# Composite graphs with stability index one

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We characterise the composite graphs with stability index one and
those with edge stability index one.

This thesis is motivated by the work of Grant, Holton, and Sims on the
semistability of various classes of graphs:  an investigation of the
relationship between the symmetry of a graph and of its vertex and edge
deleted subgraphs.  Most of the relevant definitions, results, and back-
ground are presented in Chapter 1.  A few of the concepts are defined as
follows.  A graph  $G$  is *semistable* [2] at one of its vertices  $v$  if  $G$
is trivial or the automorphism group of  $G - v$  is the stabiliser of  $v$  in
$G$ .  $G$  has *stability index one* [1] if  $G$  is semistable at one or more of
its vertices and, for all such vertices  $v$ ,  $G - v$  is not semistable at
any vertex.  $G$  is *pair transitive* (introduced as Property A in [3]) if,
for every two vertices in  $G$ , there is an automorphism of  $G$  that inter-
changes them.  A *composite* graph [4] is the cartesian product of two non-
trivial graphs.  The product is *proper* [5] if both of the factors contain
an edge.  Holton and Sims [3] have shown that, with some listed exceptions,
a pair transitive composite graph has stability index one.  They
conjectured a converse for connected composite graphs and thus a
characterisation of connected composite graphs with stability index one.
The thesis is devoted to the proof of this characterisation and its edge
analogue.  We also extend the work to disconnected composite graphs.

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Chapter 2 deals with the special case of connected *singly even* graphs: products with an unrepeated 2-path factor. We show that if such a graph has stability index one then it is pair transitive, or it is the product of the 2-path with either the 3-path or the 4-path.

All remaining composite graphs are considered in Chapter 3. We show that if they do not have a prime component then they are pair transitive or the union of one or more copies of one of the singly even exceptions. We find that, with one class of exceptions, all composite graphs with stability index one and a prime component are improper products.

The edge analogue of these results are explored in Chapter 4. We find that a composite graph with no prime component is pair edge transitive if and only if each of its non-trivial components is an $n$-cube for some fixed positive integer $n$. Also we prove that, excluding $n = 2$, these are precisely the composite graphs with edge stability index one and no prime component. Finally, we show that all composite graphs with edge stability index one and a prime component are improper products.

## References

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