\, \tan C.$$

COLIN KESSON.

The Diametric Section Axis of Two Circles.

Mr Burgess' pretty solution of the problem "to draw a circle cutting three circles diametrically" suggests a question as to the use of sign in treating of co-axial circles.

Any two circles A and B have besides their radical axis an axis with the property that any circle whose centre lies on the axis and which passes through two fixed points cuts the circles at ends of a diameter.



Let $a'a'$, $b'b'$ be diameters at right angles to AB, then a circle whose centre D is on AB passes through $a'a'$ $b'b'$ and cuts AB at two points I and I_1, and dDd_1 at right angles to AB is the required axis.

With d as centre and radius dI, construct a circle cutting B at b, then $db = dI$, and $dI^2 - dB^2 = DI^2 - DB^2 = r_1^2$,

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also $$d I_1^2 - d A^2 = D I_1^2 - D A^2 = r^2,$$
therefore $$DB^2 \sim DA^2 = r^2 \sim r_1^2 \qquad (1)$$
$A a$ and $B b$ meet on the radical axis, and
$$A X^2 \sim B X^2 = r^2 \sim r_1^2 \qquad (2)$$
if M be the mid-point of $A B$ from (1) and (2), we get
$$2 A B . M D = r^2 \sim r_1^2 \text{ and } 2 A B . M X = r^2 \sim r_1^2,$$
so that $M D = M X$, and the two axes are equidistant from M.

Again, at any point between I and I_1 a circle can be drawn which all the coaxials d_1, d_2, etc., cut at ends of diameters. When the point is outwith $I I_1$ on $A B$ the circles become the orthogonals to d_1, d_2, etc.

The question arises here, which are the real circles, A, B, etc., or the orthogonal circles, of which $D d$ is the radical axis.

Townsend, Art. 152, says: "All the circles whose centres are between I and I_1 are imaginary"; still, by foregoing they seem real enough.

WILLIAM FINLAYSON

The Limits of $\left(\cos \frac{x}{n}\right)^n$ and $\left(\sin \frac{x}{n} \Big/ \frac{x}{n}\right)^n$ when n tends to infinity.

These limits may be proved very simply by applying the following theorem in inequalities:—

If n is a positive integer and $r a$ a positive proper fraction for the values 1, 2, 3, ... n of r, then
$$1 - n a < (1 - a)^n < \frac{1}{1 + n a} \qquad (1)$$

These particular cases of the well-known inequalities generally used in connection with infinite products are easily established. Thus
$$(1 - a)^2 = 1 - 2 a + a^2 > 1 - 2 a;$$
$$(1 - a)^3 = (1 - a)(1 - a)^2 > (1 - a)(1 - 2 a)$$
$$[(1 - a)^3 = (1 - a)(1 - a)^2, \text{ etc.}]$$
since $1 - a$ and $1 - 2 a$ are both positive; therefore
$$(1 - a)^3 > 1 - 3 a + 2 a^2 > 1 - 3a,$$
and so on. The general result is easily proved by induction, though it is really obvious; thus we have the first inequality
$$1 - n a < (1 - a)^n.$$