CORRESPONDENCE.

ON THE INTEGRAL OF GOMPERTZ'S FUNCTION.

To the Editor of the Journal of the Institute of Actuaries.

SIR,—On reperusing my paper in the last Number of the *Journal*, I observe a slight misdescription of the capabilities of the table appended thereto, which I shall be glad to be allowed the opportunity of correcting.

The characteristic property of the gamma-function, expressed by the equation $\int \epsilon^{-v} v^m dv = -\epsilon^{-v} v^m + m\int \epsilon^{-v} v^{m-1} dv$, attaches also to the transformed expression $\int 10^{-10^z} \epsilon^{-nz} dz$; but in the latter case, the equation, instead of holding between successive integer values of n, or between successive values differing by *less than unity* (which latter supposition I had, by some means, erroneously entertained), applies to values of n proceeding by differences of $\log_e 10 (=2{\cdot}302 \ldots .)$. Consequently, in order to obtain the power of deducing the integral for *all* values of n, the tabulated matter must cover an interval equal to $2{\cdot}302 \ldots .$—that is to say, to be *theoretically* complete, the table must be extended in the ratio of $2{\cdot}302 \ldots .$ to 1.

The property in question may be readily demonstrated as follows:—

In $\int dz u \frac{dv}{dz} = uv - \int dz v \frac{du}{dz}$, the general formula for integration by parts, put

$$(1) \ldots . u = \left(\frac{1}{10}\right)^{10^z};$$

whence $$\frac{du}{dz} = -(\log_e 10)^2 . 10^{-10^z} . 10^z = -(\log_e 10)^2 10^{-10^z} \epsilon^{\log_e 10 . z}.$$

$$(2) \ldots . v = -\frac{\epsilon^{-nz}}{n};$$

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whence $\epsilon^{-nz} = \frac{dv}{dz}$;

giving,

$$\int 10^{-10^z}\epsilon^{-nz}dz = -10^{-10^z}\frac{\epsilon^{-nz}}{n} - \frac{(\log_e 10)^2}{n}\int 10^{-10^z}\epsilon^{-(n-\log_e 10)z}.dz.$$

I am, Sir,
Your very obedient servant,

London, 12 *June* 1873. W. M. MAKEHAM.

ERRATUM.—In the paper "On the Integral of Gompertz's Function," above referred to, there is a misprint. For the second formula on p. 309,

$$\log\frac{1}{g^{q^x}\epsilon^{-(a+\delta)x}}\int_x^\infty g^{q^x}\epsilon^{-(a+\delta)}.dx$$

should be read,

$$\log\frac{1}{g^{q^x}\epsilon^{-(a+\delta)x}}\int_x^\infty g^{q^x}\epsilon^{-(a+\delta)x}.dx.$$

ON THE RELATION BETWEEN THE NET PREMIUM AND THE RATE OF INTEREST.

To the Editor of the Journal of the Institute of Actuaries.

SIR,—In the current volume of the *Journal*, p. 227, I gave a demonstration in reference to the relation between the value of a policy and the rate of interest according to which it is calculated, and in the present letter I propose to examine in a similar way the relation between the net premium and the rate of interest.

We have $\mathrm{P}_x = \frac{1}{1+a_x} - (1-v)$,

$$\therefore \quad \frac{d\mathrm{P}}{dv} = \frac{-\frac{da}{dv}}{(1+a)^2} + 1 \text{ (omitting the subscript } x\text{)},$$

$$= \frac{(1+a)^2 - \frac{da}{dv}}{(1+a)^2}.$$

Thus, since P_x increases or decreases, when v increases, according as $\frac{d\mathrm{P}}{dv}$ is positive or negative, we have only to examine whether

$$(1+a)^2 > \text{ or } < \frac{da}{dv}.$$

Now, $(1+a)^2 = (1+a_x) + p_x v(1+a_x) + {}_2p_x v^2(1+a_x) + \ldots.$

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