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A GENERALIZATION OF THE MAPPING DEGREE

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For the single-valued case the notion of degree has been given recent expression by papers of Dold [**5**] for the finite dimensional case, and by Leray-Schauder [**8**] for the locally convex linear topological space. Klee [**7**] has removed this restriction by use of shrinkable in place of convex neighborhoods with the central role filled by a form of (2.15) below. For set-valued maps a modern formulation is, for instance, to be found in Gorniewicz-Granas [**6**]. These contributions relate the degree to the Lefschetz number, and the set-valued maps are required to map points into acyclic sets; that is to say, into "swollen points". The degree formulation presented here is valid for non-acyclic maps, and in general cannot be related to a Lefschetz number.

1. Preliminaries. The results obtained are new even for the finite dimensional situation. For simplicity of exposition we assume E is a Banach space and denote the origin by θ. In view of the maintenance of (2.15) in [**7**] it seems likely that our conclusions will extend to paracompact linear topological spaces. An *upper semi-continuous* (usc) map F takes points into closed sets and $F^{-1}(K) = \{x|F(x) \cap K \neq \emptyset\}$ for a closed set K is closed. Since the spaces involved are linear T_2, the usc condition is equivalent to the closure of the graph of F. Let D be the closure of a convex open set in E. Write

$$D_N = D \cap E_N$$

where E_N is an N-dimensional subspace of E and, as is well-known, E_N can be assumed Euclidean. A *dot* will indicate boundaries. Thus $\dot{D}$ is the boundary of D and

$$\dot{D}_N = \dot{D} \cap E_N.$$

Then D_N and $\dot{D}_N$ are topologically an N-disk and an $N - 1$ sphere, S^{N-1}, respectively. Let

$$F : D \to E$$

be *compact* in the sense that the closure of Im F, indicated by the notation $\overline{F(D)} = C$, is compact. We use the symbols $F_N = F|D_N$, $\dot{F}_N = F|\dot{D}_N$. The graph of F is

$$\Gamma(F) = \{(x, y)|y \in F(x), x \in D\} \subset D \times C.$$

Similarly $\Gamma(F_N)$ and $\Gamma(\dot{F}_N)$ are the correspondents with D replaced by D_N and by $\dot{D}_N$ respectively.

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We assume the cohomology groups are Alexander-Spanier reduced groups with *integer coefficients*. The *singular sets* are defined by

$$\sigma_r = \{x | H^r F(x) \neq 0\}$$

and

(1.01) $\sigma = \bigcup_r \sigma_r.$

The effective *bound for non-acyclicity* is

(1.02) $\mathbf{p} = 1 + \sup_r \{r + \dim \sigma_r | \sigma_r \neq \emptyset\},$

where $\dim \sigma_r$ is the maximum covering dimension (by finite covers, though this is not significant) of subsets of σ_r closed in D.

Our analysis is restricted to the following transformations:

Definition (1.1). The transformation F on D to E is *admissible if*

(1.11) F is upper semi continuous and compact;
(1.12) F is fixed point free on $\dot{D}$;
(1.13) $\mathbf{p} < \infty$; $\sigma \in E_S, S < \infty$.
(1.14) If $x \in \sigma$ then $H^*F(x)$ is finitely generated.

An admissible homotopy $h : D \times I \rightarrow E$ satisfies (1.11), . . . , (1.14) with F, D and $\dot{D}$ replaced by h, $D \times I$ and $\dot{D} \times I$ respectively.

Evidently admissibility implies (1.11), . . . (1.14) are valid also for F_N, D_N, E_N replacing F, D, E. In the sequel, F *invariably* designates an admissible transformation. Despite this, to make this work available for easy referencing, the hypotheses of some key theorems emphasize the fact by repeating this admissibility restriction.

We shall rely heavily on the following theorem.

THEOREM 1.2. *Let F_N be the restriction to D_N of an admissible transformation. Let $p(q)$ project the graph $\Gamma(\dot{F}_N)$ to $\dot{D}_N$*(Im $\dot{F}_N$). Then

(1.21) $p^* : H^{N-1}(\dot{D}_N) \rightarrow H^{N-1}\Gamma(\dot{F}_N)$

is an isomorphism for $N - 1 > \mathbf{p}$, (and $\dot{F}_N{}^*$ exists for $N - 1 > \mathbf{p}$) and

(1.22) $H^m\Gamma(F_N) \approx 0$ *for* $m \geqq N - 1 > \mathbf{p}$.

Moreover, (1.21) *and* (1.22) *remain valid if an admissible homotopy*, h, *replaces F and h_N, $\dot{D}_N \times I$ replace F_N, $\dot{D}_N$.*

This result can be derived as a consequence of [**9**] once it is established that p is closed, but this is obvious here when it is recognized that $\Gamma(\dot{F}_N)$ is compact as a consequence of (1.11) and the compactness of $\dot{D}_N \times C$. (cf. [**1**; **2**; **3**]).

2. Basic considerations. It is understood throughout this paper that the total singular set σ (cf. (1.01) is contained in E_S where S is minimal and finite. Let $R = \max(S, \mathbf{p} + 2)$ and

(2.11) $E_R \supset E_S.$

We make immediate use of (1.12); write

(2.12) $f = 1 - F$

where 1 is the identity map. Since F is usc and $\overline{F(D)}$ is compact, $f(\dot{D})$ is closed and is disjunct from θ. Accordingly there is a symmetric convex open set U about θ for which

(2.13) $f(\dot{D}) \cap U = \emptyset$.

By compactness, $\overline{F(D)}$ admits a finite cover

$$\alpha = \{x_i + U | x_i \in \overline{F(D)}\}.$$

Choose for E_N the linear extension of $\{x_i\}$ and E_R whence

(2.14) $E_N \supset E_R$.

Let Q_N be a continuous map of $\overline{F(D)}$ into E_N for which

(2.15) $z = Q_N z + u, \qquad u \in U.$

Indeed one such map is obtained by starting with a partition of the identity, $\{\pi_i\}$, subordinate to α. Define Q_N by

(2.16) $Q_N z = \sum \pi_i (z) x_i$.

That (2.15) is valid follows from the trivial observation that for $z \in \overline{F(D)}$

(2.17) $z = \sum \pi_i (z) z$.

If $z \overline{\in} x_i + U$, $\pi_i (z) = 0$. Hence combining (2.16) and (2.17), $z - Q_N(z)$ is evidently the barycenter of vectors $z - x_i$ in the convex set U and therefore lies in U.

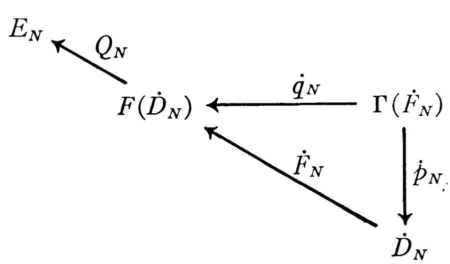
In the sequel we shall use the notation Q_N for any continuous map of $\overline{F(D)}$ to E_N satisfying (2.15). Moreover we tacitly assume that every E_N entering our results satisfies (2.14).

We introduce the notation

$$f_N = 1_N - Q_N F_N, \qquad \dot{f}_N = f_N | \dot{D}_N.$$

In order to define a cohomology homomorphism induced by f_N we use a representation of $Q_N \dot{F}_N$, namely

(2.18)



where $\dot{p}_N (\dot{q}_N)$ are the obvious projections of $\Gamma(\dot{F}_N)$.

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Remark. It might appear simpler to analyze $Q_N\dot{F}_N$ by using $\Gamma(Q_N\dot{F}_N)$ and the corresponding projections p′ and q' on $\dot{D}_N$ and on $Q_N F_N(\dot{D}_N) \subset E_N$ respectively. However, since the singular set for $Q_N\dot{F}_N$ may be quite different from that for $\dot{F}$, this alternative approach would involve serious irrelevant difficulties.

Formally,

(2.19) $f_N = (p_N - Q_N q_N)p_N^{-1} : D_N \rightarrow E_N.$

We designate the map in the parenthesis by T_N. Its domain is $\Gamma(F_N)$. Accordingly, $f_N = T_N p_N^{-1}|D_N$. Write also

$$\dot{T}_N = T_N|\Gamma(\dot{F}_N).$$

To simplify notation we write $\dot{f}_N^*$, $\dot{p}_N^*$ and $\dot{T}_N^*$ for homomorphisms on the $N - 1$ dimensional cohomology groups. Then

LEMMA 2.20. *For admissible F (when (2.14) is satisfied),*

$$\dot{f}_N^* = \dot{p}_N^{*-1}\dot{T}_N^*$$

is a homomorphism on $H^{N-1}(S^{N-1} \times K)$ to $H^{N-1}\dot{D}_N$ for K a closed segment of the reals.

We show first $\dot{T}_N$ applied to $\Gamma(\dot{F}_N)$ does not cover $\theta_N \in E_N$. Otherwise $x = Q_N y$ for some $x \in \dot{D}_N$ and $y \in F(x)$. Thus $x - y \in f(\dot{D})$, yet $y - Q_N(y) \in U$ in contradiction with (2.13). Since Γ $(\dot{F}_N)$ is closed in $\dot{D}_N \times C$, it is compact. Hence $\dot{T}_N\Gamma(\dot{F}_N)$ is compact. Accordingly the image of $\dot{T}_N$ is bounded and is disjunct from a sufficiently small disk about θ_N. It is therefore contained in $S^{N-1} \times K$, the annulus with coordinates (x, s), for $x \in S^{N-1}$ and $s \in K$, the closed interval from ϵ to M for some ϵ and $M \geqq \epsilon > 0$. Since $N \geqq \mathbf{p} + 2$, Theorem (1.2) asserts $\dot{p}_N^*$ is an isomorphism on $H^{N-1}(\dot{D}_N)$ to $H^{N-1}\Gamma(\dot{F}_N)$. The assertion of the lemma follows.

The deformation retraction r of $S^m \times K$ to S^m induces an isomorphism on the corresponding cohomology groups which will be denoted by r^* (instead of r_m^*).

3. The degree. We introduce a degree definition dependent on the key sequence,

(3.1) $$H^{N-1}(S^{N-1}) \xrightarrow{r^*} H^{N-1}(S^{N-1} \times K) \xrightarrow{\dot{T}_N^*} H^{N-1}\Gamma(\dot{F}_N) \xrightarrow{\dot{p}_N^{*-1}} H^{N-1}(\dot{D}_N)$$

where, F is of course admissible and in view of (2.14), Theorem 1.2 ensures the existence of $\dot{p}_N^{*-1}$.

Definition 3.2. Let deg denote the usual degree of a homomorphism on $H^M(S^M)$ to $H^M(S^M)$.

This is defined as the integer

$$\epsilon\ (\cap\gamma_m)\ h(m)\gamma^m$$

where γ^m and γ_m are generators of $H^m(S^m, J)$ and $H_m(S^m, J)$ whose Kronecker index is 1 so $\cap\gamma_m$ yields Poincare Duality and the augmentation homomorphism ϵ replaces an integral multiple of a generator by the integer. Thus

$$H^m(S^m) \xrightarrow{h(m)} H^m(S^m) \xrightarrow{\cap\gamma_m} H_0(S^m) \xrightarrow{\ \epsilon\ } J.$$

The *relative degree* of f requires the assumption of (2.14) and is defined as

$$(3.2)\qquad d[f, U, Q_N, E_N] = \deg(\dot{p}_N{}^{*-1}\dot{T}_N{}^*r^*) = \deg(\dot{f}_N{}^*r^*).$$

In spite of the apparent arbitrariness, the relative degree is independent of the choice of U, E_N and Q_N. For convenience, we note the known result asserting naturality of the Mayer-Vietoris sequence.

LEMMA 3.3: *If T is a map of $A, B \to A', B'$, then there is commutativity in the squares*

$$\begin{array}{ccccc} H^m(A) \oplus H^m(B) & \to & H^m(A\cap B) & \xrightarrow{\Delta} & H^{m+1}(A\cup B) \\ \uparrow & & \uparrow & & \uparrow \\ H^m(A')\oplus H^m(B') & \to & H^m(A'\cap B') & \xrightarrow{\Delta} & H^{m+1}(A'\cup B'). \end{array}$$

LEMMA 3.4. *The relative degree is independent of the choice of E_N.*

Suppose $E_{N+1} \supset E_N$. Write $e_N{}^+$ and $e_N{}^-$ for the upper and lower closed hemispherical cells of S^N determined by the equatorial cutting plane E_N. Then

$$S^{N-1} = e_N{}^+ \cap e_N{}^-, \qquad S^N = e_N{}^+ \cup e_N{}^-.$$

$$(3.41)\qquad \begin{aligned} \Gamma(\dot{F}_N) &= \Gamma(F|e_N{}^+) \cap \Gamma(F|e_N{}^-). \\ \Gamma(\dot{F}_{N+1}) &= \Gamma(F|e_N{}^+) \cup \Gamma(F|e_N{}^-). \end{aligned}$$

In view of Theorem (1.2), and recalling that (2.14) is implied,

$$0 = H^m(e_N{}^{\pm}) \approx H^m(\Gamma(F|e_N{}^{\pm})\ m \geqq N-1.$$

Hence, by exactness of the Mayer Vietoris sequence

$$(3.42)\qquad \begin{aligned} &H^{N-1}\Gamma(\dot{F}_N) \overset{\Delta}{\approx} H^N\Gamma(\dot{F}_{N+1}), \\ &H^{N-1}(S^{N-1}) \overset{\delta}{\approx} H^N(S^N). \end{aligned}$$

where Δ and δ refer to the boundary homomorphisms in the Mayer Vietoris sequence. Then

$$\begin{array}{ccccccc} H^{N-1}(S^{N-1}) & \xrightarrow{r^*} & H^{N-1}(S^{N-1}\times K) & \xrightarrow{\dot{T}_N^*} & H^{N-1}\Gamma\dot{F}_N & \xleftarrow{\dot{p}_N^*} & H^{N-1}(\dot{D}_N) \\ \approx\downarrow\delta\quad \text{I} & & \approx\downarrow\delta\quad \text{II} & & \approx\downarrow\Delta\quad \text{III} & & \approx\downarrow\delta \\ H^N(S^N) & \xrightarrow{r^*} & H^N(S^N\times K) & \xrightarrow{\dot{T}_{N+1}^*} & H^N\Gamma\dot{F}_{N+1} & \xleftarrow{\dot{p}_{N+1}^*} & H^N(\dot{D}_{N+1}). \end{array} \tag{3.43}$$

By reason of (3.42), Lemma (3.3) assures commutativity of I, II, III. Only II merits more details. Observe then that Q_N ia obviously available as a choice for Q_{N+1}. Note $C_N(\pm) = Q_N\, F\, e_N(\pm)$ denote bounded subsets of E_N. Hence

$$\dot{f}_{N+1}(e_N\pm) = e_N(\pm) - C_N(\pm) \not\supset \theta_{N+1}$$

is a collection of translates of $e_N(\pm)$ parallel to E_N (in E_{N+1}). In particular, equality is justified in

$$\dot{f}_N(e_N{}^+\cap e_N{}^-) = \dot{f}_{N+1}(e_n{}^+)\cap \dot{f}_{N+1}(e_n{}^-),$$

This is a collection of translates in E_N of $S^{N-1} = e_N{}^+\cap e_N{}^-$. Also

$$\dot{f}_{N+1}(e_N{}^+\cup e_N{}^-) = \dot{f}_{N+1}(e_N{}^+)\cup \dot{f}_{N+1}(e_N{}^-) \not\supset \theta_{N+1}.$$

In Lemma (3.3) if we identify A, B with $\Gamma(\dot{F}_{N+1}|e_N{}^+)$ and $\Gamma(\dot{F}_{N+1}|e_N{}^-)$ we have just shown that for a suitable positive interval K for the radial coordinates we can define A', B' by

$$e_N{}^+\times K = A' \supset \dot{f}_{N+1}(e_N{}^+) = \dot{T}_{N+1}\Gamma(\dot{F}_{N+1}|e_N{}^+)$$
$$e_N{}^-\times K = B' \supset \dot{f}_{N+1}(e_N{}^-) = \dot{T}_{N+1}\Gamma(\dot{F}_{N+1}|e_N{}^-).$$

Then

$$A'\cap B' = S^{N-1}\times K \not\supset \theta_N$$
$$A'\cup B' = S^N\times K \not\supset \theta_{N+1}.$$

The commutativity of II follows forthwith.

It is therefore clear that in view of the definition (3.2),

$$d[f, U, Q_N, E_N] = d[f, U, Q_N, E_{N+1}] \tag{3.44}$$

follows from consistent choices of corresponding generators in the four corner groups in (3.43). More generally, if $E_M \supset E_N$, one applies the conclusion (3.44) to the chain $E_N \subset E_{N+1} \subset \ldots \subset E_{M-1} \subset E_M$. Finally, for any pair E_N, E_M, (3.44) applies by comparing with $E_Q = E_N + E_M$.

LEMMA 3.5. *$d[f, U, Q_N, E_N] = d[f, U', Q_N, E_N]$, where U and U' satisfy* (2.13).

Assume Q_N is associated with U. Suppose first that $U'\supset U$. Since Q_N satisfies (2.15) for U, it does for U' also. This is all that is required to define $\dot{T}_N$ with range in $S^{N-1}\times K$; that is to say, all the homomorphisms in (3.1) are unaffected. For arbitrary U' and U, introduce $U'' = U'\cap U$ so the conclusion still holds.

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LEMMA 3.6.

$$d[f, U, Q_N, E_N] = d[f, U, Q'_N, E_N].$$

Let $Q(t) = tQ_N + (1 - t)Q'_N$, $t \in I$. Then $Q(t)$ is a homotopy satisfying (2.15). Accordingly there is an induced homotopy.

$$\dot{T}_t = t\dot{T}_N + (1 - t)\, \dot{T}_N'$$

and it is easy to see that the range of $\dot{T}_t$ does not include θ_N. Hence $\dot{T}_N{}^*$ and $\dot{T}_N{}'^*$ are the same, whence the asserted conclusion of the lemma.

In view of Theorems (3.4), (3.5), and (3.6) we can drop the specialization to particular choices of U and E_N and refer to the integer in (3.4) as the *degree* of f and write $d[f]$. The case of a finite dimensional Banach space $E = E_N$ is covered if $N \geqq \mathbf{p} + 2$.

4. Properties of the degree. We now establish some fundamental properties of the degree. The first, without which the definition of $d[f]$ would be of little interest, is

THEOREM 4.1. *For admissible F, if $d[f] \neq 0$, then F admits a fixed point.*

Assume the assertion untrue. Then

$$(4.11) \quad f(D) \cap \theta = \emptyset.$$

That $f(D)$ is closed follows from (4.11), and the facts that $\overline{F(D)}$ is compact and F is usc. Let U be a symmetric convex open set satisfying

$$f(D) \cap U = \emptyset.$$

Note T_N is on $\Gamma(F_N)$ to E_N, and in fact, because of (4.11), the image of rT_N is in S^{N-1}. Let j be inclusion of $\Gamma(\dot{F}_N)$ in $\Gamma(F_N)$. By Theorem (1.2),

$$H^{N-1}(\Gamma F_N) \approx 0$$

whence

$$T_N{}^*\, r^* : H^{N-1}(S^{N-1}) \rightarrow H^{N-1}(\Gamma | F_N)$$

is trivial. Since the homomorphisms are induced by maps, the diagram

$$\begin{array}{ccc} H^{N-1}\Gamma(F_N) & \xleftarrow{\;T_N{}^*\;} & H^{N-1}(S^{N-1} \times K) \\ \downarrow{\scriptstyle j^*} & & \downarrow{\scriptstyle 1} \\ H^{N-1}\Gamma(\dot{F}_N) & \xleftarrow{\;\dot{T}_N{}^*\;} & H^{N-1}(S^{N-1} \times K) \end{array}$$

commutes. Hence $\dot{T}_N{}^*$ is trivial. Therefore the degree is 0 in contradiction with our hypothesis.

The next theorem is of prime importance also.

THEOREM 4.2. *Let h be an admissible homotopy of F with* $h(\ ,0) = F$. *Then*

$$d[I - F] = d[I - F_1].$$

We repeat the essentials of our earlier constructions. Thus with I the unit segment, since h is fixed point free on $\dot{D} \times I$, if

$$g = 1 - h$$

then the image of g on $\dot{D} \times I$ is closed and does not include θ. Choose U disjunct from $g(\dot{D} \times I)$ and E_N and Q_N to satisfy (2.11) and (2.14). Write h for $h|\dot{D} \times I, \dot{P}$ for the projection of $\Gamma(h)$ onto $D \times I$ and $\dot{h}_N, \dot{P}_N$ for the analogous transformations with $\dot{D}_N \times I$. Then, just as in the comment on Theorem 1.2, it follows that $\dot{P}_N^*$ is an isomorphism. The (continuous) map $\dot{T}_N(I)$ is defined as in (2.19) with $\dot{g}_N$ and $\Gamma(\dot{h}_N)$ replacing $\dot{f}_N$ and $\Gamma(\dot{F}_N)$. Thus

$$\dot{T}_N(I) : \Gamma(\dot{h}_N) \rightarrow S^{N-1} \times K.$$

The rest of the demonstration below is essentially a transcription of that in [**3**, Theorem 2.1] where the interest is in $h^* = p^{*-1}q^*$ while here $\dot{g}_N^* = \dot{P}_N^{*-1}\, \dot{T}_N^*(I)$. (In connection with [**3**], the phrase "since $F(x)$ does" ending the proof of Theorem 3.3, is pointless.)

Let $e(t)$ be the natural map of $\dot{D}_N$ in $\dot{D}_N \times t$. Then

$$\dot{g}_N\, e(t) = \dot{T}_N(I)\, \dot{P}_N^{-1}\, e(t) : \dot{D}_N \rightarrow S^{N-1} \times K.$$

(Evidently $\dot{g}_N\, e(t)$ is usc since g_N is usc.)

Accordingly, we can define

$$(4.12)\qquad (\dot{g}_N\, e(t))^* = e(t)^*\, (\dot{P}_N^{*-1}\, \dot{T}_N(I)^*).$$

Clearly

$$(4.22)\qquad (\dot{T}_N(I)\dot{P}_N^{-1}\, e(0))^* = (\dot{T}_N(I)\, \dot{p}_N^{-1}(0))^* = e(0)^*\dot{P}_N^{*-1}\, \dot{T}_N(I)^*$$

$$(\dot{T}_N(I)\, \dot{P}_N^{-1}\, e(1))^* = (\dot{T}_N(I)\, \dot{p}_N^{-1}(1))^* = e(1)^*\dot{P}_N^{*-1}\, \dot{T}_N(I)^*.$$

It is well-known [**4**, p.177] that $e(t)^*$ is independent of t. Hence combining (4.21) and (4.22) and noting that $\dot{g}_N(0) = \dot{g}_N\, e(0)$, $\dot{g}_N(1) = \dot{g}_N\, e(1)$ we arrive at

$$\dot{g}_N(0)^* = \dot{g}_N(1)^*$$

which is tantamount to the assertion of the theorem.

LEMMA 4.3. *If F is the constant map* $F : D \rightarrow x_0 \in D \cap \dot{D}^{\sim}$, *then* $d[f] = \pm 1$.

Evidently

$$\Gamma(\dot{F}_N) = S^{N-1} \times x_0,\ x_0 \,\overline{\in}\, S^{N-1}.$$

Then

$$\dot{T}_N x = x - x_0, \qquad x \in S^{N-1}$$

or $\dot{T}_N^*$ is an isomorphism.

Remark. Our *degree of f* seems more descriptive here than the conventional index of F, [**5**].

The restriction to convex domains can be weakened somewhat. Thus

LEMMA 4.4. *Let R be a deformation retract of D with retracting function r. Suppose the interior of R is non-empty. If F is admissible on R to E, then $d[f]$ can be defined to satisfy* (4.1), ... (4.3).

Write

$$F_D x = F r x$$

for all $x \in D$. The admissibility of F implies satisfaction of 1.11, ..., 1.14 when the domain in 1.1 is R. Evidently F_D is admissible on D to E. It is understood that f is $I - F$ (and f_D is $I - F_D$). Define

$$d[f] = d(f_D).$$

The last part of the lemma is then obvious.

Added in proof. The results are valid without the restriction $\sigma \in E_S, S < \infty$ and in fact none of the proofs require this condition.

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