TRANSFORMATIONS AMONG CISs DEFINED BY VARIOUS ERP SERIES[1]

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ABSTRACT The rotational angles α_1 and α_2 for transformation between two CISs are derived from the differences of each two polar coordinate series. Then, their periodicity is analysed and they are fitted by trigonometric expansions. In terms of the fitting parameters each of them is developed into a sum of several terms with two nearly-diurnal ones involved.

1.GENERAL PRINCIPLE

As the orientation of the celestial ephemeris pole (CEP) is specified by an ERP series, the relation between two ERP series, such as that of BIH and that of ILS, defines the transformation between two conventional inertial systems (CIS) corresponding to the two CEPs. The transformation can be carried out by three very small rotations about X-axis, Y-axis and Z-axis of one of CISs (Fig.1-1). The rotational angles, denoted as α_1,α_2,α_3 respectively, can be determined by the equations (Zhu, S. Y. and Mueller, I. I., 1983) as follows:

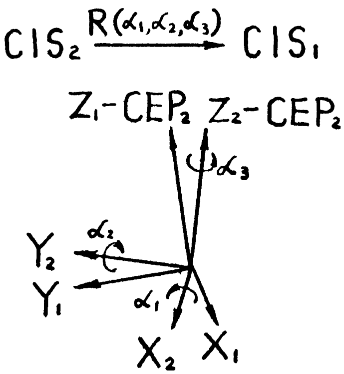

Figure 1-1. Rotational angles between two CISs

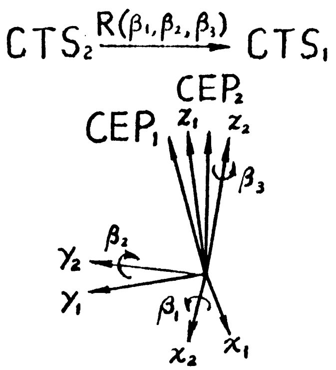

Figure 1-2. Rotational angles between two CTSs (with relative CISs)

[1]supported by the National Natural Science Foundation of China

I.I. Mueller and B. Kołaczek (eds.),
Developments in Astrometry and Their Impact on Astrophysics and Geodynamics, 391–396.

$$\Delta x = -\alpha_1 \sin\theta + \alpha_2 \times \cos\theta - \beta_2$$
$$\Delta y = \alpha_1 \cos\theta + \alpha_2 \sin\theta - \beta_1$$
$$(1+\mu)\Delta UT1 = \alpha_3 - \beta_3 \quad (1)$$

where θ is Greenwich mean sidereal time, β_1, β_2 and β_3 are the rotational angles between the two conventional terrestrial systems (CTS) which are connected with the observational stations by which the ERP series have been determined (Fig.1-2).

α_1, α_2 and β_1, β_2 can be derived with least square method from Δx, Δy series, while α_3 and β_3 closely combined can not be separated and is impossible to be resolved from UT1 data. That is to say, only can the alignment of Z-axes be realized with two rotations about X- and Y-axes.

2.DATA

Three polar coordinate series are applied in our work, as shown in Table.1.

Table.1. Polar coordinate series

series	period	normal point interval	source
ILS	1962.00–1979.95	0.05 year	Yumi,S. & K.Yokoyama,1980
BIHnew	1962 Jan.5–1981 Dec.31	5 days	Li,Z.X. 1986
BIHold	1964.00–1982.00	0.05 year	BIH, Ann.Rep. 1967–1982

For better analysis we need any two of data series with the longest possible common period, so these series have been decided on.

3.CALCULATIONS AND RESULTS

The procedure of analysis contains five steps, mainly the results between ILS and BIHnew data are to be shown here.

(1) The values of x and y at every day $0^h.0$ are derived by spline interpolation for each of series.

(2) The differences Δx and Δy at same epoch are calculated for each two of these series in the common period of data. Thus, three sets of difference series Δx and Δy are obtained corresponding to the combinations *BIHnew-ILS*, *BIHold-BIHnew*,*ILS-BIHold* (abbreviated to Comb.1,2,3 respectively, hereafter).

(3) A set of α_1,α_2,β_1,β_2 for each 15 days are derived by least square with Comb.1 Δx and Δy from Eq.1. The results are shown in Fig.2-1,2-3 and Fig.3-1,3-3. The similar results are obtained for Comb.2 and Comb.3 as well.

(4) Spectral analysis of α_1,α_2 and β_1,β_2 of Comb.1 are made by Marple autoregressive spectrum estimation (Marple, L.,1980) at order 100 and with two tolerance factors 1×10^{-3} and 1×10^{-4}. Their periodograms are shown in Fig.4-1,4-2 and Fig.5-1,Fig.5-2 and the exact periods are given in Tab.2.

https://doi.org/10.1017/S0074180900173565 Published online by Cambridge University Press

Table 2. Periods of α_1,α_2,β_1,β_2 of Comb.1. (in day)

period	for α_1	for α_2	period	for β_1	for β_2
p_1	2215	2215	q_1	366	433
p_2	198	198	q_2	366	433

Both α_1 and α_2 have two periods, approximately equal to 6 years and half a year, respectively. The deviations from the exact latter values may be due to the deviation of Chandlerian period from exact 1.2 year. Both β_1 and β_2 have also two periods, exactly equal to 1 year and Chandlerian period, which implies that there remain residual variations of their two periodic terms in the difference between two polar coordinate series.

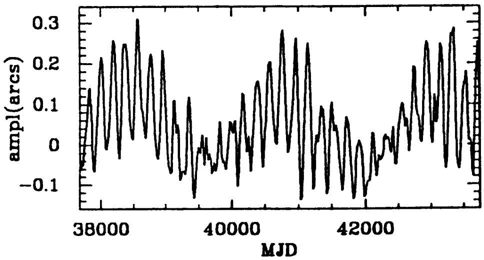

Figure 2-1. Eq.1-determined curve of α_1

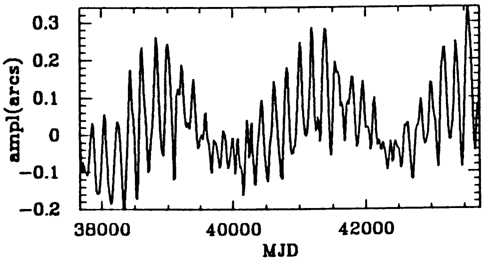

Figure 2-3. Eq.1-determined curve of α_2

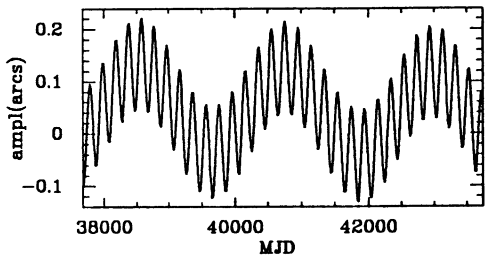

Figure 2-2. fitting curve of α_1

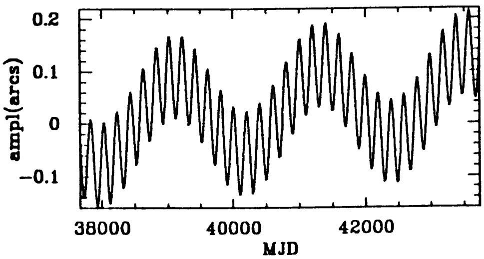

Figure 2-4. fitting curve of α_2

(5) A curve fitting process is adopted for α_1, α_2 and β_1, β_2 with following equations:

$$\alpha_j = C_j^{\alpha} + L_j^{\alpha} t + A_j^{(1)} cos(\omega^{(1)} t + a_j^{(1)}) + A_j^{(2)} cos(\omega^{(2)} t + a_j^{(2)}) \quad (j = 1, 2) \qquad (2)$$

where $\omega^{(i)} = 2\pi \times 15/p_i$, (i=1,2), p_i are taken from Tab.2

$$\beta_j = C_j^{\beta} + L_j^{\beta} t + B_j^{(1)} cos(\mu^{(1)} t + b_j^{(1)}) + B_j^{(2)} cos(\mu^{(2)} t + b_j^{(2)}) \quad (j = 1, 2) \qquad (3)$$

where $\mu^{(i)} = 2\pi \times 15/q_i$, $(i = 1, 2), q_i$ are taken from Tab.2

The results are partly given in Tab.3 and the fitting curves are shown in Fig2-2,2-4 and Fig3-2,3-4.

Table.3 Fitting parameters of α_1, α_2 curves of Comb.1

parameters	for α_1	for α_2	parameters	for α_1	for α_2
$A_j^{(1)}$	0".0828 ± 7	0".0782 ± 62	$a_j^{(1)}$	−2.4824 ± 684	2.2687 ± 71
$A_j^{(2)}$	0.0884 ± 46	0.0807 ± 35	$a_j^{(2)}$	2.9509 ± 357	1.4016 ± 615
$C_j^{\prime\alpha}$	0.0543 ± 57	−0.0016 ± 62	L_j^{α}	−0.00005 ± 2	0.00014 ± 14

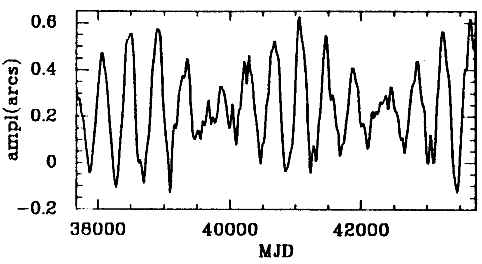

Figure 3-1. Eq.1-determined curve of β_1

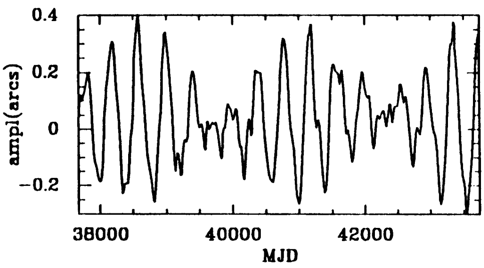

Figure 3-3. Eq.1-determined curve of β_2

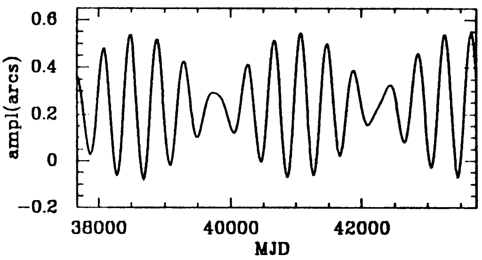

Figure 3-2. fitting curve of β_1

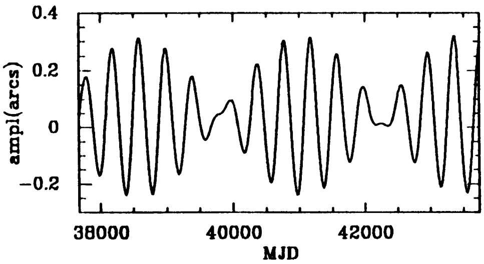

Figure 3-4. fitting curve of β_2

4. ANALYSIS OF FITTING PARAMETERS

We assume that Z_2-axis moves around Z_1-axis in two superposed circles with two different periods. Because of the rotation of the earth, these periods should be nearly-diurnal. This assumption is supported by the fact that the values listed in Tab.3 satisfy the conditions:

$$A_1^{(i)} \approx A_2^{(i)}, \qquad a_2^{(i)} - a_1^{(i)} \approx -\pi/2, \qquad (i = 1, 2) \tag{4}$$

from which the amplitudes and phases for both α_1,and α_2 can be taken as follows:

$$A^{(i)} = A_1^{(i)} = A_2^{(i)}, \qquad a^{(i)} = a_1^{(i)} = a_2^{(i)} + \pi/2, \qquad (i = 1, 2) \tag{5}$$

Then we can write down (for the sake of simplicity only periodic terms of α_j remain):

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$$\alpha_1 = \sum_{i=1}^{2} A^{(i)} cos(\omega^{(i)} t + a^{(i)}), \quad \alpha_2 = \sum_{i=1}^{2} A^{(i)} sin(\omega^{(i)} t + a^{(i)}) \tag{6}$$

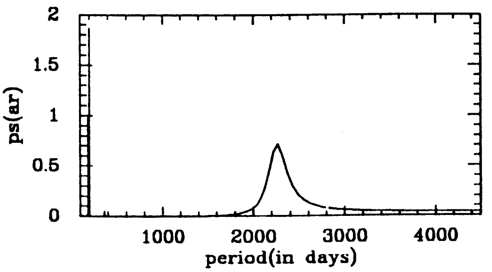

Figure 4-1. spectral power of α_1

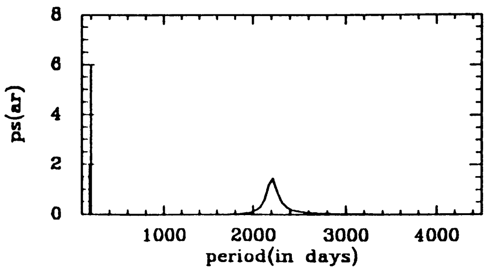

Figure 4-2. spectral power of α_2

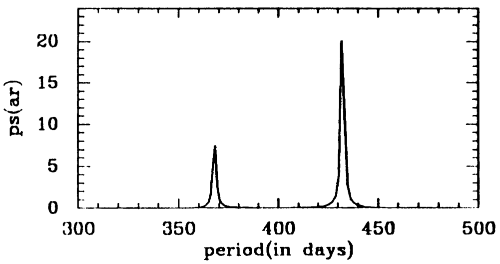

Figure 5-1. spectral power of β_1

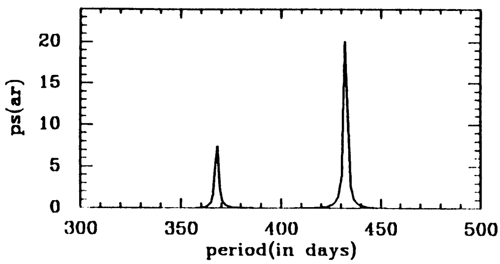

Figure 5-2. spectral power of β_2

Substituting Eq.6 into Eq.1, we get

$$\Delta x = \sum_{i=1}^{2} A^{(i)} sin[(\omega^{(i)} - \omega_0)t + a^{(i)}], \quad \Delta y = \sum_{i=1}^{2} A^{(i)} cos[(\omega^{(i)} - \omega_0)t + a^{(i)}] \tag{7}$$

where $\theta = \omega_0 t + \theta_0, \omega_0 = 7.292115 \times 10^{-5}/sec$ is the sidereal frequency of the earth rotation.

(1) for the period p_1, $\omega^{(1)} = 3.28316 \times 10^{-8}/sec$

$\omega^{(1)} - \omega_0 = -7.288832 \times 10^{-5}/sec$

(2) for the period p_2, $\omega^{(2)} = 3.67283 \times 10^{-7}/sec$

$\omega^{(2)} - \omega_0 = -7.255387 \times 10^{-5}/sec$

The similar results can be obtained for both Comb.2 and Comb.3, too, because the conditions represented by Eq.(4) exist also for these two combinations of data, which is verified by the values shown in Tab.4 and Tab.5.

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Table.4 Fitting parameters of α_1, α_2 curves of Comb.2

parameters	for α_1	for α_2	parameters	for α_1	for α_2
$A_j^{(1)}$	0".0855 ± 49	0".0826 ± 33	$a_j^{(1)}$	−0.2735 ± 322	−1.7490 ± 576
$A_j^{(2)}$	0.0828 ± 8	0.0818 ± 56	$a_j^{(2)}$	0.9354 ± 663	−0.6257 ± 111
C_j^{α}	0.0493 ± 57	−0.0193 ± 58	L_j^{α}	−0.00001 ± 2	0.00007 ± 4

Table.5 Fitting parameters of α_1, α_2 curves of Comb.3

parameters	for α_1	for α_2	parameters	for α_1	for α_2
$A_j^{(1)}$	0".0825 ± 53	0".0794 ± 38	$a_j^{(1)}$	−0.2223 ± 394	−1.7120 ± 647
$A_j^{(2)}$	0.0866 ± 6	0.0853 ± 63	$a_j^{(2)}$	0.8790 ± 707	−0.6964 ± 65
C_j^{α}	0.0428 ± 62	−0.0182 ± 63	L_j^{α}	−0.00005 ± 3	0.00007 ± 3

5. CONCLUSION

Up to now corrections between two polar motion series contributed to the transformations between the two corresponding CISs are always taken as a constant reduced from six-year data and are applied through the whole period of these six years. This is logical and reasonable because the polar coordinates are derived at every 5-day or at least 1-day separated normal points,and the diurnal components can be neglected.

It is predicted that the polar coordinates at much denser normal points, such as at every 2 hours, are to be given. Then, it is recommended to use equations like our Eq.7 of α_1 and α_2 together with Eq.3 of β_1 and β_2 instead of a constant for the corrections to these densified values, in order to keep the diurnal variations in Δx and Δy.

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