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Distribution functions for Galactic disc stellar populations in the presence of non-axisymmetric perturbations

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Abstract. The present-day response of a Galactic disc stellar population to a non-axisymmetric perturbation of the potential, in the form of a bar or spiral arms, can be treated, away from the main resonances, through perturbation theory within the action-angle coordinates of the unperturbed axisymmetric system. The first order moments of such a perturbed distribution function (DF) in the presence of spiral arms give rise to non-zero radial and vertical mean stellar velocities, called breathing modes. Such an Eulerian linearized treatment however diverges at resonances. The Lagrangian approach to the impact of non-axisymmetries at resonances avoids this problem. It is based on the construction of new orbital tori in the resonant trapping region, which come complete with a new system of angle-action variables. These new tori can be populated by phase-averaging the unperturbed DF over the new tori. This boils down to phase-mixing the DF in terms of the new angles, such that the DF for trapped orbits only depends on the new set of actions. This opens the way to quantitatively fitting the effects of the bar and spirals to Gaia data with an action-based DF.

Keywords. Galaxy: kinematics and dynamics, Galaxy: disk, Galaxy: structure

1. Introduction

The optimal exploitation of the next data releases of the Gaia mission will involve the construction of a dynamical model of the Milky Way based on a multi-component phase-space distribution function (DF), obeying the collisionless Boltzmann equation and representing each stellar population, as well as dark matter, within the gravitational potential that these populations jointly generate. The Jeans theorem tells us that such a DF for an equilibrium configuration should depend only on three integrals of motion, which we can choose to be the "action" variables $\mathbf{J}$, i.e. the DF is expressed as $f_0 = f_0(\mathbf{J})$. The dimensionless canonically conjugate variables of the actions are called the "angle" variables $\boldsymbol{\theta}$, because they are usually normalized such that the phase-space position is 2π-periodic in them [see Binney & Tremaine (2008)]. In absence of perturbations, the equations of motion are extremely simple, $\boldsymbol{\theta}(t) = \boldsymbol{\theta}_0 + \boldsymbol{\Omega} t$, where $\boldsymbol{\Omega}(\mathbf{J}) \equiv \partial H_0/\partial \mathbf{J}$ is the vector of fundamental orbital frequencies. In an equilibrium configuration, the angle coordinates of stars are phase-mixed on orbital tori that are defined by the actions $\mathbf{J}$

alone, and the phase-space density of stars $f_0(\boldsymbol{J})\mathrm{d}^3\boldsymbol{J}$ corresponds to the number of stars $\mathrm{d}N$ in a given infinitesimal action range divided by $(2\pi)^3$. Using such action-based DFs, the current best axisymmetric models of the Milky Way have been constructed [see, e.g., Cole & Binney (2017)].

The Milky Way is however not axisymmetric as it has long been known to harbour both a bar and spiral arms. Hence, non-axisymmetric DFs are needed to pin down the present structure of the non-axisymmetric components of the potential, which are important observationally [see, e.g., Dehnen (1998); Famaey *et al.* (2005)], and have enormous importance as drivers of the secular evolution of the disc [see, e.g., Fouvry *et al.* (2015)]. The outer Galactic disc is additionally vertically perturbed by the Sgr dwarf galaxy and the Large Magellanic Cloud [see Laporte *et al.* (2016)]. Hence, perturbed DFs are needed to extract all the relevant information from data. We recently developed two different approaches to the problem of the impact of non-axisymmetries on the DF, in Monari *et al.* (2016) and Monari *et al.* (2017), which we summarize herafter in Sect. 2 and Sect. 3 respectively. We conclude in Sect. 4.

2. Eulerian approach

The Eulerian approach to the problem posed by non-axisymmetry, developed in Monari *et al.* (2016), is to derive from perturbation theory explicit distribution functions for present-day snapshots of the disc as a function of the actions and angles of the *unperturbed* axisymmetric system.

Since the angle variables are defined such that the position in phase-space is 2π-periodic in them, we consider only cases where the perturbing potential Φ_1 is cyclic in the angle coordinates. Then, Φ_1 can be expanded in a Fourier series as the real part of

$$\Phi_1(\mathbf{J},\boldsymbol{\theta},t) = \mathcal{G}(t)\sum_{\mathbf{n}} c_{\mathbf{n}}(\mathbf{J})\mathrm{e}^{\mathrm{i}\mathbf{n}\cdot\boldsymbol{\theta}}, \tag{2.1}$$

where $\mathcal{G}(t)$ controls the strength of the perturbation as a function of time. It is convenient to factorize this function into two factors, $\mathcal{G}(t) = g(t)h(t)$, where $g(t)$ is a well behaved function controlling the general amplitude of the perturbation, and $h(t)$ is a periodic sinusoidal function of frequency ω_p, $h(t) = \exp(\mathrm{i}\omega_p t)$. Typically, $\omega_p = -m\Omega_p$ where m is the multiplicity of the perturber, which can thus account for a perturbing potential rotating with a fixed pattern speed Ω_p. Hereabove, $\mathbf{n}$ is a triplet of integer indexes running from $-\infty$ to ∞.

We are now interested in the first-order response of the DF to such a small perturbation Φ_1, i.e. $f = f_0 + f_1$ where f_0 is the unperturbed axisymmetric DF and f_1 the response obeying the linearized Boltzmann equation:

$$\frac{\mathrm{d}f_1}{\mathrm{d}t} = \frac{\partial f_0}{\partial \mathbf{J}} \cdot \frac{\partial \Phi_1}{\partial \boldsymbol{\theta}}. \tag{2.2}$$

We assume that the perturbation and its time derivatives are null far back in time, i.e., $\forall k, g^{(k)}(-\infty) = 0$. Moreover, we assume in the following that the amplitude of the perturbation is constant at the present time t, hence $g^{(0)}(t) = 1$, and $g^{(k)}(t) = 0$, for $k = 1, ..., \infty$. With these assumptions, we can integrate Eq. 2.2, and we show in Monari *et al.* (2016) that the general solution for f_1 is the real part of

$$f_1(\mathbf{J},\boldsymbol{\theta},t) = \frac{\partial f_0}{\partial \mathbf{J}}(\mathbf{J}) \cdot \sum_{\mathbf{n}} \mathbf{n} c_{\mathbf{n}}(\mathbf{J}) \frac{h(t)\mathrm{e}^{\mathrm{i}\mathbf{n}\cdot\boldsymbol{\theta}}}{\mathbf{n}\cdot\boldsymbol{\Omega} - m\Omega_p}. \tag{2.3}$$

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This is the general Eulerian response (within the unperturbed actions and angles coordinates) far away from resonances, but it is immediately clear that resonances lead to the problem of small divisors since f_1 diverges whenever

$$\mathbf{n} \cdot \boldsymbol{\Omega} - m\Omega_p = 0, \tag{2.4}$$

which is the actual definition of a resonance.

This Eulerian approach nevertheless has some virtues. Far away from the resonances, it allows us to explicitly compute the moments of the perturbed distribution functions. For instance, in Monari *et al.* (2016), using the epicyclic approximation to get an analytic relation between the action-angle variables and the positions and velocities, and considering a 3D spiral arm perturber with corotation in the outer Galaxy, we showed that the first order moments of the perturbed DF describe "breathing" modes of the Galactic disc in perfect accordance with simulations. Such a breathing mode might actually have been detected in the extended Solar neighbourhood (see Siebert *et al.* (2011) and Williams *et al.* (2013)), but with a larger amplitude, perhaps because the spiral arms are transient.

In the following, we will now show how to cure the problem of small divisors posed by resonances by moving from the Eulerian to the Lagrangian description of the problem, i.e. following the deformation of the orbital tori and defining a new system of action-angle variables in the resonant region.

3. Lagrangian approach

Far from resonances, in the regime where the above Eulerian treatment is valid, the orbital tori on which regular orbits are confined are simply distorted by the small perturbing potential Φ_1. But close to resonances, the tori are radically modified. Consequently, for each resonance, we should define a new set of actions and angles to describe the orbits. The key to the Lagrangian approach developed in Monari *et al.* (2017) is (i) to make two consecutive canonical transformations in order to find the relevant action variables to use in the resonant region, and (ii) to follow the prescription of Binney (2016) in populating the new tori by phase-averaging the original unperturbed DF f_0 over these new resonant tori. We limit ourselves to the 2D planar case at this stage.

The first time-dependent canonical transformation allows us to disentangle the slow and fast motion near a given resonance for which $\mathbf{n} = (l, m)$,

$$\theta_\mathrm{s} = l\theta_R + m(\theta_\phi - \Omega_p t), \quad \theta_\mathrm{f} = \theta_R, \quad J_\mathrm{s} = J_\phi/m, \quad J_\mathrm{f} = J_R - (l/m)J_\phi. \tag{3.1}$$

The angle θ_s is slow because in the unperturbed case, the definition of the resonance is such that it indeed evolves very slowly. Its physical interpretation is typically the azimuth of the apocenter of the orbit in the reference frame corotating with the perturber. The next step is to replace the real Hamiltonian of the system by a Hamiltonian averaged over the fast variable, in order to study the evolution of the slow angle and slow action, by making the fast action an approximate integral of motion. For each J_f, we can then define $J_\mathrm{s,res}$ as the J_s satisfying $\Omega_\mathrm{s}(J_\mathrm{s}, J_\mathrm{f}) = 0$. We then expand the Hamiltonian in J_s around $J_\mathrm{s,res}$ near the resonances to obtain a one-dimendional *pendulum* Hamiltonian.

The key is to then make a second canonical transformation from the slow angle and action to the actual corresponding *pendulum* action and angle $(J_\mathrm{p}, \theta_\mathrm{p})$. The trapped DF should thus be written as a function $f_\mathrm{tr}(J_\mathrm{f}, J_\mathrm{p})$.

Assuming that the perturbation has been long-lived enough for phase-mixing to be efficient, the natural outcome for the trapped DF is then, following Binney (2016),

$$f_{\rm tr}(J_{\rm f}, J_{\rm p}) = \frac{1}{2\pi}\int_0^{2\pi} f_0(J_{\rm f}, J_{\rm s}(J_{\rm p}, \theta_{\rm p}))\mathrm{d}\theta_{\rm p} \tag{3.2}$$

where f_0 is the unperturbed DF. In Monari *et al.* (2017) and in the proceeding by G. Monari *et al.* in the same volume, applications related to the velocity distribution of stars in the solar neighbourhood in the presence of a bar perturbation are presented.

4. Conclusion

Constructing action-based DFs is the best way to construct dynamical models of the Milky Way directly from data. In order to extraxt all the relevant information from future data, it is however necessary to treat the non-axisymmetries through perturbation theory. The relevant formalism, in view of the upcoming data releases from Gaia, has been recently presented in Monari *et al.* (2016) and Monari *et al.* (2017), and summarized in the present contribution (see also the proceeding in the same volume by G. Monari *et al.*). In the Eulerian approach of Monari *et al.* (2016), the linearized Boltzman equation is explicitly solved within the action and angle variables of the unperturbed system. Far away from resonances, this allows us to evaluate the impact of non-axisymmetries on stellar motions. In particular, Monari *et al.* (2016) have shown that spiral arms create breathing modes of the disc qualitatively similar to what is observed in the Solar Neighbourhood. In Monari *et al.* (2017), we then presented a Lagrangian approach to treat the impact of non-axisymmetries near resonances, where the above Eulerian treatment diverges. The idea of this Lagrangian approach is to follow the deformation of the tori outside the trapping region, and to average the distribution function over the relevant angles in the trapping region. We showed that in the trapping region the relevant action-angle variables are those of a pendulum, and averaging over those angles allows for a smooth connection with the deformed tori outside of the trapping zone. Some improvements of our formalism are however still needed. In particular, it will now be mandatory to move away from the epicyclic approximation and use more general action-angle variables. The time-dependence of the amplitude of perturbations should also be taken into account, as well as the collective effects. Once all this is will be under control, the Eulerian and Lagrangian treatments presented here will be fully complementary. Indeed, in the absence of strong resonance overlaps, each perturber should be treated with the Lagrangian approach near resonances, on top of which the impact of other perturbers can be evaluated with the Eulerian treatment.

References

Binney J. & Tremaine S. 2008, Galactic Dynamics: Second Edition. Princeton University Press
Binney J. 2016, *MNRAS*, 462, 2792
Cole D. R. & Binney J. 2017, *MNRAS*, 465, 798
Dehnen W. 1998, *AJ*, 115, 2384
Famaey B., Jorissen A., Luri X., *et al.* 2005, *A&A*, 430, 165
Fouvry J.-B., Binney J., Pichon C. 2015, *ApJ*, 806, 117
Laporte C. F. P., Gómez F. A., Besla G., *et al.* 2016, arXiv:1608.04743
Monari G., Famaey B., Siebert A. 2016, *MNRAS*, 457, 2569
Monari G., Famaey B., Fouvry J.-B., Binney J. 2017, arXiv:1707.05306
Siebert A., Famaey B., Minchev I., *et al.* 2011, *MNRAS*, 412, 2026
Williams M. E. K., Steinmetz M., Binney J., *et al.*, 2013, *MNRAS*, 436, 101

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