PHYSICAL MECHANISMS OF MIXING IN STELLAR INTERIORS

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Abstract

The different mechanisms by which mixing can take place in stellar interiors are considered : the classical Rayleigh-Benard instability with penetrative convection and over-shooting, semi-convection , gravitationnal and radiative settling,turbulent mixing . The latter mechanism is thoroughly described, from the driving force of turbulent mixing to its influence on stellar structure , stellar evolution and the analysis of the corresponding observationnal data .
Turbulent mixing has to be considered each time the building up of a concentration gradient takes place, either by gravitationnal or radiative settling or by nuclear reactions . Turbulent mixing, as a first approximation, can be described by an isotropic diffusion coefficient . The process is then governed by a diffusion equation . The behaviour of the solution of the diffusion equation needssome explanation in order to be well understood .
A number of examples concerning surface abundances of chemical elements are given (3He, 7Li, Be, 12C, 13C, 14N), as well as a discussion of the solar neutrinos problem .
The building up of a μ-barrier , which stops the turbulence allows stellar evolution towards the giant branch and explains nitrogen abundance at the surface of giants of the first ascending branch .
Turbulent mixing is also of some importance for the transfer of angular momentum and has to be taken into account for explaining the abundance of the elements in Wolf-Rayet stars .

A. Maeder and A. Renzini (eds.), Observational Tests of the Stellar Evolution Theory, 491–512.

1. Introduction

Mixing inside the stars is not a new idea. Already L . Biermann (1937) suggested the existence of some sort of turbulent diffusive mixing in stellar interiors. However, the question of the observationnal proof of chemical elements transport was not considered seriously until the discovery of the presence of the radio-active element Technecium in the atmosphere of S stars by Merrill (1952). The gravitationnal sorting of heavy elements in white dwarfs (Schatzman , 1945) , due to their large gravitationnal field was never questionned , but the extension of this physical mechanism to other stars did not come up until the work of Chapman and Aller (1960).

We are interested here in the various mixing processes which take place inside a star and the observationnal test of their presence. We shall certainly not ignore the physics which is underlying these mixing processes, as it will certainly provide us with some indication of the evolutionnary phase at which they can show up , either through surface abundance of the chemical elements, or through global properties otherwise unexplained . Let us add that the evidence of new physical processes acting inside the stars give the possibility of building up new consistent models of stellar evolution .

Let us stress this point . The hydrodynamical processes we are concerned with are not only interesting by themselves, and in many cases are worth devoting a large amount of work and efforts to understand them; but they have far reaching consequences for stellar evolution, redistribution of the elements in the galaxy , and consequently for the so-called cosmic abundances of the elements and therefore for nucleo-synthesis and our understanding of the origin of the Universe .

2. The convective zone .

The abundance of the elements at the surface of the stars has not a unique cosmic value . Enhancement of some elements, depletion of others, let us suspect the presence of a variety of physical processes leading to either one or the other of these differences. There is little doubt , nowadays that gravitationnal or radiation settling on one hand, nuclear processing on the other hand are at the origin of the observed variety of the abundance of chemical elements, when it cannot be explained by the time and space dependance of the chemical composition of the Galaxy .

However, after these two statements a number of questions are raised, which have to be answered in a consistent way. Let us consider first the question of gravitationnal or radiative settling. If we describe the outer layers of a star (spectral type later than A0 or therabout) , we have a radiative atmosphere, a convective zone, deep or shallow , eventually double , according to the spectral type, and a radiative zone below, with

https://doi.org/10.1017/S0074180900031363 Published online by Cambridge University Press

some amount of penetration of the convection and overshooting . The penetration of the convection can be of the order of one pressure scale height , according to Toomre et al (1976) for the shallow convective zone of main sequence A stars, and , according to Latour et al (1983) of the order of 1/10 of a pressure scale height for a deep convective zone , like in the Sun. The convective zone is considered as being of a uniform chemical composition . However, before any further consideration, let us study this question . To that effect, and in order to introduce orders of magnitude, take a two-dimensions plane parallel fluid , with a diffusion coefficient of diffusion D , and, at t=0 , introduce along the axis y a certain amount of a contaminent of concentration c . The perturbation can be described as

$$c(x,z) = c\ .\delta(x,z)$$

where δ is a delta function, and at t=0, c=1 . The well known solution, with the boundary conditions $\partial c/\partial z = 0$ for $z = h$ and $z = -h$ is :

$$c = (4\ D\ t)^{-\frac{1}{2}}\ e^{-(x^2/4Dt)} \sum_j \cos(j\pi z/h)\ e^{-Dj^2\pi^2 t/h^2}$$

The horizontal distance x is reached by the contaminent over a time t of the order of $(x^2/4Dt)$ and the contaminent is spread vertically over a time $(h^2/\pi^2 D)$. The Rayleigh number in any convective zone is so large that we can assume that it is highly turbulent . In the case of the solar convective zone, we can estimate D to be of the order of 10^{13} cm^2s^{-1}, which gives for the vertical spreading a characteristic time of the order of 10 days and for the horizontal spreading over the whole solar surface a characteristic time of the order of 30 years . These durations are very short compared to the stellar life time, and the assumption of a uniform chemical composition of a convective zone is physically sound .

It should be noticed that the assumption of an isotropic D can be a fair approximation for the diffusion of contaminents and a very poor one for the transport of mechanical quantities like angular momentum.

3.Elements sorting.

Let us come back to the problem of gravitationnal or radiation settling . The convective zone can be considered as a reservoir where the contaminent under consideration is stored . Gravity or radiation pressure can the contaminent downwards, out of the convective zone or push the contaminent upwards inside the convective zone (Michaud et al, 1976).

The effect of the penetration of the motions from the convectively stable zone into the radiatively stable zone is to generate a a fully mixed region which is larger than the convective zone stricto sensu. Let us assume a sharp boundary between the radiative zone and the fully mixed region. Consider the case of a test element : no retroaction of the test element on the background. Then, the equation of continuity in the radiatively stable region can be written

$$\frac{\partial}{\partial t}\rho c + \mathrm{div}\, \rho v\, c = 0$$

where v includes the diffusion and the drag , and can be written:

$$cv = - D \left\{ \frac{\partial c}{\partial z} + \frac{c}{h} \right|$$

the flow - D (c/h) represents the drag due to the gravity, the pressure gradient, the electric field and the radiation pressure. The sign oh (D/h) depends whether the gravity exceeds the radiation pressure ((D/h)> 0), or the reverse ((D/h)< 0); and the quantity (D/h), which is independant of the concentration for a test element, is concentration dependent through saturation effects in the radiative transfer, for a finite concentration (and nevertheless small with respect to the background).

The boundary condition

$$- \rho\ c\ v = m \frac{\partial c}{\partial t}$$

where m is the mass, per unit surface, of the fully mixed region, expresses the fact that the depletion of the convective zone is due to the gravitationnal drag . Conversely, it can express the fact that the filling of the convective zone is due to radiation pressure .

When the diffusion is only due to the microscopic diffusivity,the major effect is the drag, -(D/h) . The time scale turns out to be (Schatzman, 1969) of the order of $t_{mic} = (H\ h\ /\ D_{mic})$. H is the equivalent height of the fully mixed region .

In the case of pure gravitationnal drag,

$$h = H_p/(1.692\ i^2 + 3.870\ A - 2.203\ i - 2.666)$$

where the sorting is supposed to take place in pure hydrogen , and i is the degree of ionization and A the atomic weight .

Considering the value of D_{mic} we find that as soon as the convective zone is sufficiently shallow, the time scale of depletion, t_m , becomes very short, compared to the stellar life time. We are therefore facing the situation that gravitationnal or radiative settling can become a major phenomenon for all main sequence stars earlier than F2. The question is not so much to include or not to include the sorting of the elements but to understand the reason for which the phenomenon is observed in such a small number of stars, and why it turns out that in "normal" stars the effect is negligible .

Let us consider again a simplified problem in which the essential of the physics is present, which is mathematically simple and can be solved exactly . We have a fully mixed region with a uniform chemical composition; at the lower boundary we find the presence of graviationnal sorting and turbulent diffusion, with a coefficient D_t. Assuming that the turbulent diffusion coefficient and the gravitationnal drag are independant of z, the evolution of the concentration c is governed by the equation

$$D_t \frac{\partial^2 c}{\partial x^2} + f \frac{\partial c}{\partial x} - \frac{\partial c}{\partial t} = 0$$

The boundary condition at the bottom of the convective zone says that the rate at which the element flows downwards is equal to the rate of depletion of the convective zone :

$$- D_t \frac{\partial c}{\partial x} - f\, c = H \frac{\partial c}{\partial t}$$

f is the (D/h) used above , and we can write $f = (D_{mic}/h)$. When $D_{turb} >> D_{mic}$ the change of the concentration in the convective zone as a function of time is given for $(t/t_{turb}) << 1$ by

$$c = c_0 \left(1 - \frac{2}{\sqrt{\pi}} (t/t_{turb})^{1/2} \right)$$

and for $(t/t_{turb}) >> 1$ by

$$c = c_0 \frac{2}{\sqrt{\pi}} (t/t_{turb})^{-1/2} \exp(-t/t_{turb})$$

It should be noticed that for t/t_{turb} small, the turbulent diffusion mixing slows down the sorting, wheras for t/t_{turb} large, the gravitationnal sorting is accelerated ! This is due to the disappearance of a concentration gradient .

The characteristic time t_{turb} is given by

$$t_{turb} = (4\, h\, H\, D_{turb} / D_{mic}^2)$$

Let us consider the behaviour, as a function of the mass, of turbulent diffusion mixing. To that effect, we obtain values of $(1/2) t_{star}/t_{turb}$, taking for D_{turb}, $D_{turb} = Re^* \nu$, $Re^* = 100$ and ν being the microscopic viscosity (molecular plus radiative) as a function of the mass M. For Pop.I stars the behaviour is given in fig 1.

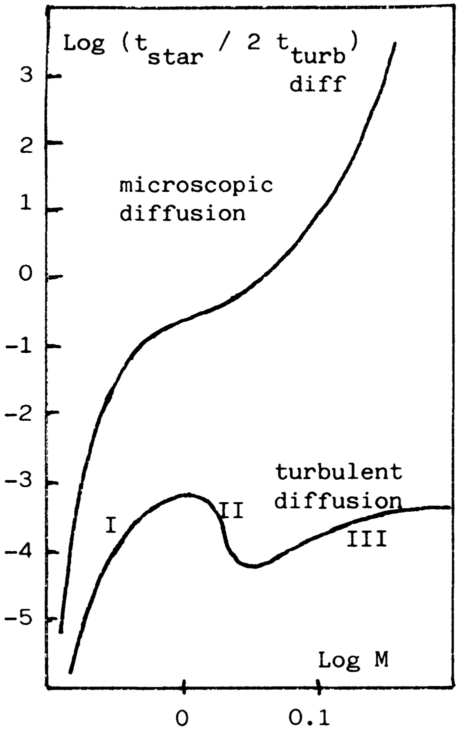

Fig.1. The time scale of element (Li) sorting in the presence of microscopic diffusion only and with the presence of turbulent diffusion mixing. In the section I of the curve, the size of the convective zone decreases and the viscosity is the molecular viscosity. In section II the viscosity changes to the radiative viscosity, the turbulent diffusion coefficient increases , in section III, the radiative viscosity dominates, the convective zone is shallow. For population I stars with turbulent diffusion mixing the element separation does not take place

We shall come later to the question of the origin of this turbulence. For the time being, we have still to consider the effect of penetrative convection and overshooting, with a slight difference in the two terms. Penetrative convection is the penetration of large scale lines of flow in the radiatively stable zone; overshooting is the penetration of a turbulent element which, reaching the boundary with a finite velocity, is there slowed down in the radiatively stable region by negative boyancy force and friction . A phenomenological description of overshooting has been given by Shaviv and Salpeter (1973), Roxburgh (1978),Maeder (1975). Overshooting launches turbulent elements over a finite distance, and it seems likely that beyond that point the convective zone does not induce any mixing. Similarly, it has been shown by Latour et al (1983) that the lines of flow do not penetrate beyond a finite distance from the boundary of the convective zone. Due to the very large value of the turbulent diffusion coefficient in the convective zone, the stochastic motions produced in the radiatively stable zone generate a large turbulent diffusion coefficient. Even if the energy density of the motion is small, and such that the radiative transfer still dominates, the effect of the motion is to keep the region of penetration chemically homogeneous. This is the reason for introducing the concept of fully mixed region, different from the convective zone it self.

https://doi.org/10.1017/S0074180900031363 Published online by Cambridge University Press

4.Lithium

Let us consider now another astrophysical puzzle, the case of the Lithium abundance. The problem of the surface abundance of Lithium has been considered now for a long time, starting with Herbig (1965). The abundance of Lithium in clusters (Duncan,1981), in field stars (Boesgard, 1976), show that the abundance is mass dependant and time dependant. For one solar mass stars, except perhaps a $\sqrt{t}$ dependance at small t (Skumanich,1972), the obvious law, as given by Duncan (1981) is dLog(n(Li))/dLog T = -0.63. The range of spectral type where the Li λ 6707 doublet can be observed is relatively small, from about 6000°K to 5300°K (Cayrel et al, 1983). The lines are observed in dwarfs and in giants. The abundance of Lithium in giants, according to Scalo and Miller (1980) can be interpreted as the result of the dredge up taking place in stars which have destroyed their Lithium on the main sequence, confirming Schatzman picture (1977). Further questions are raised by the measure by Spite and Spite (1982) of the abundance of Lithium in old population II stars , in the range 6200°K -5200°K. They find a quasi uniform abundance, Log Li = 1 (for Log NH = 12) wheras for other stars, the abundance varies from Log Li = 3 to very small values.

Is it possible to put some order in this disorder ? The idea of pre-main sequence destruction of Li has been considered by Bodenheimer (1965) and more recently by Mazitelli(1983) . It is assumed that the convective zone before receding to its main sequence extension , reaches the temperature of Lithium burning with the right time scale . The interesting result of Mazitelli is that , depending on the extension of the overshooting, and with an extreme sensitivity to that extension, either pre-main sequence burning does not take place, or the Lithium is entirely burnt .

This result can easily be understood if one considers the rate of Lithium burning . At T = 2.5 . 10^6 °K, the reaction rate t_R is highly temperature dependant, $t_R \sim T^{-20}$. The range of temperature at the bottom of the convective zone of pre-main sequence stars, the time spent at the maximum teperature would require such a minute adjustment to obtain the final abundance that we can definitely consider that Li burning at the bottom of the convective zone is ruled out . Similarly, the Li deficiency on the main sequence cannot be explained by the rate of nuclear reactions at the bottom of the fully mixed region . The result of Cayrel et al (1983) for the main sequence stars of the Hyades shows beautifully that another explanation has to be found .

We are then bound to consider again turbulent diffusion mixing. The equations governing the problem are (1) the diffusion equation with chemical reactions; (2) the boundary conditions . The search of a solution of the form exp(- s t) leads to an eigen value

problem for s .
It turns out , as shown by numerical study of the problem by Baglin and Morel (1983) that the z dependance of ρ and D is relatively unimportant . We can therfore consider the system

$$D \frac{\partial^2 c}{\partial z^2} + (s - K \rho T^n) = 0$$

$$D \frac{\partial c}{\partial z} = s H c \text{ , boundary of the fully mixed region}$$

c = 0 deep inside
An approximate solution of the WKB type has been obtained (Schatzman, 1983). In the solar case, we have the following values H (equivalent height of the fully mixed region) = 5.6 10^9 cm , T(bottom fully mixed region) = 2 . 10^6 °K, T(Li burning, $t_{nucl} = t_*$) = 2.5 . 10^6 °K . The distance from the bottom of the fully mixed region to T($t_{nucl} = t_*$) is 1.5 10^9 cm .
As a function of the depletion factor, we have the following results :

Depletion factor	$s\ t_*$	D	Re*
1/250	5.521	1100	55
1/100	4.6	650	35

In the case of the Hyades, Schatzman (1983) has obtained a solution which is model free, as it is expressed as a function of the temperature at the bottom of the fully mixed region (fig.2).

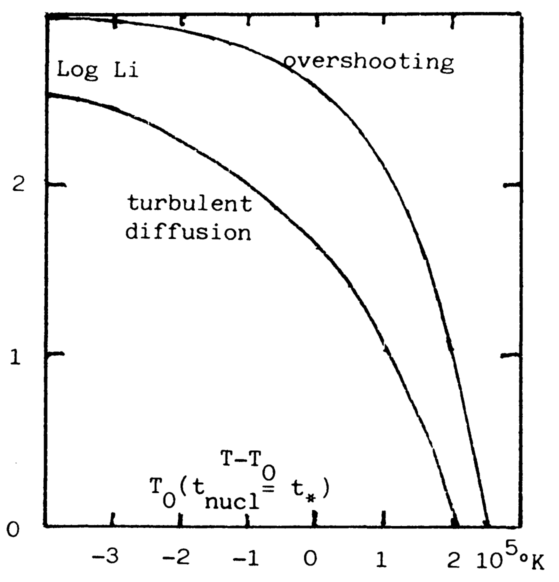

Fig.2. Li depletion

For the same temperature at the bottom of the fully mixed region, turbulent diffusion gives more depletion than burning ; as a function of the temperature, diffusion gives a depletion which varies more slowly than burning . This is in agreement with the results of Baglin and Morel and fits with the results of Cayrel et al. The exact solution obtained by Baglin and Morel (1983) shows that the exact fitting of the theoretical curve with the observed points is possible .

We are not through the mystery story of the Lithium destruction and hiding . Coming back to the old population II stars observed by Spite and Spite (1982), it is very likely that the presence of Lithium is due to the fact that, as a consequence of the low metal content, the convective zone is not as deep as the convective zone of Population I stars. Turbulent diffusion, if it is there, cannot, even in 10^{10} years carry the Lithium to the burning region .However this not the whole story, as gravitationnal drift of Lithium takes place slowly, but takes place, as well as metal deposition, even in the presence of turbulent diffusion .

This reasoning leads to the conclusion that the Li abundances found by Spite and Spite (1982), with respect to the primordial Li abundances, may be underestimates. The constraint on the big-bang nucleosynthesis can therefore be considered as less rigid. No final conclusion can be reached before quantitative calculations have been carried with well established models of the outer layers of these stars .

5. The solar Helium ^{3}He .

The problem of the surface abundance of He is closely related to the problem of the primordial abundance of the elements . Geiss et al (1972) have shown that there is a significant increase in the ^{3}He trapped in the lunar ashes, beginning something like $3 . 10^9$ y ago . The surface abundance of ^{3}He in the Sun, $(^3He/^4He) = 4$ to 5.10^{-4} includes the product of deuterium burning in the convective zone. The low value of the present ^{2}D abundance (Laurent, 1983), $(^2D/^1H) \simeq 5.10^{-6}$ suggest that the surface abundance ratio $(^3He/^4He)$ which has to be explained is about 3.5 to $4.5 . 10^{-4}$.

In the presence of turbulent diffusion mixing, the distribution of the main isotopes in the Sun is greatly modified . Fig.3 shows, according to Schatzman and Maeder (1981) the internal distribution of ^{1}H and ^{3}He at the age of $4.57 . 10^9$ y, assuming a turbulent diffusion coefficient $D = Re^*\nu$, where ν is the microscopic viscosity and Re^* is respectively 0,100 and 200.

The internal distribution of ^{3}He presents in the standard model a peaked distribution which is illustrated in fig 3. During the evolution, the peak moves outwards and increases in size. At $t=4.57 . 10^9$ y the maximum reaches $2.8 . 10^{-3}$ at $M_r/M = 0.58$. With turbulent diffusion mixing, the distribution of ^{3}He below $M_r/M = 0.4$ is mainly determined by the rapid equilibrium between creation and destruction, as in the standard case. Above $M_r/M = 0.4$ (T 8.3 . 10^6 °K), the transport by turbulent diffusion has a tendency to uniformise the ^{3}He distribution : the peak is reduced and ^{3}He is spread through the star and its abundance considerably increased in the outer layers. At $t = 4.57 . 10^9$ y the surface abundance in ^{3}He is $8.5 . 10^{-4}$ for $Re^* = 200$ and $3.8 . 10^{-4}$ for

for Re* = 100. An interpolation formula, applicable for the case of diffusion from a source,

$$He = (A/Re^{*1/2}) \exp(-B/Re^*)$$

gives the following results :

Re*	$^3He/^4He$
30	$2.80 . 10^{-4}$
31	$3.12 . 10^{-4}$
32	$3.44 . 10^{-4}$

The measured surface abundance of ^3He fits with a turbulent diffusion coefficient given by Re*≃ 30

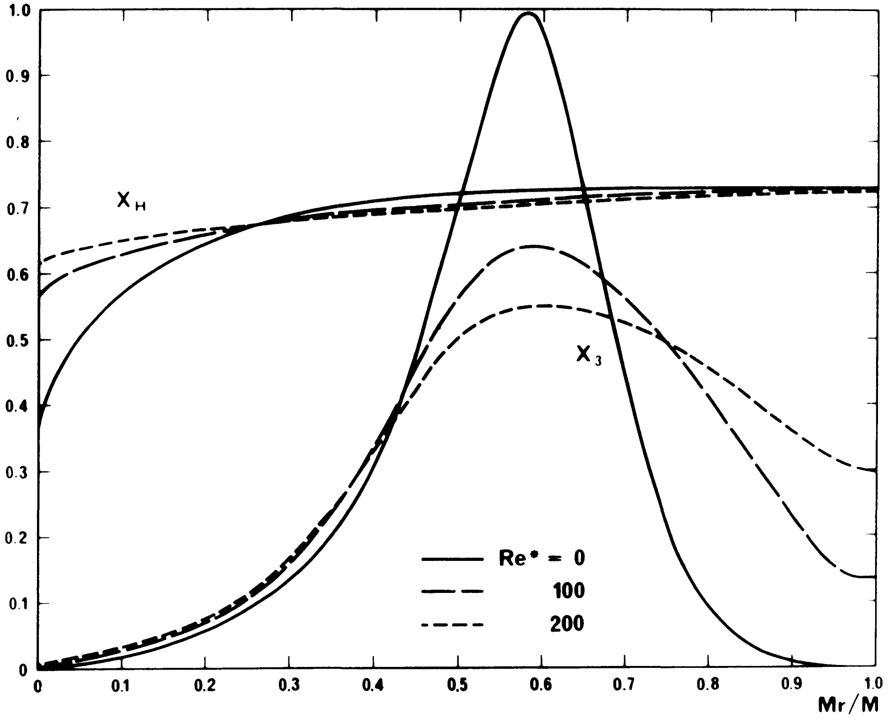

Fig.3 The hydrogen ^1H and ^3He concentrations in a 1 solar mass star at the age of 4.6 . 10^9 y (according to Schatzman and Maeder, 1981). The curves are plotted for various values of the parameter Re* which describes the efficiency of the turbulent diffusion .

6. The Carbon isotopes .

The explanation of the abundance ratio ($^{12}C/^{13}C$) meets in the classical models a number of difficulties which can be solved by introducing the turbulent diffusion mixing .

https://doi.org/10.1017/S0074180900031363 Published online by Cambridge University Press

Let us consider a one solar mass star at the end of its evolution on the main sequence (Dearborne et al , 1976). Outside the region $q = (M_r/M^*) = 0.4$, ^{13}C has been buildt at the expense of ^{12}C. Inside the region $q = (M_r/M^*) = 0.3$, ^{13}C is destroyed and the first two reactions of the Carbon cycle are followm by the production of ^{14}N.

It has been noticed for a long time (see, for example Iben, 1977) that in the first ascending branch, when the deep convective zone is generated, the dredge up of ^{13}C decreases the ($^{12}C/^{13}C$) ratio; however, starting with an initial abundance ratio of the order of 80 to 100, the mixing in the deep convective zone cannot make the abundance ratio lower than 40 or therabout , and in any case cannot bring it to the observed value of 10 to 25 (Lambert et al 1980).

The difficulty concerning the interpretation of the ($^{12}C/^{13}C$)ratio is due to the fact that one finds,in the same area of the HR diagram stars which have different masses and are in different evolutionnary stages:(i) stars on the first ascending branch,(ii) stars of the horizontal branch : according to Faulkner (1966) for population I stars, the stars of the ZAHB are gathered in a clump which sits very close to the first ascending branch, (iii) stars on the giant asymptotic branch . For stars belonging to the first ascending branch, the observations lead to an abundance ratio ($^{12}C/^{13}C$) of the order of 15 to 25, definitely smaller than the abundance ratio predicted by the standard models.

The idea that turbulent diffusion mixing is important is obvious if on looks at the ^{13}C concentration as a function of the radius instead of the lagrangian variable, the mass. The concentration gradient is larger and the expectancy is that the flow of ^{13}C ,

$$- 4 \pi r^2 \rho D_{turb} \frac{\partial(^{13}C)}{\partial r}$$

outside the region of ^{13}C production will be appreciable.During the life of the star on the main sequence, ^{13}C is carried slowly, driven by the concentration gradient outside of the region of ^{13}C burning, enriching the outer radiative zone in ^{13}C. When the dredge up takes place, the ^{13}C which has been stored in the outer radiative zone shows up and the abundance ratio ($^{12}C/^{13}C$) is smaller than in the standard models.After the preleminary results of Genova and Schatzman (1979), Bienaymé et al (1983) have obtained the exact solution of the diffusion equation for stationnary stellar models with 1;1.5; 2 and 3 solar masses.It appears from the exact solution that turbulent diffusion mixing cannot bring the ($^{12}C/^{13}C$) ratio to values smaller than 15 to 20. The need of stopping some time the turbulent diffusion mixing in the stellar core is obvious if one considers the physics of the evolution towards the giant branch. This is confirmed by the fact that continuous turbulent diffusion mixing brings ^{14}N in the outer radiatively stable regions, wheras a proper inhibition of turbulent diffusion mixing prevents the appearance, at the giant stage, of anomalous abundances of Nitrogen .

The inhibition of turbulence is likely to be due to the generation of a gradient of molecular weight. As soon as the gradient $\nabla\mu$ (d log μ/d log P) exceeds some critical value, the turbulence stops, and turbulent diffusion mixing meets an unpenatrable barrier. Bienaymé et al have explored for a 1.6 solar mass star the effect on abundance ratios (C/N) and ($^{12}C/^{13}C$) of the critical value of the gradient of molecular weight (fig.4). The best fit

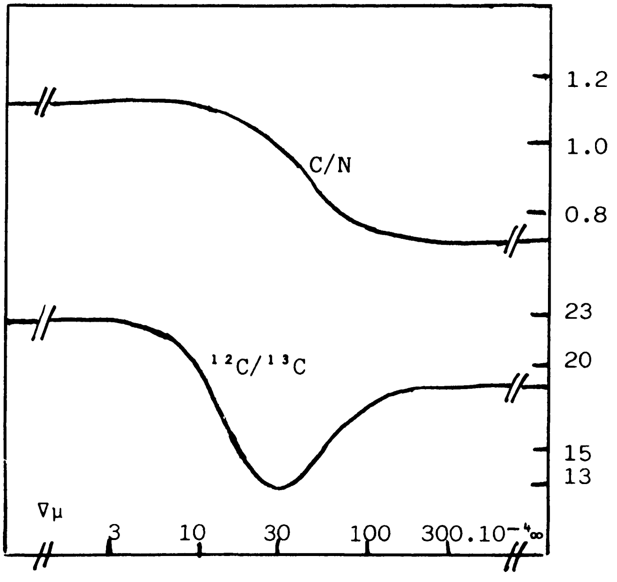

Fig.4. Surface abundance ratios for 1.6 solar mass as a function of $\nabla\mu_{crit}$. $\nabla\mu=\infty$: no inhibition of diffusion $\nabla\mu= 0$: identical to Re* = 0 (from Bienaymé et al 1983)

has been obtained for Re* = 100.

The case of the more evolved stars is different. Sweigart and Mengel 1979) noticed that the Carbon anomalies become more prominent with increasing luminosity. They suggest that, as far as the Carbon anomalies are concerned, there is no compelling observationnal evidence for an additionnal mechanism operating beyond the Red Giant Branch phase .

In order to explain the anomalies, they suggest that the Eddington Sweet flow carries matter from the convective envelope to the level of Carbon burning, where the ^{13}C is generated at the expense of ^{12}C. This is obtained by assuming a very fast meridional circulation and consequently requires a swift rotation.

However, it is possible to explain the mixing from the convective envelope to the Carbon burning region by turbulent diffusion mixing. The gradient of $\mu, \nabla\mu$ is very small in the region going from the bottom of the convective envelope to the Carbon burning region . The time scale of the turbulent diffusion mixing is of the order of

$$t_{diff} = [\int D_{turb}^{-1/2} dz]^{-2}$$

Using the tables of Sweigart and Gross (1978) it is possible to obtain estimates of t_{diff} . The table gives t_{diff} as a function of (t_{fl} - t) for one of the population I models of Sweigart and Gross.

t_{diff} as a function of (t_{fl}-t)
(t_{fl} : He flash epoch)
(M;Y;Z) =(0.90; 0.30; 0.01) ; Re* = 100

t_{fl}- t	t_{diff}
23 . 10^6 y	3.4 . 10^6 y
3	0.67
0	0.55

It appears clearly that almost until the Helium flash the diffusion time can be shorter than the evolution time scale . The result would be then a reduction of the Carbon abundance and an increase of the ($^{12}C/^{13}C$) ratio, like in the model of Sweigart and Mengel (1979), but with a different physical processwhich has the advantage of being more consistent with the general picture of stellar evolution with turbulent diffusion mixing .

7. The solar neutrinos problem.

Compared to standard models, turbulent diffusion has two consequences :

(i) it increases the central concentration of hydrogen by turbulent diffusion mixing along the gradient of concentration of hydrogen;
(ii) it increases the central concentration of the isotope ^{3}He by turbulent diffusion mixing along the gradient of concentration of the isotope (fig.3). This last effect is not included in partially mixed models of Shaviv and Salpeter (1968), Ezer and Cameron (1968) Shaviv and Beaudet(1968), Bahcall et al (1968) .

The effect of turbulent diffusion is then to make available more energy per proton than in the standard models. The small amount of ^{3}He which is brought by turbulent diffusion mixing to the central regions, due to the large amount of energy involved in the ^{3}He reactions, contributes appreciably to the energy generation in the solar core; the final result is a decrease of the central temperature and consequently a considerable decrease in the neutrino flux.

Three models have been calculated by Schatzman and Maeder (1981), for a turbulent diffusion coefficient D = Re*ν , with Re* resp. equal to 0,100 and 200.

More models are needed . However, it is possible to obtain, by an interpolation formula the neutrino flux as a function of Re*.The solution of the diffusion equation with chemical reactions show that for D close to zero the partial derivative $\partial/\partial D$ is very large. We then obtain the following table :

Re*	Φ_ν (SNU)		
0	11.6	6	4.6
20	5.76	2.97	2.28
30	4.95	2.56	1.96
50	3.83	1.98	1.33
100	2.39	1.23	0.95
200	1.43	0.74	0.57

whe the interval 11.6 to 4.6 for the predicted flux is about the error interval given by Bahcall et al (1982) . The observed neutrino flux (Davis, quoted by Bahcall et al, 1980),

$$\Phi_\nu = 2.2 \pm 0.4$$

gives as a central value $Re^* = 40 {}^{+100}_{-\ 20}$.

8.The origin of turbulent diffusion mixing .

Zahn(1983) has given a simple and natural explanation of the slow turbulence which is at the origin of the mixing process which we have considered here. With a turbulent diffusion coefficient $D = Re^* \nu$, $Re^* = 25$ we obtain for the Sun characterestic velocities of the order of 20 μ $s^{-1}$and characteristic lengths of the order of 200 meters.

The basic idea is to consider the baroclinic instability. When a star does not rotate cylindrically,$(\partial\Omega/\partial z) \neq 0$, the surfaces of constant entropy do not coincide with the level surfaces, and there are always motions, those inside the wedge between the isentropic surface and the horizontal which are unstable. The instability generates a turbulence which is first dominated by the almost horizontal nature of the motion, and is a two-D turbulence. This holds for all sacles for which the Coriolis force dominates over the inertial terms,

$$\Omega > 2.5\ (u/l) \qquad \text{(I)}$$

the numerical coefficient being issued from an experiment carried by Hopfinger et al (1982).The cascade towards smaller scales is accompanied by a transition to 3-D turbulence, when the condition (I) is violated.

Assuming that the kinetic energy of the differential rotation,fed itself by advection of the angular momentum,is converted into

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turbulent energy. Zahn obtains the following value for the vertical turbulent diffusion coefficient, due only to the 3-D turbulence ,

$$D = Re^* \nu$$

with

$$Re^* = \frac{K}{\nu} \frac{\Omega^2 r}{g} (\nabla_{ad} - \nabla_{rad})^{-1}$$

where K is the thermal diffusivity, ν the microscopic viscosity, (ν/K) is the Prandtl number, which is small . In the solar case, this gives near the region of Li-burning , $Re^* = 60$, assuming Ω to be the same than at the surface of the Sun.

This can be considered as a reasonable estimate, the exact value depending, on the one hand on the precise choice of numerical coefficients of the order of 1, and on the other hand, on the complete solution of the flow diagram, including the feed back mechanisme rotation → turbulence → rotation and a proper choice of the rate of loss of angular momentum by the star.

The question of the stabilizing μ-gradient has been extensively discussed recently by Knobloch and Spruit (1982,1983). When the μ-gradient is small, the Goldreich-Schubert-Fricke instability is the most important. This is suppressed for a very small μ-gradient, $\nabla\mu \cong (H_p \, \Omega^2 / g) << 1$. However, if the turbulent motion is present, we do not know under which conditions it will stop, either by a change in the distribution of Ω, or by a change in the μ-gradient. Knobloch and Spruit (1982) suggest that the turbulent motions, which have been generated at an earlier phase of stellar evolution, could remain present for a long time after the linear condition of stability has been reached during stellar evolution .

In this respect, the phenomenological theory of turbulent diffusion mixing, by the quantitative astrophysical estimates which are obtained, can be considered as a guide to a full understanding of the generation and disappearance of turbulence in a rotating star .

9. He deposition .

The problem of the discrimination between Am stars and δ Scuti stars, which sit in the same area of the H-R diagram,has been discussed several times, for example by Baglin et al (1973) . Breger (1979). Let us recall the suggestion of Baglin (1972) that the ^{4}He settling in a non turbulent star leads to the disappearance of the He-convective zone (Vauclair et al , 1974) and allows then the dripping in of the heavy elements . Mixed stars, being in the instability strip, can pulsate, wheras demixed stars, will show the Am characteristics.

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This discrimination is perhaps the most striking indication of the presence or absence of turbulent diffusion mixing .

The baroclinic instability which appears to be at the origin of the turbulent mixing in stars, in a star of uniform chemical composition, for a very slow rotation. As an order of magnitude, for a star where the viscosity is the radiative viscosity,

$$\Omega^2 \geqq 4 \frac{g^2}{c^2} (\nabla_{ad} - \nabla_{rad}).$$

For Log g = 4.5, $(\nabla_{ad} - \nabla_{rad}) = 0.1$,

$$P \leqq 60 \text{ days}.$$

However, this does not mean that the 2-D turbulence is generated. We need to have a shear sufficiently strong to generate the 2-D turbulence; when the viscosity is the radiative viscosity,

$$\Omega^2 \geqq \frac{g^2}{2\, c^2} R_c \, (\nabla_{ad} - \nabla_{rad})$$

where R_c is the critical Reynolds number. In other terms, we have

$$\Omega(\text{2-D turbulence}) \cong (R_c/8)^{1/2} \Omega(\text{baroclinic instability})$$

For $R_c = 2000$,

$$P(\text{2-D turbulence}) \leqq 4 \text{ days}$$

$$V_{eq} \geqq 20 \text{ km s}^{-1}$$

The order of magnitude is comparable to the estimate of Baglin (1972) : 50 km s^{-1}. The difference rests in fact on a different value of the critical Reynolds number .

This result suggests that all Am stars are slow rotators. There are, however, a few fast rotating stars (Abt and Moyd, 1973).This can be explained in the following way. According to Abt (1961, 1967) all Am stars belong to a binary system. For close binaries, as shown by Zahn (1977),"solid body roatation" can be achieved, the axial period of rotation becoming equal to the period of the orbital motion . In this cas, the velocity of the circulation is, with respect to the ratio $\chi = (\Omega^2 R^3 / G M)$, one order higher . We then have for the instability of the shear (for radiative viscosity):

$$\Omega > \left(\frac{g}{R}\right)^{\frac{1}{2}} \left[\frac{g R}{2 c^2} R_c \, (\nabla_{ad} - \nabla_{rad})\right]^{1/4}$$

$$P \leqq 1 \text{ day}$$

$$V_{eq} \geqq 70 \text{ km s}^{-1}$$

We then can conclude to the possibility of two classes of Am stars, depending on the period of the binary. In wide binaries, the star behaves as if it were isolated; and only slow rotators can present the Am characterestics . In close binaries, on the contrary, almost solid rotation is present, the circulation is much more slow and relatively fast rotation can be present without preventing the gravitationnal settling to take place.

10. Wolf Rayet stars .

The time scale for turbulent diffusion mixing of large mass stars has been considered by Maeder (1982) taking the turbulent diffusion coefficient $D = Re^* \nu$, with $Re^* = 100$. If we take the expression of Re^* given by Zahn (1983) and the privileged distribution of angular velocity given by Spruit and Knobloch (1982) we obtain

$$Re^* \cong \frac{4\ r\ g\ \mu}{(k_B/m_H)T}$$

which gives for M = 30 solar masses , $Re^* \cong 2.10^2$ in the middle of the star . We can the compare , with Maeder , the time scale of mixing, mass loss and nuclear main sequence time scale (fig.5). It is quite clear that the time of mixing is likely to be smaller than any other characteristic time for M > 50 M(sun) . Evolutionnary models for initial masses of 30 and 60 M(sun) have been computed by Maeder (1982) including mass loss, overshooting and turbulent diffusion mixing, leading to the evolutionnary tracks shown on fig.6.As a consequence of mixing, the 60 M(sun) star remains almost homogeneous and definitely turns to the left of the HR diagram before the end of he core burning phase.

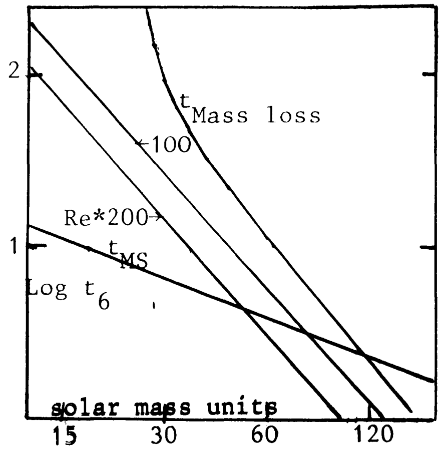

Fig. 5. The different time scales for large mass stars : mass loss, mixing with Re* =100 and 200, main sequence life time. Mixing is faster than other scales for M>50 solar masses.

The frequancy of the stars generated by mixing is of the order of five times larger than when only mass loss is present . Maeder notices that mixing appears necessary in view of explaining the observed frequency of transition WN 7-9 stars.

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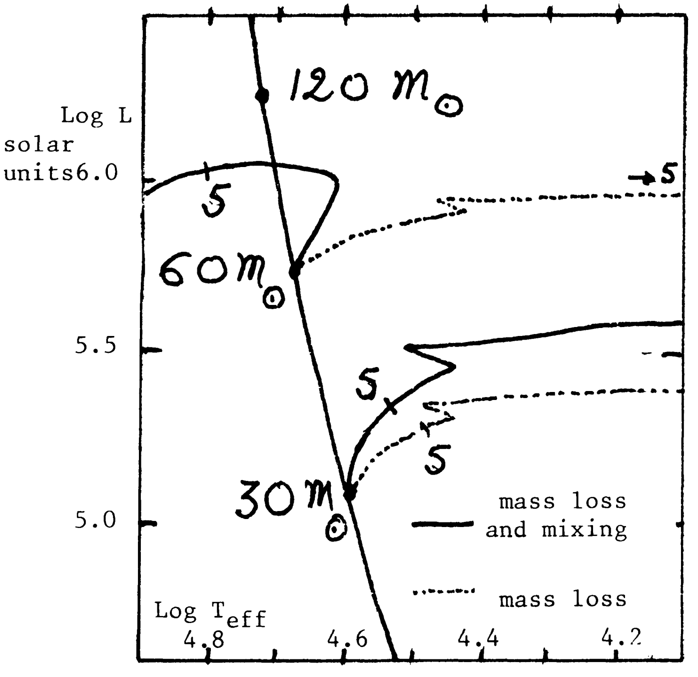

Fig.6. Evolutionnary tracks of massive stars with mass loss (dotted line) and with mass loss and TDM (continuous line).

11.Conclusion

Turbulent diffusion mixing seem to be able to explain a variety of stellar properties in a consistent way . It is not an ad hoc hypothesis, but is based on a consistent physical model. We shall conclude from the above given examples that turbulent diffusion mixing (TDM) is there and that it is necessary to deal with it .

More work has to be done, to determine exactly the conditions under which TDM appears and disappears, its possible connection with the magnetic field, the dynamo mechanism, the transfer of angular momentum, its anisotropy and its relation with meridional circulation and differential rotation ...

For the time being, we shall consider that the existence of a turbulent diffusion coefficient ,

$$D_{turb} = Re^*$$

is established, with values of Re* in the range 30 to 200 .

https://doi.org/10.1017/S0074180900031363 Published online by Cambridge University Press

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DISCUSSION

Cox: Does the He lifting problem limit the value of Re^*? Can that energy come from the ^3He burning?

Schatzman: The problem of the transport of ^4He upwards from the place where it has been formed is certainly a difficulty. The amount of energy available in the unburnt ^3He is at least one order of magnitude larger than the energy which is necessary to lift ^4He.

Vanbeveren: I thought that already with standard evolution one is able to produce WR stars. I therefore do not understand your fourth conclusion where you state that "T.D. is a must" in order to form WR stars!

Schatzman: It is what I have understood from Maeder's paper: it is much easier to obtain Conti's scenario with mixing than without, and it is much easier to compel with the number of transition WN 7-9 stars.

Renzini: Since we don't have a theory of stellar mixing in non-fully convective regions, it is legitimate to parametrize the mixing and ask whether such parametrized models give a better account for the pertinent observations. However, when doing so all the isotopes which can potentially be affected by mixing should be explicitly included in model calculations. For instance, in the case of the Sun, besides ^7Li and ^3He, also ^6Li, Be, B and the CNO isotopes should be considered, and the results compared with the observational data. To my knowledge this has never been performed, and fitting one observable with one free parameter does not say much about what is going on.

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Tayler: The internal conditions in the different types of stars which you have discussed differ greatly and whatever process gives rise to the turbulence must also be very different. Presumably you would be unhappy if your parameter Re^* was too similar from star to star.

Schatzman: There is no doubt that the turbulent diffusion coefficient varies from star to star and is certainly not a constant inside a star. However, if turbulence results from a feed-back mechanism, in which we are always very close to the instability limit, we can expect little variation from place to place and from star to star.

Gurm: For any such formulation in the continuum, the presence of the magnetic field, particularly in the case of Am and Sun, is going to affect the turbulence and mixing.

Schatzman: The presence of a magnetic field can certainly affect mixing in various ways. In the presence of a weak magnetic field, the viscosity of the medium will be increased. In fact, it can easily be recognized that turbulence by itself, being able to carry chemical contaminents, is unable to carry enough angular momentum. A factor of the order of 10 is actually missing, and the way of solving this discrepancy is to introduce a turbulent magnetohydrodynamic viscosity.
On the other hand, the effect which you mention, the rising of magnetic loops exists certainly in a convective zone: see the Sun! But this is related to the strong dynamo effect which is present in the solar convective zone. Do we have the same kind of instability taking place in radiatively stable regions? Is the differential rotation strong enough to generate unstable loops of magnetic fields? A weak mixing due to upward motion of flux tubes may eventually explain some of the difficulties of Am stars: in the absence of any mixing, some elements would be overabundant or very underabundant by a large factor, which is not the case. On the other hand, it does not seem to be possible to explain these difficulties with a unique turbulent diffusion coefficient, that the mechanism you suggest would provide. Weak mixing in the outer layers of Am stars must be there, but it must be a very delicate and complex mechanism.

R. Cayrel: Si je peux intervenir sur la question du rapport $^6Li/^7Li$ dans les Hyades je peux préciser que le ^6Li n'est certainement pas dominant et que l'on a une limite supérieure de ce rapport de l'ordre de 1/4.

Andersen: In your model of Li depletion, would you expect the $^6Li/^7Li$ isotope ratio to go down with the Li abundance, and is that observed in the Hyades?

Schatzman: The isotope ratio would go down with the lithium abundance, as the surface of ^{6}Li burning is closer to the boundary of the fully mixed region.