Preface

Solitons were first discovered by a Scottish engineer, J. Scott Russell, in 1834 while riding his horse by a water channel when a boat suddenly stopped. A hump of water rolled off the prow of the boat and moved rapidly down the channel for several miles, preserving its shape and speed. The observation was surprising because the hump did not rise and fall, or spread and die out, as ordinary water waves do.

In the 150 years or so since the discovery of Scott Russell, solitons have been discovered in numerous systems besides hydrodynamics. Probably the most important application of these is in the context of optics where they can propagate in optical fibers without distortion: they are being studied for high-data-rate (terabits) communication. Particle physicists have realized that solitons may also exist in their models of fundamental particles, and cosmologists have realized that such humps of energy may be propagating in the far reaches of outer space. There is even speculation that all the fundamental particles (electrons, quarks etc.) may be viewed as solitons owing to their quantum properties, leading to a "dual" description of fundamental matter.

In this book I describe the simplest kinds of solitons, called "kinks" in one spatial dimension and "domain walls" in three dimensions. These are also humps of energy as in Scott Russell's solitons. However, they also have a topological basis that is absent in hydrodynamical solitons. This leads to several differences e.g. water solitons cannot stand still and have to propagate with a certain velocity, while domain walls can propagate with any velocity. Another important point in this regard is that strict solitons, such as those encountered in hydrodynamics, preserve their identity after scattering. The kinks and domain walls discussed in this book do not necessarily have this property, and can dissipate their energy on collision, and even annihilate altogether.

Why focus on kinks and domain walls? Because they are known to exist in many laboratory systems and may exist in other exotic settings such as the early universe. They provide a simple setting for discussing non-linear and non-perturbative

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physics. They can give an insight into the dynamics of phase transitions. Lessons learned from the study of kinks and domain walls may also be applied to other more complicated topological defects. Domain walls are good pedagogy as one can introduce novel field theoretic, cosmological, and quantum issues without extraneous complexities that occur with their higher co-dimension defects (strings and monopoles).

The chapters of this book can be approximately categorized under four different headings. The first two chapters discuss solitons as classical solutions, the next three describe their microscopic classical and quantum properties, followed by another three chapters that discuss macroscopic properties and applications. The very last chapter discusses two real-world systems with kinks and, very briefly, Scott Russell's soliton. The book should be accessible to a theoretically inclined graduate student, and a large part of the book should also be accessible to an advanced undergraduate. At the end of every chapter, I have listed a few "open questions" to inspire the reader to take the subject further. Some of these questions are intentionally open-ended so as to promote greater exploration. Needless to say, there are no known answers to most of the open questions (that is why they are "open") and the solutions to some will be fit to print.

Every time I think about research in this area, I feel very fortunate for having unwittingly chosen it, for my journey on the "soliton train" has weaved through a vast landscape of physical phenomena, each with its own flavor, idiosyncrasies, and wonder. I hope that this book, as it starts out in classical solitons, then moves on to quantum effects, phase transitions, gravitation, and cosmology, and a bit of condensed matter physics, has captured some of that wonder for the reader.

This is not the first book on solitons and hopefully not the last one either. In this book I have presented a rather personal perspective of the subject, with some effort to completeness but focusing on topics that have intrigued me. Throughout, I have included some material that is not found in the published literature. Prominent among these is Section 4.5, where it is shown that the leading quantum correction to the kink mass is negative. The discussion of Section 6.5, with its emphasis on a bifurcation of correlation scales, also expresses a new viewpoint. I had particular difficulty deciding whether to include or omit discussion of domain walls in supersymmetric theories. On the one hand, many beautiful results can be derived for supersymmetric domain walls. On the other, the high degree of symmetry is certainly not realized (or is broken) in the real world. Also, non-supersymmetric domain walls are less constrained by symmetries and hence have richer possibilities. In the end, I decided not to include a discussion of supersymmetric walls, noting the excellent review by David Tong (see below). Some other must-read references are:

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9. Tong, D. (2005). *TASI Lectures on Solitons*, [hep-th/0509216].

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