MAPPING COORDINATE SYSTEM IRREGULARITIES ON THE CELESTIAL SPHERE

Paul D. Hemenway and Arthur L. Whipple
Astronomy Department and Center for Space Research
The University of Texas at Austin
Austin, Texas 78712

Raynor L. Duncombe
Department of Aerospace Engineering and Center for Space Research
The University of Texas at Austin
Austin, Texas 78712

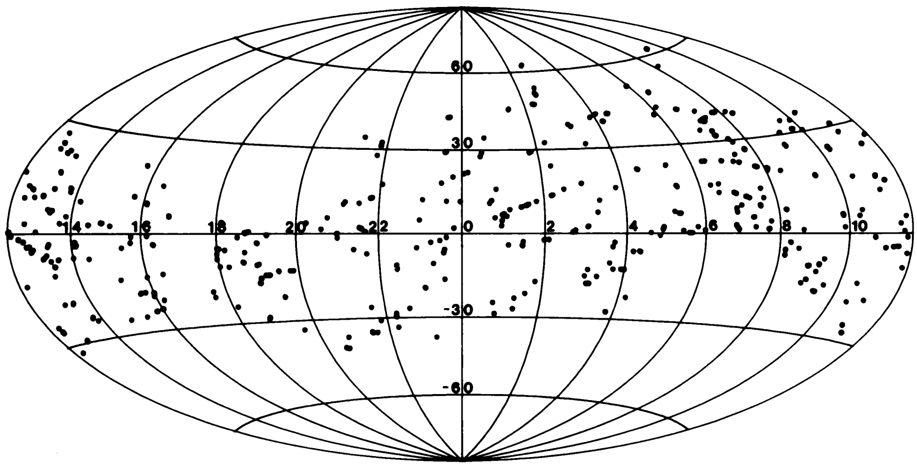
ABSTRACT. Various techniques have been used to evaluate and describe coordinate system irregularities. The basic techniques have developed from simple differences to differences in coordinate "bins" to full spherical and cylindrical harmonic treatments. We are undertaking a 15-year program of minor planet observation with the express purpose of finding basic parameters of the adopted fundamental system with respect to a dynamical reference frame. The program is expected to provide a few hundred observations per year of 34 minor planets selected for their distribution of physical and orbital characteristics. The ability of the program to contribute to our knowledge of the rotation of the fundamental system and systematic irregularities within the system will depend on the accuracies of the observations and the distribution of the observations over the orbits and over the celestial sphere. We are considering the use of splines as a method of evaluating systematic corrections to the extant fundamental system. The initial development of the formalism and prospects for evaluation are presented.

Celestial coordinate systems provide a basis for observation of various phenomena and for the determination of various physical and observational properties of individual objects. For example, the uses of the coordinate system from antiquity still obtain for the determination and prediction of the motions of the solar system bodies in space, for civil time keeping and for navigation (extending to celestial navigation within the solar system). Another example is the determination of the relationship between a radio map and an optical map of the same object in order to determine the object's physical properties. One of the uses of an accurate coordinate system is to study the motions of classes of objects within that system to discover the dynamical basis for their motions (e.g., the motions of a set of A stars within the galaxy). A final example is to discover the physical basis as to how the coordinate system itself behaves because of the properties of the objects in it, and the structure of Space-Time itself. Each of these "uses" of the coordinate system calls on different aspects of the coordinate system structure for the analysis. The relations between observations of the same object at the different frequencies requires that the single coordinate system be realized in the observed bandpasses; the relation will be limited by the local and global *relative* errors of both wavelength observations. In trying to measure

S. Débarbat et al. (eds.), Mapping the Sky, 159–164.

the motions of objects within the coordinate system, major questions involve the global and temporal coherence of the *system*. Thus, different observations put different requirements on the coordinate system, even though (usually) the coordinate system will have been derived by techniques quite independent of its ultimate uses. Therefore, the coordinate system should be tested in as many independent ways as possible. (Here, *independence* refers to the techniques of testing, as well as independence from the methods and instruments used in the system formation.)

We are undertaking a 15-year project of minor planet observation with the express purpose of finding basic parameters of the adopted fundamental system with respect to a dynamical reference frame. In the end, we expect to have about 3,000 observations of a selected set of 34 minor planets. The positions will be with respect to a practical realization of the extant fundamental system, at the 0.01 to 0.1 arcsecond level, per observation. The orbits will be tied together by "crossing point observations" at the 0.02 to 0.002 arcsecond level per observation. The following figure shows the positions of plates already obtained from 1983 to 1987.



Observations of the 34 minor planets made between July 1983 and May 1987 by the Texas Minor Planet Project. Hammond-Aitoff equal area projection of geocentric equatorial coordinates.

Historically, systematic differences have been derived by comparing differences of positions in "bins" on the sky. Two such examples are the corrections to the GC and Yale zones derived by Pierce (1974) using the ideas of Brouwer (e.g., Brouwer, 1935) upon which our project is based (e.g., Hemenway, 1980). More classically, tables of systematic differences between catalogues in α and δ as functions of position and magnitude are routinely published with a new catalogue. A second method of finding catalogue corrections is analytical, and usually global. The formalism is given by Eichhorn (1974) as

$$\Delta\alpha = \Delta\alpha_\alpha + \Delta\alpha_\delta + \Delta\alpha_m$$

$$\Delta\delta = \Delta\delta_\alpha + \Delta\delta_\delta$$

https://doi.org/10.1017/S0074180900139567 Published online by Cambridge University Press

for meridian circle catalogues. Given the problem of color and magnitude for a general observational system, we need to include color terms in both coordinates and a magnitude term in declination.

The development of the terms in α and δ have been made analytically or graphically (by smoothing the bins by hand, for example) in order to find the corrections at a particular point in the sky. Brosche (1966) developed an analytical procedure for finding $\Delta\alpha$, $\Delta\delta$ as a function of position, using associated Legendre polynomials in δ and orthogonal sinusoidal functions in α; the system is a sum of spherical harmonics. The system has been used not only for star catalogue corrections, but for other problems including finding the coefficients of flexure models for the telescopes at McDonald Observatory. Again, following Eichhorn (1974), the functional form is

$$\Delta(\alpha, \delta) = \sum_{m,n} P_{nm}(\delta)\,(S_{nm}\sin(m, \alpha) + C_{nm}\cos(m, \alpha))$$

We have begun to look again at the question of "catalogue corrections" as pertains to the development of systematic irregularities in the initial coordinate system against which the minor planets are observed, and with respect to which the systematic corrections are to be found.

The general problem is as follows. Given: 1) a set of measured minor planet positions with respect to a fundamental catalogue, and 2) a set of ephemeris positions at the times of observation, based on integration of the best available orbits and solar system model (DE200, for example). Then: What is the "best" or "most meaningful" way to derive systematic corrections to the system of star positions and motions?

Classically, the answer would be to solve for the orbits and the system corrections simultaneously (Brouwer, 1941; Pierce, 1974). However, one problem we see is that the use of global functions forces symmetric or asymmetric components to "respond" to local irregularities in a global way. Thus, since any series must be truncated *someplace*, per force, adequate modeling of an arbitrary local irregularity at the correct spatial wavelengths is not obvious. One problem with "bins" is that they have discontinuous boundaries. Therefore, we were led to the possibility of using two-dimensional piecewise continuous polynomial approximations (splines) as an alternative. Our discussion follows the development of Schultz (1973) with appropriate changes in notation to conform somewhat with Jefferys (1980, 1981).

Suppose that a function is evaluated at 5 points, $K = 0, 1, 2, 3, 4$. We will refer to these points as "nodes." Also, let us define a set of "basis functions:"

$$s_i(\alpha) = \sum_{k=0}^{4} [D\,\omega_i\,(\alpha_{i+k})]^{-1}\,(\alpha_{i+k} - \alpha)_+^3$$

and similarly for δ, μ_α, and μ_δ at $(\hat{\alpha}, \hat{\delta})$. Then, in the interval considered, the spline function is a sum of the basis functions evaluated at each point between nodes, but appropriately multiplied by a coefficient which is the result of an adjustment procedure.

The area of sky covered by the selected minor planets is basically the half sphere between +30° and −30° ecliptic latitude. The minor planets have inclinations of at least 20° and an even distribution of the longitude of the ascending node. In equatorial coordinates, the classical condition equations are:

$$\overset{o}{\alpha} - [\overset{c}{\alpha}(\bar{a}, \overset{o}{t}) + \Delta\alpha(\bar{a}, \overset{o}{t})] = 0 \equiv \mathbf{D}\alpha$$

where

$$\Delta\alpha = \sum_{j=1}^{6} \frac{\partial\alpha}{\partial c_j} \Delta c_j + \text{system corrections}$$

Here the super "o" stands for observed, the super "c" stands for computed, and $\Delta\alpha$ is the residual at the time of observation. If $\Delta\alpha$ is taken at (α, δ) to represent the catalogue correction, then it is really made up of:

$$\Delta\alpha(\bar{a}, \overset{o}{t}) = \Delta\alpha(\bar{a}, T_*) + \Delta\mu_\alpha(\overset{o}{t} - T_*)$$

to first order, where T_* is the mean epoch of the catalogue being corrected.

The crossing point observation equations are:

$$\mathbf{D}\alpha_2 - \mathbf{D}\alpha_1 = 0$$

$$\mathbf{D}\delta_2 - \mathbf{D}\delta_1 = 0$$

where minor planet 2 is observed at t_2 and minor planet 1 is observed at t_1. Note that the correction to the coordinates at epoch cancels because the observations are in the same star field; the local reference frame is common. This is the advantage of the crossing points: Only the term $\Delta\mu_2(t_2 - t_1)$ remains.

We could consider the observations individually. However, if we put our observations in bins, one possibility is:

$$\bar{a} = [\{(a_j, e_j, i_j, \omega_j, \Omega_j, M_{o_j} \mid 1 \le j \le 34\},$$

$$\{b_{i_j}^{\Delta\alpha}(\alpha, \delta) \mid 0 \le i \le 23 \text{ and } 1 \le j \le 5\},$$

$$\{b_{i_j}^{\Delta\delta}(\alpha, \delta) \mid 0 \le i \le 23 \text{ and } 1 \le j \le 5\},$$

$$\{b_{i_j}^{\Delta\mu_\alpha}(\alpha, \delta) \mid 0 \le i \le 23 \text{ and } 1 \le j \le 5\},$$

$$\{b_{i_j}^{\Delta\mu_\delta}(\alpha, \delta) \mid 0 \le i \le 23 \text{ and } 1 \le j \le 5\}]$$

or 684 parameters altogether. With 3,000 observations (of both α and δ), the system is nicely overdetermined. But does it do what we want? In fact, the b_{ij}'s are piecewise analytic functions over the interval α_i to α_{i+4} and δ_j to δ_{j+4}, and are computed using the functions s.

In the functions s, we have the following definitions:

$$D\,\phi(x) \equiv \frac{d\phi}{dx},$$

$$w_i = \prod_{k=0}^{4} (x - x_{i+k}),$$

$$\xi_+^3 \equiv \begin{Bmatrix} \xi^3, & \xi > 0 \\ 0, & \xi \le 0 \end{Bmatrix}$$

where the b_{ij}^f are defined as the two-dimensional extension of the s_i:

$$f(\alpha, \delta) = \sum_{j=1}^{5} \sum_{i=0}^{23} b_{ji}^{l}\, s_i^l(\alpha)\, s_j^l(\delta)$$

https://doi.org/10.1017/S0074180900139567 Published online by Cambridge University Press

where $s_i(\alpha)$, $l=1$; $s_i(\delta)$, $l=2$; $s_i(\Delta\mu_\alpha)$, $l=3$; and $s_i(\Delta\mu_\delta)$, $l=4$. Thus, the splines provide an interpolation between the individual data points.

A major question is whether or not one is trying a) to force the "corrections" to fit a piecewise smooth function, or b) to fit a model to the residuals in a global statistical adjustment sense. Whether the least squares or spherical harmonic adjustment to a model really separates random from systematic errors, or whether the procedure simply smooths the systematic and random errors over the scaling implicit in the model is not entirely clear. The spline solution will produce accurate local representation between individual nodes, weighted to be continuous and n times differentiable at the nodes ($n=3$ with 5 nodes, in the case given). Whether such a representation is an advantage over the other methods is our next area of investigation. The orbit correction program is working, including crossing point observations. Next we will add the star system parameters, in both the analytical (spherical harmonic) and spline fashion, and investigate the differences in the solutions. We will have the advantage of knowing what the "true" solution should be, because those data, plus noise, will be the input. We will then have a direct test of the ideas presented here.

Acknowledgments. We are indebted to Professor William H. Jefferys for suggesting the spline approach. We are indebted to many people for discussions about the form of the catalogue corrections, including Professor Walter Fricke, Dr. Jean Kovalevsky, Dr. Peter Brosche, Dr. David Pierce, and our late colleague, Dr. Paul Herget. We alone are responsible for the content of this paper. We gratefully acknowledge the support of the National Science Foundation for all aspects of this work for the past 8 years.

REFERENCES

Brosche, P. 1966 *Veroff. des Astron. Rechen-Institut*, Heidelberg, #17.
Brouwer, D. 1935 *Astron. J.*, **44**, 1022.
Brouwer, D. 1941 in *Annals of the New York Academy of Sciences*, **XLII**, 133.
Eichhorn, H. 1974 *Astronomy of Star Positions*, Fredrick Ungar Publishing Co., New York.
Hemenway, P. 1980 *Celest. Mech.*, **22**, 89.
Jefferys, W. H. 1980 *Astron. J.*, **85**, 177.
Jefferys, W. H. 1981 *Astron. J.*, **86**, 149.
Pierce, D. 1971 *Astron. J.*, **76**, 177.
Schultz, M. H. 1973 *Spline Analysis*, Prentice Hall, Inc., Englewood Cliffs, New Jersey.

Discussion:

KRISTENSEN For the numerical experiment comparing spline-functions versus sperical harmonics I propose that the FK5 data rather than artifical data be used.

HEMENWAY We might be able to compare the spline technique to the spherical harmonic technique on the FK5 data. However, the solution for the spline function coefficients will be made simultaneously with the solution for the orbital elements. Therefore, a direct comparison of the techniques on the FK5 data will require a separate program.

GEFFERT Have you considered in your project the possibility of tying the minor planets directly to optical bright radiosources with precise positions from VLBI?

HEMENWAY Yes. In our original discussions (1978-1980) we pointed out the possibility of comparing an extragalactic and a dynamical reference frame. Because of the small cross-section of the minor planets passing close to radiosources that have good optical objects (position calibrators in both the radio and optical), the comparison will be made using the HIPPARCOS reference frame as an intermediary. We plan to observe HIPPARCOS with respect to QSOs and ultimately reduce the minor planets to the HIPPARCOS system.

HUGHES Do you characterize each star field by a single mean epoch?

HEMENWAY No. Since our observations will span 15-20 years, we must worry about the motions of the reference stars, and the systematic motions in the backgound stars in the case of the crossing point observations.

MURRAY 1) What reference star catalogue do you use for the observations?

2) Are your minor planets the same as those in the HIPPARCOS list?

3) There must be several hundred observations of the HIPPARCOS minor planets made with the Carlsberg Automatic Meridian Circle in the last three years.

HEMENWAY 1) We are planning to reduce our plates with respect to the IRS when it becomes availabe, and obviously, re-reduce the plates with respect to HIPPARCOS when that catalogue becomes available.

2) Except for (51) Nemausa, our minor planets are too small and hence too faint to be observed by HIPPARCOS. They were mostly chosen to be small enough to be observed by the interferometric guiding system on the Hubble Space Telescope, which is smaller than about $0\overset{''}{.}04$ or 30 km (at 1 A.U.). However, we are considering a crossing point program for the HIPPARCOS minor planets.

https://doi.org/10.1017/S0074180900139567 Published online by Cambridge University Press