pedal circle intersects the N.P.C. in points which depend on the directions of OS, OS'. If S, S' coalesce at I (the in-centre), or are in line with O, then these two directions coincide and the circles touch.

R. F. Davis.

Geometrical Proofs of the Trigonometrical Ratios of 2θ and 3θ.

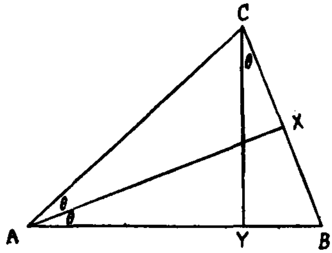

Fig. 1.

1. *Ratios of* 2θ.

$\angle BAX = \angle XAC = \theta$.

CB is drawn perpendicular to AX, and CY perpendicular to AB: then $\angle YCB = \theta$.

$$\sin 2\theta = \frac{YC}{AC} = \frac{YC}{BC} \cdot \frac{BC}{AC}$$

$$= \frac{YC}{BC} \cdot \frac{2XC}{AC}$$

$$= \cos\theta \, . \, 2\sin\theta$$

$$= 2\sin\theta \, . \, \cos\theta.$$

$$\cos 2\theta = \frac{AY}{AC} = \frac{AB - YB}{AC} = 1 - \frac{YB}{AC}$$

$$= 1 - \frac{YB}{BC} \cdot \frac{BC}{AC}$$

$$= 1 - \frac{YB}{BC} \cdot \frac{2XC}{AC}$$

$$= 1 - \sin\theta \, . \, 2\sin\theta$$

$$= 1 - 2\sin^2\theta.$$

The other forms for $\cos 2\theta$ and that for $\tan 2\theta$ can readily be deduced by transformation.

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2. *Ratios of* 3θ.

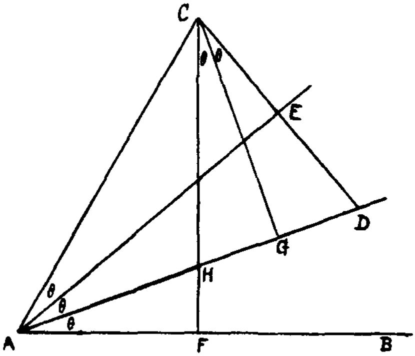

Fig. 2.

$B\widehat{A}D = D\widehat{A}E = E\widehat{A}C = \theta$.

CF, CG, CD are drawn perpendicular to AB, AD, AE, respectively. Then $H\widehat{C}G = G\widehat{C}D = \theta$.

$$\sin 3\theta = \frac{FC}{AC} = \frac{FH}{AC} + \frac{HC}{AC}$$
$$= \frac{FH}{AH} \cdot \frac{AH}{AC} + \frac{DC}{AC}$$
$$= \frac{FH}{AH}\left(\frac{AD - HD}{AC}\right) + \frac{2\,EC}{AC}$$
$$= \frac{FH}{AH}\left(1 - \frac{2\,GD}{AC}\right) + \frac{2\,EC}{AC}$$
$$= \frac{FH}{AH}\left(1 - \frac{2\,GD}{DC} \cdot \frac{DC}{AC}\right) + \frac{2\,EC}{AC}$$
$$= \frac{FH}{AH}\left(1 - \frac{2\,GD}{DC} \cdot \frac{2\,EC}{AC}\right) + \frac{2\,EC}{AC}$$
$$= \sin\theta\,(1 - 2\sin\theta \,.\, 2\sin\theta) + 2\sin\theta$$
$$= 3\sin\theta - 4\sin^3\theta.$$

$$\cos 3\theta = \frac{AF}{AC} = \frac{AF}{AH} \cdot \frac{AH}{AC}$$
$$= \cos\theta \,.\, (1 - 4\sin^2\theta) \text{ as above}$$
$$= \cos\theta\,(4\cos^2\theta - 3)$$
$$= 4\cos^3\theta - 3\cos\theta.$$

ALEX. D. RUSSELL.