On the Use of COBE Results

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Abstract

The aim of this brief report is to find a lower limit of the Hubble parameter using the COBE's detected fluctuations in the temperature of the CMBR.

1.Mathematical Considerations

If T is the temperature of the CMBR and $T_\mu = \frac{\partial T}{\partial x^\mu}$ is a time-like covariant gradient vector, then it is possible to define the temporal variation of the magnitude of the gradient of T as follows [1]:

$$F = \frac{d}{dt}(g^{\mu\nu}T_\mu T_\nu)^{1/2} = \frac{dG}{dt}, \tag{1}$$

where t is the cosmic time, $g^{\mu\nu}$ is the contravariant metric tensor which represents the background gravitational field in which the microwave background radiation exists and $\mu, \nu = 0,1,2,3$.It is possible to prove that F can be written as follows (Melek (1992)):

$$F = \frac{1}{G}g^{\mu\nu}T_{\mu;\sigma}T_\nu U^\sigma, \tag{2}$$

where $T_{\mu;\sigma}$ is the usual covariant derivative with respect to x^σ and $U^\sigma = \frac{dx^\sigma}{dt}$.

2.Physical and Cosmological Considerations

Since the COBE results confirmed that the universe's anisotropy, regarding the CMBR, is of the same order as the anisotropy which can be produced by a small spatial perturbations of the FRW metric ([2], [3],

M. Kafatos and Y. Kondo (eds.), Examining the Big Bang and Diffuse Background Radiations, 461–463.

[4], [5] and references therein) therefore it is siutable to represent the universe's gravitational field by the spatially perturbed FRW metric, which is given as follows in the spherical polar coordinates:

$$d\tau^2 = R^2(t)dt^2 - \frac{R^2(t)(1+h_{11})}{1-kr^2}dr^2 - R^2(t)h_{12}rdrd\Omega - R^2(t)r^2(1+h_{22})d\Omega^2, \tag{3}$$

where Ω is a solid angle defined in terms of θ,ϕ and $R(t)$ is the scale (expansion) factor. The parameter k is a constant which takes the values +1 or 0 or -1, if the universe is closed or flat or open respectively. Finally h_{11}, h_{12} and h_{22} are the spatial perturbat- ions which are assumed to be small quntities, i.e. their qudratic terms are neg- ligable.

Since the FRW universe is an isotropic and homogeneous,therefore

$$F_{(FRW)} = \frac{d}{dt}(g^{\mu\nu}{}_{(FRW)}T_\mu T_\nu)^{1/2} = 0, \tag{4}$$

where $g^{\mu\nu}{}_{(FRW)}$ is the metric tensor of the FRW world line.

Since the general motion in the universe is only due to its general expansion, then:

$$\frac{dr}{dt} = \frac{d\Omega}{dt} = 0, \tag{5}$$

Besides that, at any fixed cosmic time since the CMBR was separated from the matter, its temprature is independent of the radial coordinate r, i.e.

$$T_1 = \frac{\partial T}{\partial r} = 0, \tag{6}$$

Using (5), (6) and the fact that the temprature of the CMBR is decreasing only due to the expansion of the universe, one can conclude that:

$$\frac{dT_1}{dt} = 0 \tag{7}$$

3.Expression of F in the Spatially Perturbed FRW Universe and its Consequences

Using the metric (3) and inserting the physical and the cosmological constraints (4), (5), (6) and (7) in (2), therefore the expression of F is given as follows:

$$F = \frac{h_{22}}{R^3(t)Gr^2} T_2 [\frac{dT_2}{dt} - HT_2], \tag{8}$$

This expression gives the temporal variation of the magnitude of the angular gradient of the temprature of the CMBR. This magnitude should decrease with respect to the cosmic time due to the universe's expansion. Therefore the expression (8) of F should be negative. Since T_2 is positive, then F will be negative iff the following inequality is satisfied:

$$\frac{\frac{dT_2}{dt}}{T_2} < H, \tag{9}$$

where H is the Hubble parameter.

4.Conclusion

The iequality (9) gives a lower limit of the Hubble parameter in terms of the ratio between the cosmic time variation of the angular gradient and the angular gradient itself of the temperature of the CMBR. The author hopes that, using the COBE data, it might be possible to calculate the left hand side of the iequality (9).

References

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[4] Sachs R.K. and Wolfe A.M. (1967), Ap.J. 147, 73.

[5] Wright E.L., etal. (1992), Ap.J. 396, L13.