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ARTICLE

The plague, the skill-premium, and the road to modern economic growth†

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Abstract
When bubonic plague arrived in Britain in the mid-14th century, it caused dramatic economic and structural change. Within 50 years, the skill-premium was reduced by half, and another 50 years on, agriculture's share of the labor force had declined by more than 20 percentage points. This paper develops a two-sector pre-industrial growth model and draws on recent data sources covering Late Medieval and Early Modern Britain to explain these and the ensuing developments. Our main findings are that the skill-premium's decline was related to the guild and apprenticeship system and that it and the other post-Plague adjustments were crucial determinants of the British trajectory toward industrialization. In particular, prior sectoral transformation and the skill-premium's determination were important when the Early Modern population boom (1525–1654) threatened to reverse the adjustments caused by the Plague.

Keywords: Unified growth theory; skill-premium; physical-to-human capital ratio; sectoral transformation; pre-industrial economic development; long-run economic history

1. Introduction

When bubonic plague spread from Asia to Europe in the late 1340s, it devastated population levels and caused dramatic economic and institutional change. Labor shortage and revolt led to the eventual demise of serfdom in Western Europe and the birth of modern labor markets [Gottfried (1983), Allen (2008), and van Zanden (2009a)]. The Plague's impact on relative prices and income levels was also profound: Across Europe, interest rates fell by about half [Clark (1988)], and in Western Europe, the relative price of skilled to unskilled workers (the skill-premium) declined from an average of 128% before the Plague to an average of 55% between the early 15th and late 19th centuries [van Zanden (2009b), p. 127]. This paper's main focus is Britain whose historical skill-premium is shown in Figure 1a. Strikingly, after adjusting to a much lower level after the Plague, the skill-premium proceeded to maintain "this level for 500 years till about 1900" [Clark (2005b), p. 1309], and the interest rate likewise remained range bound.[1] As seen in Figure 1b, income per capita meanwhile settled in a (broad) range whose mean sat some 40% above the

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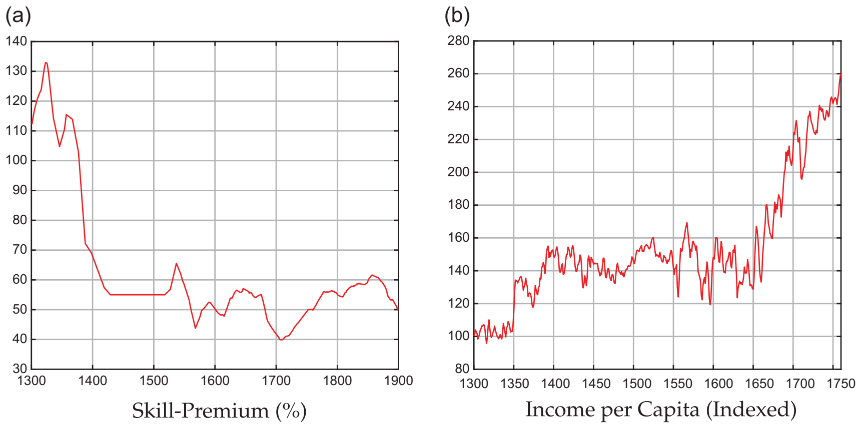

Figure 1. Skill-premium and income per capita in England.
Notes: Skill-premium calculated from Allen (2001)'s data for construction workers. Depicted curve is the 30-year moving average. Income per capita is from Broadberry et al. (2015) [BoE (2017), Table A21]. Indexed to base year 1300, and depicted as a 3-year moving average.

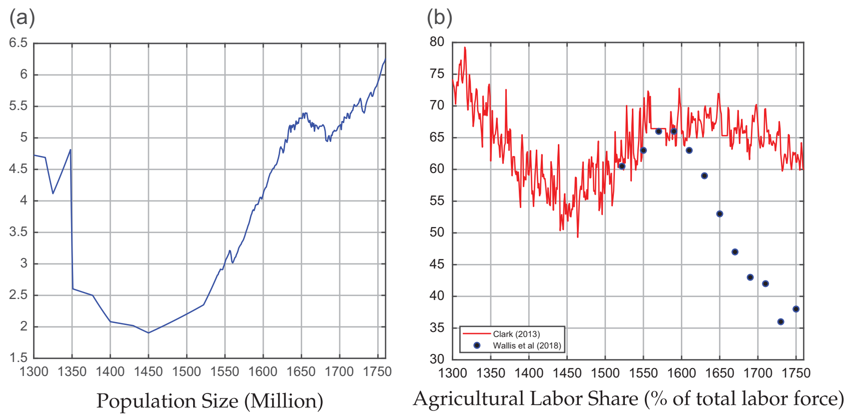

Figure 2. Population and sectoral transformation in England.
Notes: Population data are from Broadberry et al. (2015) [BoE (2017), Table A2]. The agricultural labor shares are from Clark (2013) and Wallis et al. (2018).

pre-plague average. Here it remained until it broke out in the mid-17th century, with the Industrial Revolution following not long after.

The 1300–1760 picture, then, is one of dramatic adjustments followed by 200 years of relative stability and a break-out and subsequent move into the modern growth regime. What is puzzling about this picture is that already by the early 1600s, the population size had completely recovered from the decline that set the adjustments off in the first place; see Figure 2a, noting especially the Early Modern population boom between 1525 and 1654. But if population decline was the fundamental *cause* of the initial adjustments—and no one doubts this—why did the Early Modern population boom not cause their reversal? Absent any hint of a technological revolution in this period [Mokyr (2005)], even less of a human capital take-off [Clark (2005b) and Galor et al. (2009)], one would have expected rising population to have diluted agricultural economies of scale back to pre-plague levels and therefore to have caused income per capita to decline and the relative costs of physical and human capital/skills to spike back up. What had changed in the meantime to prevent this from happening in Britain, and how is this related to the skill-premium's decline and stability pattern?

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To address this question, we develop a very long run, or "unified" [Galor and Weil (2000) and Galor and Moav (2004)], model of growth that specifically targets Late Medieval and Early Modern Britain (ca. 1300–1660). We then, in a manner similar to for example Leukhina and Turnovsky (2016), proceed to study the historical data structurally. Throughout, we draw heavily on economic historians' insights and data [Allen (2001), Broadberry et al. (2015), Clark (2005b, 2008, 2013), van Zanden (2009b), Shaw-Taylor and Wrigley (2014), Humphries and Weisdorf (2019), and Wallis et al. (2018)].

The answer we give begins with the skill-premium decline which we, like van Zanden (2009b), attribute to a reduction in the relative opportunity cost of human capital accumulation. Put simply, our explanation is that this caused a shift in human capital supply which overwhelmed demand and caused the relative price (skill-premium) to decline. In more detail, the Plague improved agricultural economies of scale, raised incomes and savings, and led to scarcity of human capital in comparison with physical capital. The resulting decline in the physical-to-human capital price ratio made it more attractive to invest in human capital, all else being equal. The supply-side response was, however, distorted by the apprenticeship system whose guardians were trade and craft guilds [Epstein (1998), Wallis (2008), and Wallis (2019)]. Due to this system, individual human capital levels were "stuck" at a level corresponding to the length of an apprenticeship, and this prevented diminishing returns to human capital production from setting in [*e.g.* see Galor and Moav (2004), p. 1002]. Instead, the decline in the physical-to-human capital price ratio affected the external margin of human capital accumulation and, all-else equal, made it more attractive to become skilled than to remain unskilled. This caused an "outsized" shift in the human capital supply curve. Although improved incomes increased the relative demand for non-agricultural goods (hence firms' demand for human capital), demand could not meet such an increase in supply, and the consequence was a sharp decline in the skill-premium.

As we show, this description explains both the direction and the quantitative magnitude of the decline after the Plague. Empirically, it also fits the agricultural labor share decline seen in Figure 2b. In fact, our explanation of the skill-premium decline implies a change in labor shares of precisely this magnitude because when individual-level human capital is "stuck," an increase in human capital requires a one-for-one decline in unskilled workers. In this way, our model both accounts for the skill-premium decline and the sectoral changes that followed.

To the best of knowledge, van Zanden (2009b) is the only other paper that seeks to explain the decline and subsequent stability of the skill-premium. Van Zanden focuses on the (partial equilibrium) supply of human capital: "the most straightforward explanation for the post-1350 decline of the skill-premium in construction is that interest rates in Europe declined sharply in this period. This induced households to increase their investment in human capital, which led to the observed change in the skill-premium." [van Zanden (2009b), p. 135]. Our model can be viewed as an elaboration of this explanation provided we interpret van Zanden as speaking about the number of skilled workers (as opposed to individual human capital accumulation). In addition, we model the demand side and explain the skill-premium decline in general equilibrium, and we link this to the sectoral labor shares. It should also be mentioned that, because the explanation does not invoke a Galor and Moav (2004)-type individual human capital transition, it is consistent with the conventional view that such a transition did not occur until hundreds of years after the Plague.[2]

Turning to the "why no reversal" question, our answer again begins with the skill-premium which had at this point stabilized. In our model, such stability sets in the moment individuals acquire *more* human capital than the guild-imposed minimum because at that point, any decline in the physical-to-human capital price ratio is "neutralized" by diminishing returns to human capital accumulation (Section 2.3.1 contains a detailed exposition of this key element of our model). In particular, at the start of the Early Modern population boom, a considerable increase in the physical-to-human capital price ratio was required for it to exit the zone where the skill-premium remained stable. In addition, the sectoral transformation discussed a moment ago reduced economy-wide diseconomies of scale, further blunting the reversal. The transformation also moved production into sectors with more potential for technological advancement. However, consistent with Mokyr (2005), the latter's impact was according to our empirical analysis very modest—the economy-wide productivity only grew by around 15% cumulatively over the entire 1400–1660 period (for comparison, the population size approximately tripled during the Early Modern Population Boom). Even so, when combined with reduced diseconomies and the dampening effect of the skill-premium stability, the productivity improvements become critical: In our counterfactual analysis, we show that absent the productivity increases, all of the changes caused by the Plague unravel toward the end of the Early Modern population boom. Interestingly, this counterfactual almost perfectly matches Southern Europe's historical record. As we discuss more in Section 4.3, this provides a new input into the "Why Britain" and "Little Divergence" debates [*e.g.* see Voigtländer and Voth (2006) and part 2 of van Zanden (2009a)].

What is the wider significance of these results? The study of pre-industrial Europe, and especially pre-industrial Britain and the lead-in to the Industrial Revolution, is central to economic history because understanding these developments informs our understanding of growth and development today. Thus, the "Kaldor-inspired" view (still widespread among non-historians) holds that at the dawn of the Industrial Revolution, Britain was economically not meaningfully different from Britain before the bubonic plague: a largely agrarian society whose per-capita income had until then been kept at subsistence levels by preventive and positive Malthusian checks (with a possible allowance for a "golden" 15th century).[3] Borrowing a phrase from development economics, British industrialization—and with some delay industrialization elsewhere—thus amounted to 'shock therapy', simultaneously transforming agrarian societies into industrialized nations and setting them on a virtuous path of capital accumulation, innovation, and technological adoption. A corollary of this view is that anything happening prior to the mid-18th century is of secondary importance to our understanding of economic growth and development today. This paper's findings have very different implications. In particular, any 'shock therapy' development strategy is misguided to the extent it is thought to mimic Britain's path to industrialization. British industrialization was preceded by much earlier skill-premium and sectoral transformations—and if we accept this paper's findings, either one might well not have happened at all if not for the historical coincidence of the Plague.[4]

The paper is organized as follows: Section 2 lays out the model and clarifies the determination of the skill-premium. Section 3 solves the model and establishes our main results on the relationship between the physical-to-human capital ratio and the skill-premium. Section 4 contains our empirical analysis along with a fully developed account of the findings sketched above. Finally, Section 5 summarizes and concludes. The appendices contain details of the data sources, calibration, model testing, and a discussion of how adopting the data in Clark (2004, 2010, 2013) instead of Broadberry et al. (2015) affects the empirical results.

2. A two-sector pre-industrial growth model

The model is based on Galor et al. (2009), which we extend in two ways to better account for the developments taking place in the period between the onset of the Plague and the end of the Early Modern Population boom (ca. 1300–1660). First, we do not assume that the sectors produce the same homogenous good. Second, we explicitly model the occupational choice between skilled and unskilled work in an attempt to capture the institutional framework laid out in for example Epstein (1998) and Wallis (2008, 2019). The first of these allows an interplay between demand patterns and the sectoral composition similar to what we see in the data. The second is key to accounting for the skill-premium's decline and stability pattern. Note that to minimize distractions, all discussion of the historical context is placed in a separate subsection (Section 2.4).

2.1. Production

There are two sectors: an *agricultural* sector and a *non-agricultural* sector comprising both manufacturing and services. Output (real GDP) at date t, Y_t, is the sum of agricultural sector output Y_t^{P} and non-agricultural sector output Y_t^{S} with the non-agricultural sector's output price as the numeraire:

$$Y_t = p_t Y_t^{\mathrm{P}} + Y_t^{\mathrm{S}}. \tag{1}$$

The agricultural sector uses unskilled labor U_t and land X_t for production. Its productivity, $A_{X,t}$, is land embodied and the production function Cobb-Douglas with land share $\theta \in (0, 1)$:

$$Y_t^{\mathrm{P}} = (A_{X,t} X_t)^{\theta} U_t^{1-\theta}. \tag{2}$$

The non-agricultural sector's inputs are capital K_t, unskilled labor U_t^{S}, and human capital H_t. Its productivity A_t is Hicks neutral and its production function Cobb-Douglas with capital and unskilled labor shares $\alpha > 0$ and $\beta > 0$, respectively; $\alpha + \beta < 1$:

$$Y_t^{\mathrm{S}} = A_t K_t^{\alpha} H_t^{1-\alpha-\beta} (U_t^{\mathrm{S}})^{\beta}. \tag{3}$$

Firms maximize profits given the relative price of the agricultural output p_t, the unskilled wage w_t^{U}, the price of human capital w_t^{H}, the land rent ρ_t, and the interest rate r_t. To simplify, we assume full depreciation throughout so that the interest rate r_t equals the households' rate of return on assets/capital.

Letting k_t denote the *physical-to-human capital ratio*, K_t/H_t, the inverse demand functions can be written compactly as follows:

$$w_t^{\mathrm{U}} = p_t (1-\theta)(A_{X,t} X_t)^{\theta} U_t^{-\theta} = \beta A_t k_t^{\alpha} (H_t / U_t^{\mathrm{S}})^{1-\beta} \tag{4}$$

$$w_t^{\mathrm{H}} = (1-\alpha-\beta) A_t k_t^{\alpha} (H_t / U_t^{\mathrm{S}})^{-\beta} \tag{5}$$

$$\rho_t = p_t \theta A_{X,t}^{\theta} X_t^{\theta-1} U_t^{1-\theta} \tag{6}$$

and

$$r_t = \alpha A_t k_t^{\alpha-1} (H_t / U_t^{\mathrm{S}})^{-\beta}. \tag{7}$$

Dividing equation (7) with equation (5), we see that the marginal rate of technical substitution between physical and human capital equals the corresponding price ratio if

$$\frac{r_t}{w_t^{\mathrm{H}}} = \frac{\alpha}{1-\alpha-\beta} \frac{1}{k_t}. \tag{8}$$

In particular, the physical-to-human capital ratio is inversely related to the physical-to-human capital price ratio, one-for-one in percentage terms.

2.2. Land and labor markets and the skill-premium

The market for land always clears, $X_t = X_t^S$ where X_t^S is the supply. Denoting (total) labor supply by L_t, the unskilled labor market clears if $U_t + U_t^{\mathrm{S}} = L_t - S_t$ where S_t is the number of skilled workers at date t.

The demand for human capital is equal to the supply if $H_t = \sum_{i=1}^{S_t} h_t^i$ where h_t^i is the human capital level of skilled worker i.[5] Throughout, we interpret the human capital market as an explicit market for skilled workers with (individually) embodied human capital levels. If a firm wishes to increase its total human capital input, it must thus hire a specific skilled worker i, say, to obtain his human capital h_t^i, and in return, it must pay the wage

$$w_t^{\mathrm{S}}(h_t^i) = w_t^{\mathrm{H}} h_t^i. \tag{9}$$

The *average skilled wage* is thus $w_t^{\mathrm{S}} = \sum_{i=1}^{S_t} w_t^{\mathrm{S}}(h_t^i))/S_t$.

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The *skill-premium* is the average skilled wage divided by the (average) unskilled wage $w_t^{\mathrm{U}} > 0$:

$$\frac{w_t^{\mathrm{S}}}{w_t^{\mathrm{U}}} = \frac{\sum_{i=1}^{S_t} w_t^{\mathrm{S}}(h_t^i))/S_t}{w_t^{\mathrm{U}}} = \frac{w_t^{\mathrm{H}} \cdot (\sum_{i=1}^{S_t} h_t^i/S_t)}{w_t^{\mathrm{U}}}. \tag{10}$$

Using equations (4) and (5), we obtain the associated marginal rate of technical substitution:

$$\frac{w_t^{\mathrm{S}}}{w_t^{\mathrm{U}}} = \frac{1-\alpha-\beta}{\beta}\frac{U_t^{\mathrm{S}}}{S_t}. \tag{11}$$

Thus, at the production side, the skill-premium pins the non-agricultural sector's relative demand for skilled workers down. In particular, if the skill-premium declines, the non-agricultural sector substitutes skilled for unskilled workers. It is also seen that, all-else being equal, the smaller the human capital-to-unskilled labor share in the production function $(1-\alpha-\beta)/\beta$, the lower the skill-premium.

It can thus be seen that, unless the Plague changed the production function in the non-agricultural sector—for which there is no evidence—it must have affected the non-agricultural sector's ratio of unskilled-to-skilled workers, or else the skill-premium would have remained the same. While the Plague killing more skilled than unskilled workers could possibly explain an increase in U_t^{S}/S_t in the short run (hence a short-run decline in the skill-premium), it cannot account for the prolonged stability of the skill-premium following the decline. What follows develops a mechanism through which the Plague would have persistently altered the demand for and supply of skills and the ratio U_t^{S}/S_t.[6]

2.3. Households

Individuals' lifespans are divided into two periods, youth and adulthood. To simplify, we assume that young individuals do not work and that all adults work. Hence at date t, the number of adults is equal to the labor supply L_t.

At date t, young individual $i \in \{1, \ldots, L_{t+1}\}$ receives bequest $b_t^i \geq 0$, inherits land holdings $X_t^i \geq 0$, and decides how much to invest $e_t^i \geq 0$ into human capital with the human capital production function

$$h_{t+1}^i = B \cdot (e_t^i)^\gamma \text{ where } B > 0,\ 0 < \gamma < 1. \tag{12}$$

The difference from Galor et al. (2009) is that for an individual to enter the skilled workforce, she must invest (weakly) more than an exogenously given threshold level $\bar{e} > 0$ which is throughout referred to as the *skilled workforce entry requirement*. If $e_t^i \geq \bar{e}$, individual i will in adulthood become skilled and earn the wage in equation (9), that is, $w_{t+1}^H \cdot B \cdot (e_t^i)^\gamma$. If $e_t^i < \bar{e}$, she becomes an unskilled worker and earns w_{t+1}^{U}.

Young individuals do not consume, and the input into the human capital production function is the non-agricultural output. The income of adult i at date $t+1$ is thus:

$$I_{t+1}^i(e_t^i) = \begin{cases} w_{t+1}^{\mathrm{H}} B \cdot (e_t^i)^\gamma + r_{t+1}(b_t^i - e_t^i) + \rho_{t+1} X_t^i & \text{if } e_t^i \geq \bar{e} \\ w_{t+1}^{\mathrm{U}} + r_{t+1}(b_t^i - e_t^i) + \rho_{t+1} X_t^i & \text{if } e_t^i < \bar{e}. \end{cases} \tag{13}$$

Note that e_t can be interpreted both as a "resource cost" (as done so far) and as a "time cost." For the latter, we interpret e_t as years of training and assume that the master owns the human capital technology and must be paid one unit of the numeraire commodity per year of the apprenticeship.[7]

Adults give birth to an exogenously determined number of children and allocate their income between agricultural consumption c_{t+1}^i, non-agricultural consumption $c_{t+1}^{S,i}$, and bequests $\bar{b}_{t+1}^i$

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(bequests are, along with any land holdings, transferred to the offspring; see Section 2.3.2). Taking e_t^i and consequently $I_{t+1}^i(e_t^i)$ from equation (13) as given, an adult's objective is thus to maximize utility $U_{t+1}^i = U(c_{t+1}^i, c_{t+1}^{S,i}, \bar{b}_{t+1}^i)$ given the budget constraint

$$p_{t+1}c_{t+1}^i + c_{t+1}^{S,i} + \bar{b}_{t+1}^i \leq I_{t+1}^i(e_t^i).$$

Note that this is a two-stage decision problem: When young, the individual chooses how much to invest in human capital in order to maximize adult income (in particular, she chooses whether to become a skilled worker). Then in adulthood, she takes that income as given and chooses how much to consume and bequest to maximize utility. We now characterize these two stages.

2.3.1. The Young's human capital investment decisions and the skill premium

By equation (13), it is clear that a young individual will never choose e_t^i such that $0 < e_t^i < \bar{e}$. So if individual i joins the unskilled workforce, she will remain entirely unskilled, $e_t^i = 0$. For this reason, "remaining unskilled" and "becoming an unskilled worker" are used interchangeably from now on.

Lemma 1. *If $r_{t+1}/w_{t+1}^H > B\bar{e}^{\gamma-1}$, then every young individual will at date t remain unskilled. If $r_{t+1}/w_{t+1}^H \leq B\bar{e}^{\gamma-1}$, then any young individual who at date t decides to become skilled will invest*

$$e_t = \max\left\{\bar{e}, \left[\frac{1}{\gamma B}\frac{r_{t+1}}{w_{t+1}^H}\right]^{\frac{1}{\gamma-1}}\right\} \tag{14}$$

into human capital accumulation. In particular, skilled workers' human capital investments are independent of their land holdings and bequests, decreasing in the physical-to-human capital price ratio, and strictly greater than the skilled workforce entry requirement $\bar{e}$ if and only if $r_{t+1}/w_{t+1}^H < \gamma B\bar{e}^{\gamma-1}$.

Proof. "$r_{t+1}/w_{t+1}^H > B\bar{e}^{\gamma-1}$": By equation (13), a young individual obtains strictly greater income from $e_t^i = 0$ than from $e_t^i = \bar{e}$ if and only if $w_{t+1}^{\mathrm{U}} + r_{t+1}b_t^i + \rho_{t+1}X_t^i > w_{t+1}^{\mathrm{H}}B\bar{e}^{\gamma} + r_{t+1}(b_t^i - \bar{e}) + \rho_{t+1}X_t^i$. Because w_{t+1}^{U} is positive, it is therefore clear that if $0 > w_{t+1}^{\mathrm{H}}B\bar{e}^{\gamma} - r_{t+1}\bar{e}$, then the individual will strictly prefer $e_t^i = 0$ to $e_t^i = \bar{e}$. But since $r_{t+1}/w_{t+1}^H > B\bar{e}^{\gamma-1} \Leftrightarrow 0 > w_{t+1}^{\mathrm{H}}B\bar{e}^{\gamma} - r_{t+1}\bar{e}$, this must hold under the condition of the lemma and we conclude that $e_t^i = 0$ is strictly preferred to $e_t^i = \bar{e}$. That $e_t^i = 0$ is optimal now follows because $e_t^i \in (0, \bar{e})$ is never optimal (see the discussion just prior to the lemma), and $w_{t+1}^{\mathrm{H}}B \cdot (e_t^i)^{\gamma} - r_{t+1}e_t^i$ is strictly decreasing in e_t^i when $e_t^i \geq \bar{e}$ (the derivative $\gamma w_{t+1}^H B(e_t^i)^{\gamma-1} - r_{t+1} < w_{t+1}^H B(e_t^i)^{\gamma-1} - r_{t+1} \leq w_{t+1}^H B\bar{e}^{\gamma-1} - r_{t+1} < 0$, where the first inequality holds because $0 < \gamma < 1$, the second inequality holds because $e_t^i \geq \bar{e}$ and $\gamma < 1$, and the last inequality holds because $r_{t+1}/w_{t+1}^H > B\bar{e}^{\gamma-1} \Leftrightarrow 0 > w_{t+1}^{\mathrm{H}}B\bar{e}^{\gamma} - r_{t+1}\bar{e}$ which implies $w_{t+1}^{\mathrm{H}}B\bar{e}^{\gamma-1} - r_{t+1} < 0$ since $\bar{e} > 0$). "$r_{t+1}/w_{t+1}^H \leq B\bar{e}^{\gamma-1}$": The expression in equation (14) maximizes the income in equation (13) under the constraint that $e_t^i \geq \bar{e}$, that is, $e_t = \arg\max_{e_t^i \geq \bar{e}} w_{t+1}^{\mathrm{H}}B \cdot (e_t^i)^{\gamma} + r_{t+1}(b_t^i - e_t^i) + \rho_{t+1}X_t^i$. It is clear that e_t does not depend on X_t^i and b_t^i, and since $0 < \gamma < 1$, that e_t is decreasing in $r_{t+1}/w_{t+1}^{\mathrm{H}}$. Finally $e_t > \bar{e} \Leftrightarrow ((\gamma B)^{-1}r_{t+1}/w_{t+1}^{\mathrm{H}})^{1/(\gamma-1)} > \bar{e} \Leftrightarrow r_{t+1}/w_{t+1}^{\mathrm{H}} < \gamma B\bar{e}^{\gamma-1}$ where the second bi-implication holds since $\gamma < 1$. □

Imagine that there are no guilds to enforce the (skilled workforce) entry requirement. Then, it follows immediately from the previous proof that young individuals will invest the second term within the curly bracket in equation (14), as this is the unique optimal human capital investment without the constraint $e_t^i \geq \bar{e}$. Let us denote this investment level by e_t^*. By Lemma 1, $r_{t+1}/w_{t+1}^H \in (\gamma B\bar{e}^{\gamma-1}, B\bar{e}^{\gamma-1}) \Rightarrow 0 < e_t^* < \bar{e}$, that is, when the physical-to-human capital price

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ratio is between $\gamma B\bar{e}^{\gamma-1}$ and $B\bar{e}^{\gamma-1}$, then the (unconstrained) optimizer is strictly positive but below the entry requirement. This has two consequences: First, for physical-to-human capital price ratios in this range, skilled workers prefer to invest strictly less than the entry requirement $\bar{e}$. Second, for there to be a positive equilibrium supply of both skilled and unskilled workers when the entry requirement is enforced, the unskilled workers must also strictly be preferring e_t^* to their actual decision $e_t^i = 0$ (this is because young individuals must be indifferent between choosing $e_t^i = e_t$ as in Lemma 1 and choosing $e_t^i = 0$). When $r_{t+1}/w_{t+1}^H \in (\gamma B\bar{e}^{\gamma-1}, B\bar{e}^{\gamma-1})$, the entry requirement thus drives a "wedge" between everyone's actual and preferred actions: Unskilled workers would prefer to become skilled (except not as skilled as required by $\bar{e}$) and skilled workers prefer to be less skilled than $\bar{e}$ requires them to. As we now show, this "wedge" has important implications for how the skill-premium is determined (as we discuss in Section 2.4, it also captures several key features of the guild system of Late Medieval and Early Modern Britain).

Lemma 2. *Suppose that $r_{t+1}/w_{t+1}^H \leq B\bar{e}^{\gamma-1}$ and let e_t denote the human capital investment of Lemma 1. Also, denote the corresponding average skilled wage at date $t+1$ by w_{t+1}^S. Then, at date t, young individuals will be indifferent between becoming skilled and remaining unskilled if and only if*

$$\frac{w_{t+1}^S}{w_{t+1}^U} = \frac{1}{1 - \frac{r_{t+1}}{w_{t+1}^H} \cdot \frac{e_t^{1-\gamma}}{B}} = \begin{cases} \frac{1}{1-\frac{r_{t+1}}{w_{t+1}^H}\cdot\frac{\bar{e}^{1-\gamma}}{B}} & \text{if} \quad r_{t+1}/w_{t+1}^H > \gamma B\bar{e}^{\gamma-1}, \quad \text{and} \\ \frac{1}{1-\gamma} & \text{if} \quad r_{t+1}/w_{t+1}^H \leq \gamma B\bar{e}^{\gamma-1}. \end{cases} \tag{15}$$

Proof. Beginning with the first equality in equation (15), individuals' lifetime utility is strictly increasing in the adult income, hence the lemma's statement is equivalent to $I_{t+1}^i(e_t) = I_{t+1}^i(0)$. By equation (13), this holds if and only if $w_{t+1}^{\mathrm{S}} = w_{t+1}^{\mathrm{U}} + r_{t+1}e_t \Leftrightarrow w_{t+1}^{\mathrm{S}}/w_{t+1}^{\mathrm{U}} = 1/(1 - r_{t+1}e_t/w_{t+1}^{\mathrm{S}})$. By equations (9) and (12), $w_{t+1}^{\mathrm{S}}(h_{t+1}^i) = w_{t+1}^{\mathrm{H}} h_{t+1}^i = w_{t+1}^{\mathrm{H}} B \cdot (e_t^i)^\gamma$. By Lemma 1, we can remove the index i from e_t^i and it is also clear that $w_{t+1}^{\mathrm{S}}(h_{t+1}^i) = \sum_{i=1}^{S_{t+1}} w_{t+1}^{\mathrm{S}}(h_{t+1}^i))/S_{t+1} \equiv w_{t+1}^{\mathrm{S}}$. Hence, $w_{t+1}^{\mathrm{S}} = w_{t+1}^{\mathrm{H}} B \cdot (e_t)^\gamma$. Substituting for w_{t+1}^{S} in $1/(1 - r_{t+1}e_t/w_{t+1}^{\mathrm{S}})$, we obtain the first equality in equation (15). The second equality in equation (15) follows by substituting e_t from Lemma 1 into the second expression. □

As illustrated in Figure 3, Lemma 2 says that the human capital supply curve is horizontal at the level $SP_a = w_{t+1}^{\mathrm{S}}/w_{t+1}^{\mathrm{U}}$ in equation (15) for any given physical-to-human capital price ratio. If the skill-premium is above this level, every young individual will at date t prefer to become skilled and if the skill-premium is below this level, every young individual will at date t prefer to remain unskilled.

As can also be seen, the skill-premium determination is very different depending on whether the entry requirement binds or not. If it does not, that is, if $r_{t+1}/w_{t+1}^{\mathrm{H}} < \gamma B\bar{e}^{\gamma-1}$ (Lemma 1), then the skill-premium is stable at the level $1/(1-\gamma)$. If $r_{t+1}/w_{t+1}^H \in (\gamma B\bar{e}^{\gamma-1}, B\bar{e}^{\gamma-1})$ and therefore $e_t = \bar{e}$ (Lemma 1 again), then a decline in the physical-to-human capital price ratio will cause a skill-premium decline.

To explain, let us begin with the first case ($r_{t+1}/w_{t+1}^{\mathrm{H}} < \gamma B\bar{e}^{\gamma-1}$). In this situation, if $r_{t+1}/w_{t+1}^{\mathrm{H}}$ declines, young individuals respond by accumulating more human capital (Lemma 1), and, specifically, they accumulate precisely so much more that diminishing individual returns to human capital accumulation [Galor and Moav (2004)] exhausts the income increase: The stable level $1/(1-\gamma)$ guarantees that the unskilled wage, w_{t+1}^{U}, equals the rents, $(1-\gamma)w_{t+1}^{\mathrm{S}}$, which a young individual will obtain from utilizing the human capital production function to become skilled ($w_{t+1}^{\mathrm{U}} = (1-\gamma)w_{t+1}^{\mathrm{S}} \Leftrightarrow w_{t+1}^{\mathrm{S}}/w_{t+1}^{\mathrm{U}} = 1/(1-\gamma)$).[8] This reflects that in our model, there are no frictions and young individuals free to choose whether to remain unskilled or become skilled. Note that in this "stability regime," a declining physical-to-human capital price ratio will of course

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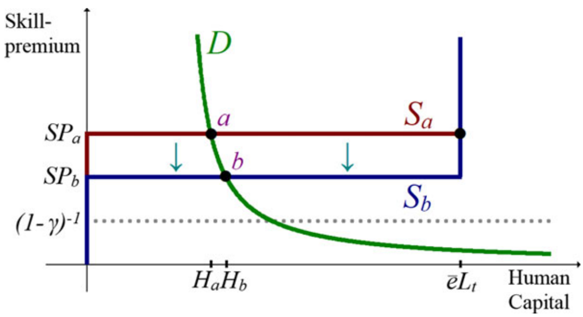

Figure 3. When $r_{t+1}/w^H_{t+1} \in (\gamma B\bar{e}^{\gamma-1}, B\bar{e}^{\gamma-1})$, a decline in physical-to-human capital price ratio shifts the supply curve from S_a to S_b and reduces the skill-premium from SP_a to SP_b. For fixed skill-premium SP_a, human capital supply increases from H_a to $\bar{e}L_t$. Demand curve is assumed to be fixed for simplicity.

still affect human capital accumulation (intuitively, it causes a "move to the right" along the horizontal dotted curve in Figure 3). In particular, this regime is compatible with persistent human capital growth and a fixed ratio of skilled to unskilled workers [which is the standard description of modern human capital-driven growth, for example, see Galor and Moav (2004)].

However, if $r_{t+1}/w^H_{t+1} \in (\gamma B\bar{e}^{\gamma-1}, B\bar{e}^{\gamma-1})$, then the previous diminishing returns mechanism is deactivated because skilled workers' human capital investments will be "stuck" at $\bar{e}$. And because the "wedge" described above forces skilled individuals to invest more than the (unconstrainted) optimal human capital investment e^*_t, becoming skilled would lead to a loss in comparison with remaining unskilled if the skilled workers were only paid the competitive rents $(1-\gamma)w^S_{t+1}$. For a positive supply of human capital, the skill-premium must therefore be greater than $1/(1-\gamma)$. Moreover, if r_{t+1}/w^H_{t+1} declines—as long as it does not reach the level where the entry requirement no longer binds—all that happens is that the return to investing $\bar{e}$ increases. If before the decline, there is a positive supply of both skilled and unskilled workers as in the initial equilibrium in Figure 3 (marked *a*), then after the decline, the skill-premium will consequently have to decline to a level such as SP_b in the figure, or everyone would now prefer to become skilled. In terms of the adjustment, if we imagine that the skill-premium initially remains the same when r_{t+1}/w^H_{t+1} declines, then the human capital supply increases from H_a to $\bar{e}L_t$ in Figure 3. The resulting excess supply then forces the skill-premium down.[9] Crucially, note that in this "wedge regime," any growth in human capital must be due to an increase in the number of skilled workers (as opposed to an increase in the individual human capital levels).[10]

A few clarifying comments are in order. First, Lemma 2 and Figure 3 capture the very long run (*e.g.* in our simulation, the period length is 20 years). In the short run, a decline in r_{t+1}/w^H_{t+1} will more realistically cause a "large" shift in the human capital supply through the previous mechanism, but one that is limited both by the adjustment time and various frictions and rigidities. As long as the demand curve is not "too flat," however, the skill-premium will decline for the same reasons. Second, with a caveat, the previous explanation can be viewed as a general equilibrium version of the explanation in van Zanden (2009b), who focuses on the supply side in partial equilibrium (corresponding to a vertical demand curve). The caveat is that we must interpret van Zanden's explanation as referring to the *number* of skilled workers (as opposed to an increase in the individual human capital supply). Adopting this interpretation, van Zanden's simulation exercise clearly shows that the interest rate reduction (the decline in the physical-to-human capital price ratio) would have increased skilled workers' incomes significantly, all else being equal. This is also the critical element in our quantitative analysis; in particular, it is what explains the *magnitude* of the skill-premium decline (Section 4.2). Third, note that if the rent-accruing (fixed) factor's share in human capital accumulation $1-\gamma$ is "high"—if, intuitively, there is little to be

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usefully learned to complement the fixed factor—then the skill-premium will be "low." This might explain why the earliest measures of the skill-premium we have tend to be low. Finally, it is clear that any subsidy to, or mandatory accumulation of (etc.), human capital investments will require us to modify the previous description, the skill-premium determination in particular. In the era, we are concerned with here, those aspects of human capital accumulation played negligible roles however.

2.3.2. The adults' consumption and bequest decisions

Denote the adult income in equation (13) by $I^i_{t+1} := I^i_{t+1}(e^i_t)$, $i = 1, \ldots, L_{t+1}$, where e^i_t is individual i's human capital investment when young.[11] For our quantitative analysis, it is crucial that consumption-bequest decisions reflect the inelastic nature of agricultural consumption during early phases of development. To capture this without introducing difficult-to-handle income effects, we assume that demand for the agricultural good (food) is a non-increasing function $\bar{c}(\cdot)$ of the relative price of the agricultural output conditional on affordability,

$$c^{P,i}_{t+1} = \min\{\bar{c}(p_{t+1}), I^i_{t+1}/p_{t+1}\}. \tag{16}$$

The remaining income $I^i_{t+1} - p_{t+1}c^{P,i}_{t+1} \geq 0$ is divided between non-agricultural consumption and bequest:

$$c^{S,i}_{t+1} = (1 - \sigma(p_{t+1})) \cdot (I^i_{t+1} - p_{t+1}c^{P,i}_{t+1}) \tag{17}$$

$$\bar{b}^i_{t+1} = \sigma(p_{t+1})(I^i_{t+1} - p_{t+1}c^{P,i}_{t+1}) \tag{18}$$

where $\sigma : \mathbb{R}_{++} \to (0, 1)$ is a decreasing function.[12]

This description ensures that if the relative price of the agricultural output declines and an individual's disposable income after agricultural consumption therefore increases, then the individual will spend a relatively lower fraction of income on non-agricultural consumption, and bequeath a relatively larger fraction of income. It also follows that

$$\sum_{i=1}^{L_{t+1}} \bar{b}^i_{t+1} = \sum_{i=1}^{L_{t+1}} \sigma(p_{t+1})(I^i_{t+1} - p_{t+1}\bar{c}(p_{t+1})) = \sigma(p_{t+1})(Y_{t+1} - p_{t+1}\bar{c}(p_{t+1})L_{t+1}) \tag{19}$$

where $\sum_{i=1}^{L_{t+1}} I^i_{t+1} = Y_{t+1}$ (the real GDP). As a fraction of aggregate income, aggregate bequest is thus equal to $(\sum_{i=1}^{L_{t+1}} \bar{b}^i_{t+1})/Y_{t+1} = \sigma(p_{t+1})(1 - p_{t+1}\bar{c}(p_{t+1})L_{t+1}/Y_{t+1})$. In particular, it is increasing in the real per-capita GDP.

Naturally, $\sum_{i=1}^{L_{t+1}} \bar{b}^i_{t+1} = \sum_{i=1}^{L_{t+2}} b^i_{t+1}$, that is, aggregate bequests given equals aggregate bequests received. As a consequence of Lemma 1, precisely how adults allocate bequests and land holdings to their offspring does not impact the economy's aggregate development (because skilled workers will acquire the same human capital levels and hence earn the same wage in adulthood). Since we focus on the aggregate development in our empirical analysis, it is therefore not necessary to specify how many children individual i gets and how $(\bar{b}^i_{t+1})_{i=1}^{L_{t+1}}$ maps into $(b^i_{t+1})_{i=1}^{L_{t+2}}$, or how the land holdings $(X^i_{t+1})_{i=1}^{L_{t+1}}$ change hands over time.

2.4. Discussion

2.4.1. Guilds and the skilled workforce entry requirement

In Medieval Britain, skilled workers were employed primarily in building and construction (masons, architects), shipbuilding (shipwrights), and various mercantile trades. To enter these professions, an individual would enter into a guild-regulated apprenticeship contract of, typically, 7 years duration [Epstein (1998), Wallis (2008), and van Zanden (2009b)]. Anyone who did not complete his apprenticeship was barred from the profession, and this was vigorously enforced by guilds who by various means prevented "quits" from working as "semi-professionals." The skilled workforce entry requirement $\bar{e}$ is a succinct way of capturing the resulting gap in the market for human capital.

With the Late Medieval sectoral transformation (ca. 1350–1450), guilds' power and influence arguably weakened, as much of the manufacturing industry located itself outside of towns [*e.g.* Cope (1939)]. Also, quits appear to have been commonplace [Wallis (2008)].[13] On the other hand, there is evidence that guild regulations strengthened during the 1350–1450 period in towns [Clark (2005b), p. 1316], and with the Statute of Artificers' adoption in 1562, guild regulations were written into legislation and violations became criminal offenses [Wallis (2019)].[14] There is also circumstantial evidence that even prior to the Statute of Artificers, semi-professionals were not employed outside of guilds' traditional domains to such an extent that it affected skilled wages.[15] Finally, it should of course be kept in mind that hiring a semi-professional implied the loss of recourse to guild courts. Therefore, enforcement need not have been that problematic even outside of guilds' traditional domains.

For our analysis to hold up to historical scrutiny, what is required precisely, is that until approximately 1400 and again for a 100–150 years period after 1560, guilds were sufficiently effective in enforcing standards for any "undercutting" semi-professionals' impact on the market salaries of skilled workers (journeymen and masters) to have been negligible. This is hardly a controversial reading of the historical evidence; in particular, the adoption of the Statute of Artificers coincides almost perfectly with the start of the second of these periods.

Finally, note that in the empirical section, we keep the skilled workforce entry requirement fixed. Hence, consistent with Clark (2005b), p. 1316 as well as for example van Zanden's works cited previously, variations in the skill-premium are not explained by variations in guilds' power and influence (*e.g.* enforcement capabilities).

2.4.2. The structure of the workforce

In the model, unskilled workers are homogenous and employable in either sector. Similarly, skilled workers are homogenous; for example, we make no distinction between a mason and a shipwright, or a shipwright and a skilled manufacturing worker.

Economic history is surprisingly supportive of such a simple framework. Wage data show that agricultural workers and unskilled construction workers earned almost exactly the same daily wage throughout the entire period considered in this paper [see Clark (2005a), Figure 3]. From this, it must arguably follow that unskilled workers in other non-agricultural industries (*e.g.* manufacturing) earned a similar wage too, since if unskilled wages were equalized between urban and rural areas, it seems very unlikely that different unskilled workers would earn different salaries within these areas. A similar pattern is also clear from the data on skilled workers' relative wages [van Zanden (2009b)], and more broadly, the consensus amongst economic historians is that relative wage changes in Late Medieval and Early Modern Britain were caused by underlying trends in the demand for and supply of skills [van Zanden (2009a, b) and Clark (2005b)].

Finally, note that it is a mistake to think of Early Modern manufacturing as predominantly a low-skill occupation, and it is also a mistake to sharply separate skilled manufacturing workers from, say, skilled construction workers in terms of human capital levels. Textile production, for example, required very substantial human capital until the 18th Century when innovations such as the Spinning Jenny made traditional manufacturing skills redundant.

2.4.3. Exogeneity of the labor supply

In pre-Modern Britain, as today, birth and death rates depended on a combination of exogenous events (wars, epidemics, the weather, . . .) and economic variables such as real incomes and wages

[Malthus (1798)]. When we treat the labor supply as exogenous, this corresponds to assuming that decisions impacting birth and death rates (how many children to have, how much to feed them, how much to spend on health care, etc.) can be written as a function of the consumption and human capital decision problem laid out in Section 2.3 (as opposed to being an integral part, requiring us to write down a model where all decisions are made simultaneously).[16] For example, the number of young individuals at date $t+1$, L_{t+2}, could be a function of the lifetime income of the adults at date $t+1$ (the parents), $I^i_{t+1}(e^i_t)$, death rates a function of consumption levels, and so on:

$$L^S_{t+2} = \sigma(L^S_{t+1}, I^i_{t+1}(e^i_t), c^{P,i}_{t+1}, c^{S,i}_{t+1}, \tau_t)$$

where (τ_t) is a stochastic process capturing exogenous factors and σ accounts for both birth and death rates.

Empirically, the interpretation is then that we measure the left-hand side (the labor supply) and remain agnostic about the right-hand side. To repeat, this is of course only valid in and so far as such a relationship holds and is exhaustive. The crucial point is that this does not mean that there is not such a relationship—the limitation is that, by design, this approach offers no explanation (corresponding to estimating σ and, in particular, identifying the appropriate right-hand side variables in the previous equation). Since a complete understanding of the issues treated in this paper does require such an explanation, we must, as highlighted in the Introduction, rely on other works such as Allen (2008), Voigtländer and Voth (2013), and Clark (2013).[17]

3. Dynamic equilibrium and the equilibrium skill-premium

In this section, we reduce the model to a single state variable, characterize the dynamic equilibrium path, and establish a key result on the relationship between the physical-to-human capital ratio and the skill-premium.

Because, in equilibrium, the physical-to-human capital ratio is one-to-one with the physical-to-human capital price ratio [refer to equation (8)], we can also write the skilled workers' human capital investments in Lemma 1 as a function of the physical-to-human capital ratio:

$$e_t = e(k_{t+1}) \equiv \max\left\{\bar{e}, \left[\frac{\gamma B(1-\alpha-\beta)k_{t+1}}{\alpha}\right]^{\frac{1}{1-\gamma}}\right\}. \tag{20}$$

In particular, skilled workers will invest strictly more than the skilled workforce entry requirement ($e_t > \bar{e}$), if and only if the physical-to-human capital ratio is above a specific *threshold* level $\bar{k}$ as follows:

$$k_{t+1} > \bar{k} \equiv \frac{\alpha \bar{e}^{1-\gamma}}{\gamma B(1-\alpha-\beta)}. \tag{21}$$

Combining with Lemma 2, we therefore have:

Proposition 1. *Suppose that in equilibrium the physical-to-human capital ratio is* $k_t \geq \frac{\alpha \bar{e}^{1-\gamma}}{B(1-\alpha-\beta)}$ *at date t. Then, the equilibrium skill-premium will at date t be*

$$\begin{array}{lll} \left[1 - \frac{\alpha \bar{e}^{1-\gamma}}{B(1-\alpha-\beta)} \frac{1}{k_t}\right]^{-1} & \text{if} \quad k_t < \bar{k}, & \text{and} \\ \frac{1}{1-\gamma} & \text{if} \quad k_t \geq \bar{k}. & \end{array} \tag{22}$$

In particular, the skill-premium is a continuous and strictly decreasing function of the physical-to-human capital ratio below the threshold $\bar{k}$*, and it is constant if the physical-to-human capital ratio is above this threshold.*

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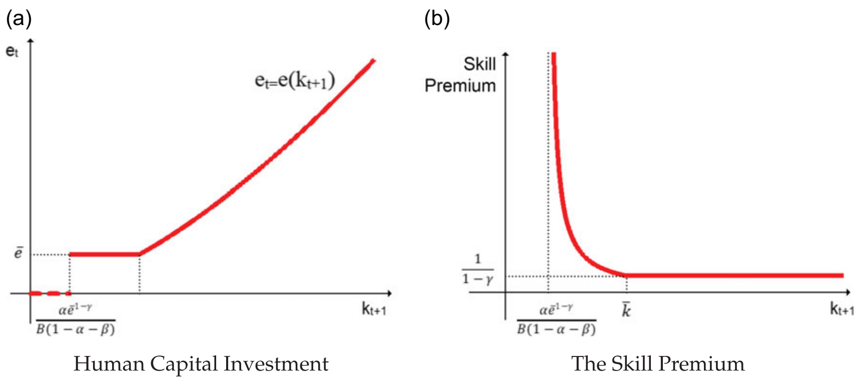

Figure 4. Human capital investment and skill premium.

Proof. By equation (8), $k_t \geq \frac{\alpha\bar{e}^{1-\gamma}}{B(1-\alpha-\beta)}$ if and only if the condition of Lemmas 1 and 2 holds. Given k_t, use equation (8) and Lemma 1 to determine e_t. Then, insert this and equation (8) into equation (15) and evaluate. The remaining claims follow from the fact that $1 - \frac{\alpha\bar{e}^{1-\gamma}}{B(1-\alpha-\beta)}\frac{1}{k}$ is strictly increasing in k with limit $1-\gamma$ as $k \to \bar{k}$. □

Note that in the case not covered by the proposition, $k_t < \frac{\alpha\bar{e}^{1-\gamma}}{B(1-\alpha-\beta)} = \gamma\bar{k}$, the economy is purely agrarian because there will be no skilled workers (compare with Lemma 1). In particular, the skill-premium is then not well-defined.

Let us briefly discuss the consequences of this proposition (the underlying economic mechanisms were outlined in Section 2.3.1). Most importantly, it shows that the model can account (qualitatively) for the decline and stability pattern seen in Figure 1a, assuming the Plague raised the physical-to-human capital ratio. Thus as seen in Figure 4, the skill-premium declines when $k_t < \bar{k}$ and human capital investments therefore are stuck at the skilled workforce entry requirement.

Another consequence of Proposition 1 that will play a key role later is that the skill-premium's elasticity with respect to the physical-to-human capital ratio is negative and strictly increasing in k_t below the threshold (*i.e.* whenever $k_t < \bar{k}$).[18] So a one percent increase in the physical-to-human capital ratio implies a greater skill-premium decline, the farther below the threshold the physical-to-human capital ratio is to begin with. This is simply due to decreasing returns to human capital production: as k_t gets closer to the threshold $\bar{k}$, the physical-to-human capital price ratio gets closer to the threshold in Lemma 1, and consequently, the optimal human capital investment e_t^* increases more slowly because the marginal gain is reduced.

The next lemma plays mostly a technical role (it allows us to reduce the model to a single state variable).

Lemma 3. *Suppose that in equilibrium the physical-to-human capital ratio is $k_t \geq \frac{\alpha\bar{e}^{1-\gamma}}{B(1-\alpha-\beta)}$ at date t. Then, at date t, the non-agricultural sector's equilibrium human capital-to-unskilled labor ratio H_t/U_t^S is equal to*

$$\varsigma(k_t) \equiv \begin{cases} \frac{B(1-\alpha-\beta)\bar{e}^{1-\gamma}}{\beta} - \frac{\alpha}{\beta}k_t^{-1}\bar{e} & \text{if} \quad k_t < \bar{k} \\ (\gamma^{-1}-1)\frac{\alpha}{\beta}\left[\frac{\alpha}{B\gamma(1-\alpha-\beta)}\right]^{\frac{1}{\gamma-1}} k_t^{\frac{\gamma}{1-\gamma}} & \text{if} \quad k_t \geq \bar{k}. \end{cases} \tag{23}$$

In particular, the equilibrium human capital-to-unskilled labor ratio is a continuous and strictly increasing function of the physical-to-human capital ratio.

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Proof. Under the condition of the lemma, skilled and unskilled workers earn the same lifetime income and so $w_t^{\mathrm{U}} = w_t^{\mathrm{S}} - r_t e(k_t) = (1-\alpha-\beta)A_t k_t^{\alpha}(H_t/U_t^{\mathrm{S}})^{-\beta}B(e(k_t))^{\gamma} - \alpha A_t k_t^{\alpha-1}(H_t/U_t^{\mathrm{S}})^{-\beta}e(k_t)$ where, for the last equality, we use the inverse demand functions and $e(\cdot)$ is the human capital investment of Lemma 1. Combine with equation (4) to obtain $(H_t/U_t^{\mathrm{S}}) = \frac{B(1-\alpha-\beta)}{\beta}(e(k_t))^{\gamma} - \frac{\alpha}{\beta}k_t^{-1}e(k_t)$. Finally, insert $e(\cdot)$ and evaluate to get equation (23). Since $\varsigma(\cdot)$ is a composition of continuous functions, it is continuous, and it is clearly an increasing function. □

Using this lemma, we can express the wage of unskilled workers as a function of the physical-to-human capital ratio by substituting $\varsigma(k_t)$ into equation (4). We can then calculate the agricultural sector's labor demand function [again using equation (4)] and obtain the agricultural sector's (unskilled) labor demand given k_t and p_t:

$$U_t(k_t, p_t) = p_t^{\theta^{-1}}(1-\theta)^{\theta^{-1}}A_{X,t}X_t(\beta A_t k_t^{\alpha}(\varsigma(k_t))^{1-\beta})^{-\theta^{-1}}. \tag{24}$$

This formula enables us to pin down the relative price of the agricultural output, the agricultural sector's labor share, and the skilled labor share, all in equilibrium as a function of k_t:

Lemma 4. *Suppose that the equilibrium physical-to-human capital ratio equals $k_t \geq \frac{\alpha \bar{e}^{1-\gamma}}{B(1-\alpha-\beta)}$ at date t. Let $\bar{c}(\cdot)$ be the function defined prior to equation (16) and $\varsigma(\cdot)$ be the non-agricultural sector's human capital-to-unskilled labor ratio of Lemma 3. Then, in equilibrium at date t, the relative price of the agricultural output equals the (unique) solution to the equation*

$$p_t(\bar{c}(p_t))^{\frac{1}{1-\theta^{-1}}} = \left(\frac{A_{X,t}X_t}{L_t}\right)^{\frac{1}{1-\theta^{-1}}}(1-\theta)^{-1}\beta A_t k_t^{\alpha}(\varsigma(k_t))^{1-\beta}. \tag{25}$$

Denote this solution by $p_t(k_t)$. Then, the agricultural sector's equilibrium labor share U_t/L_t is

$$\mathcal{P}_t(k_t) = (1-\theta)^{\theta^{-1}}(p_t(k_t))^{\theta^{-1}}\frac{A_{X,t}X_t}{L_t}(\beta A_t k_t^{\alpha}(\varsigma(k_t))^{1-\beta})^{-\theta^{-1}}. \tag{26}$$

Finally, the equilibrium share of skilled workers in the workforce S_t/L_t is

$$\mathcal{S}_t(k_t) = \left[\frac{1}{1+(\varsigma(k_t))^{-1}B(e(k_t))^{\gamma}}\right]\cdot[1-\mathcal{P}_t(k_t)]. \tag{27}$$

Proof. The market for the agricultural good clears if and only if $\bar{c}(p_t)L_t = (A_{X,t}X_t)^{\theta}U_t^{1-\theta}$. Insert equation (24) into this equation and rearrange to get equation (25). The solution is unique because $\bar{c}$ is non-increasing. Inserting the unique solution $p_t(k_t)$ into equation (24) and dividing by L_t yields the equilibrium labor share in equation (26). Finally, since $\varsigma(k_t) = \frac{S_t}{U_t^{\mathrm{S}}}B(e(k_t))^{\gamma}$ and labor markets clear, $L_t - U_t = U_t^{\mathrm{S}} + S_t = [1 + ((\varsigma(k_t))^{-1}B(e(k_t))^{\gamma}]S_t \Rightarrow$ (27). □

Theorem 1. *Let $e(k_t)$, $\varsigma(k_t)$, and $\mathcal{S}_t(k_t)$ be the functions defined in equations (20), (23) and (27), respectively. Also, let $p_t(k_t)$ denote relative equilibrium price of Proposition 4. Fix $k_0 > \frac{\alpha \bar{e}^{1-\gamma}}{B(1-\alpha-\beta)}$ as well as productivity, land, and labor supply paths $(A_t)_{t=0}^{\infty}$, $(A_{X,t}X_t)_{t=0}^{\infty}$ and $(L_t)_{t=0}^{\infty}$, and consider the unique sequence $(k_t)_{t=1}^{\infty}$ determined recursively by*

$$[k_{t+1}B + e(k_{t+1})^{1-\gamma}]\mathcal{S}_{t+1}(k_{t+1})L_{t+1}e(k_{t+1})^{\gamma} = \sigma(p_t(k_t))A_t k_t^{\alpha}\mathcal{S}_t(k_t)L_t B e(k_t)^{\gamma}(\varsigma(k_t))^{-\beta}. \tag{28}$$

Then if $k_t \geq \frac{\alpha \bar{e}^{1-\gamma}}{B(1-\alpha-\beta)}$ for all t, $(k_t)_{t=0}^{\infty}$ is the economy's unique dynamic equilibrium path.

Proof. By equations (1), (3) and clearing of the agricultural market, $Y_t = p_t\bar{c}(p_t)L_t + A_t k_t^{\alpha}S_t B e(k_t)^{\gamma}\ (\varsigma(k_t))^{-\beta}$. Substitute this into equation (19) to express total bequests as a function of k_t and p_t: $\sum_{i=1}^{L_t}\bar{b}_t^i = \sigma(p_t)A_t k_t^{\alpha}S_t B\, e(k_t)^{\gamma}(\varsigma(k_t))^{-\beta}$. Since young individuals either invest

Table 1. Model parameters

Name	Variable	Estimate
Land share in agricultural sector*	θ	0.4000
Physical capital share in non-agricultural sector*	α	0.3300
Unskilled labor share in non-agricultural sector	β	0.2900
Human capital production function scale variable	B	3.7500
Human capital production function elasticity*	γ	0.3548
Skilled workforce entry requirement	$\bar{e}$	0.0650
Slope of $\sigma(p)$	η_1	−0.057
Intercept of $\sigma(p)$	η_2	0.1023
Scale parameter of $\bar{c}(p)$	ϕ_1	1.6185
Elasticity of $\bar{c}(p)$	ϕ_2	−2.06

*θ is from Voigtländer and Voth (2013); γ is pinned down by the stable average value of the skill-premium, $1/(1-\gamma)=1.55$; α is the conventional value.

their bequests in human capital or save it, savings equals physical capital investments, and there is full depreciation: $K_{t+1}=\sum_{i=1}^{L_t}\bar{b}_t^i - e(k_{t+1})S_{t+1} = \sigma(p_t)A_t k_t^{\alpha} S_t Be(k_t)^{\gamma}(\varsigma(k_t))^{-\beta} - e(k_{t+1})S_{t+1}$. Rewrite and use Lemma 4 to get equation (28). The dynamic path is uniquely determined because the left-hand side of equation (28) is strictly increasing in k_{t+1}. □

For a given initial state, k_0, and given exogenous sequences and parameters, Theorem 1 shows how to compute the state variable's path in equilibrium, $(k_t)_{t=1}^{\infty}$. From there, all other relevant variables' dynamic paths follow straight forwardly, for example, the human capital investment can be calculated from equation (20) and the skill-premium from equation (22).

4. Quantitative analysis

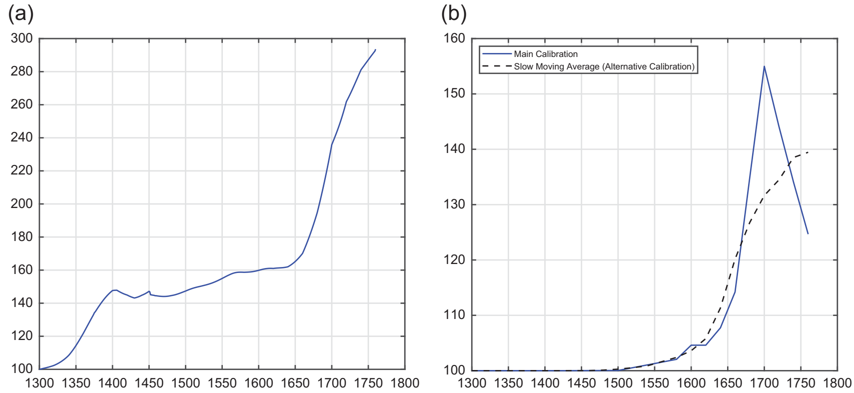
4.1. Calibration

This subsection provides a brief summary of the calibration and discusses the productivity estimates. Additional details can be found in Appendix B, where we also compare predicted prices with the data (external validity testing). All data sources as well as the mapping between the model and the data are detailed in Appendix A. The primary data source is Broadberry et al. (2015). To check the model's robustness/sensitivity, we also do a comparison analysis based on Clark's data [Clark (2004, 2010, 2013)] as discussed more at the end of this subsection.

The time period we consider is 1300–1760, and the calibration/simulation period length is set to 20 years. The description of consumer demand in Section 2.3 requires us to impose functional forms on σ and $\bar{c}$. We take σ to be linear, $\sigma(p)=\eta_1 p+\eta_2$, and $\bar{c}$ to exhibit constant price elasticity, $\bar{c}(p)=\phi_1 p^{\phi_2}$.[19] Our parameter estimates are summarized in Table 1 with "deep" parameters marked by asterisks.

As explained in more detail in the appendix, agricultural productivities are determined endogenously in a fixed point loop via the agricultural output data of Broadberry et al. (2015). The variables we must calibrate are thus the non-agricultural productivities (A_t) and the non-deep parameters.

The calibration takes two steps: First, we calibrate the parameters by focusing on the 1300–1420 subperiod under the identifying assumption that there is no non-agricultural productivity growth during this time interval. Second, fixing the parameters from the first step, the non-agricultural productivities after 1420 are calibrated by targeting income per capita (only). Because the model is extremely over-identified, we are able to perform extensive external validity testing by holding



Hicks Neutral Agricultural Productivity Estimates ($A^{\theta}_{X,t}$)

Non-agricultural Productivity Calibration (A_t)

Figure 5. Agricultural and non-agricultural sectors' productivities.

predicted price series for wages, the land rent, and the relative price up against the actual data (Appendix B).

The rest of the paper makes frequent references to the productivities depicted in Figure 5, so it is useful to briefly overview these.

Beginning with the non-agricultural productivities, we find approximately zero growth until the turn of the 15th century when there was a small uptick which persists until the early 17th century. Non-agricultural productivity then accelerates sharply, followed by a partial reversal. Between 1454 and 1660, which is the most important subperiod for what follows, the cumulative increase is 5% or 0.02% p.a. Between 1620 and 1760, the cumulative increase is 19% (0.1% p.a). The sharp reversal seen in Figure 5b is implied by our calibration which matches the income per-capita data. The dashed curve in Figure 5b shows a slow-moving average that nets this reversal out. Because our main focus is the period prior to 1660 and the dashed curve is seen to coincide with the solid curve (the preferred calibration) in 1660, running our simulation with the slow-moving average productivities makes no difference for anything that follows.[20] This shows that our results are not sensitive to this (possibly slightly worrying) aspect of the calibrated non-agricultural productivities.

The non-agricultural productivities we find prior to 1600 are what one would expect from, for example, Mokyr (2005). The acceleration of productivity growth in the 17th century precisely coincides with the account of Clark (2005b), specifically with what Clark refers to as "the first appearance of modern growth in the years 1630–1690" [Clark (2005b), p. 1313].

As for agriculture's productivity, this improves sharply after the onset of plague: By just over 30% in Hicks neutral terms within 50 years of 1348 (0.5% p.a.).[21] Productivity growth then levels off—over the 200 years where population grows between 1454 and 1660, the cumulative increase is 17%, or 0.08% p.a. From the mid-17th century, our simulation detects another sharp increase (approximately 40% between 1650 and 1760, or 0.3% p.a.).

In order to rely on Broadberry et al. (2015) to the greatest extent possible, we estimate agricultural productivities without using external price data. We then check the estimates by comparing them with the structural estimates one instead obtains by using price data [Clark (2001), Bar and Leukhina (2010) and Leukhina and Turnovsky (2016)]. We do this in Appendix B. The bottom line is that our approach is fully consistent with an approach based on (external) price data if the price-based estimation uses the unskilled (farm) wage of Humphries and Weisdorf (2019) (what land rent data is used makes little difference). If instead one uses the farm wage of Clark (2013), a somewhat different picture emerges. In the appendix, we address the resulting sensitivity

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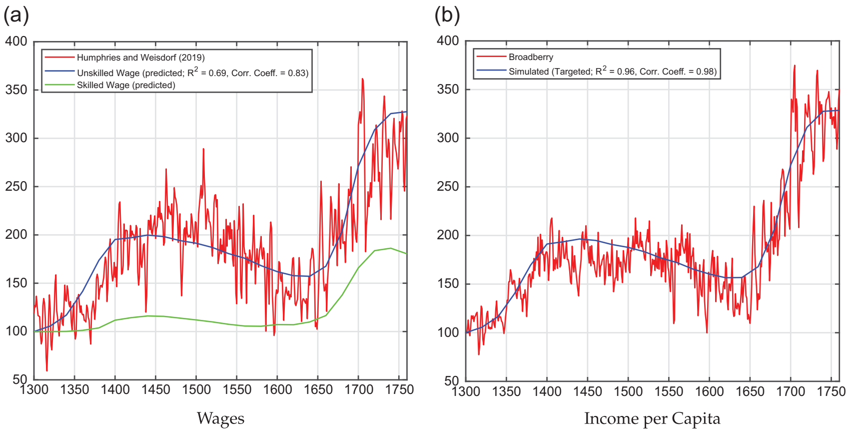

Figure 6. Wages and income per capita (indexed, 1300 = 100).
Notes: The unskilled real wage is from Table A48 of BoE (2017) which is the annual male earnings from Humphries and Weisdorf (2019) deflated by the Allen CPI index.

concerns by redoing the entire empirical analysis using Clark's data in place of our main data source [Broadberry et al. (2015)]. The conclusion is that our main findings remain valid.

4.2. 1300–1454: plague, skill-premium decline, and Late Medieval sectoral transformation

The Plague arrived in Britain in 1348, and within 4 years known as "the Great Mortality" (1348–1351) it reduced the British population by 46 %. This was followed by additional outbreaks and population declines until Britain's population size bottomed some 60% below its pre-plague level in 1454 (Figure 2a). Because agriculture employed roughly three-quarters of the workforce in 1348, the direct impact of the initial population declines is in our simulation completely dominated by the usual Malthusian mechanisms [*e.g.* see Galor (2005)]—the declining workforce improves agricultural economies of scale which drives up wages and income per capita.[22] Figure 6 depicts this together with the historical data.

Note that the simulation's period length is 20 years and whenever we speak of a year that is not a multiple of 20 below, we are referring to the linear interpolation of our 20-yearly data output (e.g. the income in 1450 is the average of the simulated income between 1440 and 1460).

Within 50 years of the Great Mortality, unskilled workers earned 55% more than they did before the Plague. In 1450, a century after the Great Mortality, they earned 58% more.[23] The effect on other sources of income is discussed below but in light of agriculture's dominant position at this point, it is not surprising that the income per-capita series display broad similarity with the unskilled wage series in the Late Middle Ages (for income per capita, the 50 and 100 years-on numbers are, respectively, 53% and 56%). As we discuss more in Appendix B, the predicted unskilled wage series' excellent fit with Humphries and Weisdorf (2019) provides strong (external) support for our analysis.

Another effect of improved agricultural economies of scale in the model is to reduce the relative price of food.[24] In our simulation, it falls by 16% between 1340 and 1380. Since we target this in the calibration, it (obviously) aligns with the historical data. We discuss relative price changes in more detail in Section 4.3.

On the demand side, the price and income changes lead in our simulation to an increase in per-capita agricultural consumption of 32% within 50 years of the onset of plague, falling slightly to 28% one hundred years on as the relative price of food begins to increase again at the turn of the century. Because of the inelastic nature of food demand (p. 15), the increases in per-capita

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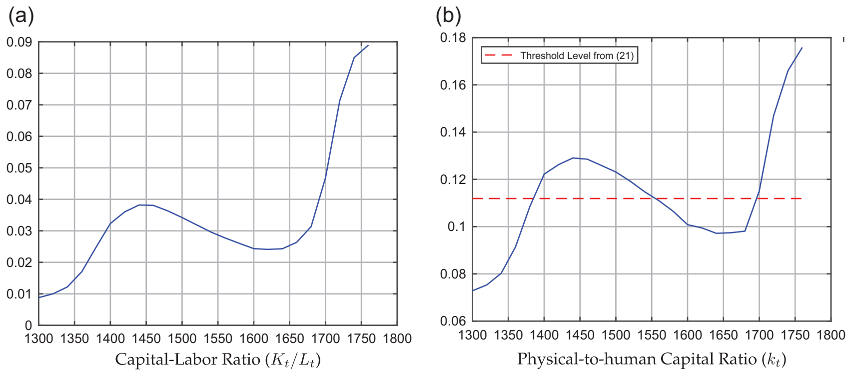

Figure 7. Capital-labor and physical-to-human capital ratios.

demand for non-agricultural goods and per-capita bequests are even more pronounced: 50 years on the two stand at 69% and 111%, respectively, and after a further 50 years, non-agricultural per-capita demand has grown by 90% and per-capita bequests by 132% in comparison with 1347. It should be remarked that the latter changes reflect small absolute changes from very low initial levels (in level terms, the increase in per-capita agricultural consumption far outweighs them).

In equilibrium, an increase in per-capita bequests causes substitution toward physical capital in production and a decline in the rate of interest through the usual channels. Since per-capita bequests equals per-capita savings, this maps directly into the capital-labor ratio depicted in Figure 7a.[25] Crucially, the increasing (physical) capital-to-labor ratio is not matched by the increase in the human capital-to-labor ratio in our simulation; that is to say, the physical-to-human capital ratio increases (Figure 7b).

The growing scarcity of human capital in comparison with physical capital is reflected in the relative price: Within 50 years of the onset of plague, the physical-to-human capital price ratio declines by 45%, with the following 50 years adding a more modest 8 percentage points decline to this.[26]

In Section 2.3.1, we discussed in detail how a decline in the physical-to-human capital price ratio affects human capital accumulation and how this determines the skill-premium in our model. Importantly, for roughly 50 years after the Great Mortality, the optimal human capital investment is below the skilled workforce entry requirement (Figure 8a). We are thus in the situation illustrated in Figure 3 where a decline in the physical-to-human capital price ratio causes the skill-premium to decline. Until around the year 1400, our simulation thus indicates that skilled workers would have preferred to finish apprenticeships early and work as semi-professionals but that guilds were effective in preventing this from impacting relative wages (refer to Section 2.4.1).[27] In more detail, it can be seen from the figures above that the skill-premium decline takes the form of a (substantial) increase in the unskilled wage and a more limited increase in the skilled wage. Significantly, consistent with, for example, Clark (2013), the skill-premium decline was according to our analysis not driven by increased individual human capital investments but by an increase in the supply of skilled workers.[28,29]

Another thing that is clear from the explanation in Section 2.3.1, as well as Figure 3, is that the quantity (the stock of human capital) is in equilibrium determined primarily from the human capital demand side. Indeed, any increase in human capital demand can be met given prices as long as the skill-premium takes the value in Lemma 2. This limits the direct causes of equilibrium changes in the human capital stock to factors affecting the demand side. There are two such demand-side factors: The first is the increase in the relative per-capita demand for non-agricultural goods (all-else equal, this raises the relative demand for human capital because only the non-agricultural

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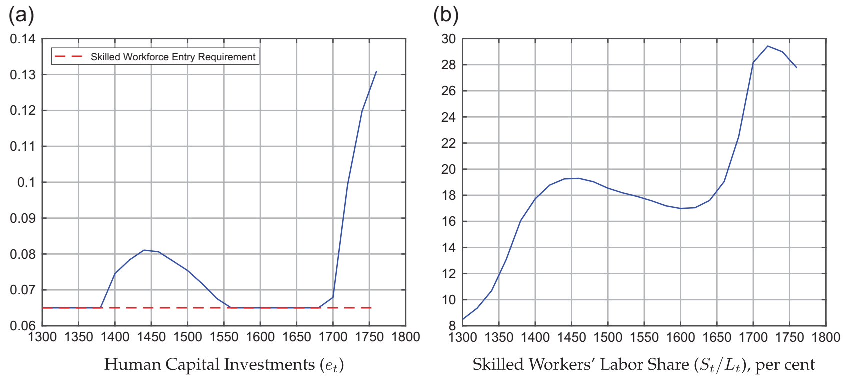

Figure 8. Individuals' human capital and the skilled labor share.

sector uses human capital for production). The second is substitution toward skilled workers in production caused by the skill-premium decline [in equilibrium, the skill-premium equals the marginal rate of technical substitution between skilled and unskilled workers; see equation (11)]. So in terms of Figure 3, the demand curve is decreasing and shifts "to the right.. Importantly, while increased demand is significant in itself (as returned to in a moment), it is quantitatively insignificant in comparison with the dramatic impact the Plague has on per-capita savings and therefore the physical capital-to-labor ratio in our simulation (as explained previously). This, then, is why in the first place, the physical capital stock increases much faster than the human capital stock. Note that this "closes" our explanation of the skill-premium decline because the previous explanation began with the increasing physical-to-human capital ratio.

Figure 9 presents the simulated skill-premium together with the historical skill-premium calculated from the data in Allen (2001) as well as Clark (2010). Note that we here depict the yearly data (for the 30-year moving average of Allen's data, see Figure 1a in the Introduction). It should be mentioned that this data is representative of the labor market more generally [for example, see Figure 2 in Clark (2005b) as well as van Zanden (2009a, b)].[30] Combining this figure with everything else said so far, we conclude that the change in the physical-to-human capital ratio required to produce an empirically realistic skill-premium decline, precisely corresponds to changes in the physical-to-human capital price ratio, unskilled/skilled labor demands and the physical and human capital stocks which imply empirically realistic time series for income per capita and wages (and much more besides, as we shall see). In a line, this shows that our general equilibrium explanation of the skill-premium's decline holds up quantitatively when all factors are taken into account.

In contrast to the skill-premium's decline, its stabilization is much more easily explained in the model. This is because as long as the human capital production function remains the same,[31] a stable skill-premium is automatic once guilds cease to influence human capital investment decisions (Section 2.3.1). Of course, this just shifts the explanatory problem elsewhere—we must then answer why the Early Modern population boom did not set off enough of a reversal in the physical-to-human capital (price) ratio to cause reversals. That is the main topic of the next subsection.

Let us finally consider the Plague's impact on the sectoral composition in Britain.

In our model, a skill-premium decline as substantial as the one seen in Figure 10 shifts, equally substantially, the economy's cost structure in favor of the non-agricultural sector. The reason is simply that the non-agricultural sector employs skilled workers and the agricultural sector does not. So even for fixed input ratios and fixed demand, the decline in the skill-premium benefits the non-agricultural sector relative to the agricultural sector. This direct (relative) cost effect is

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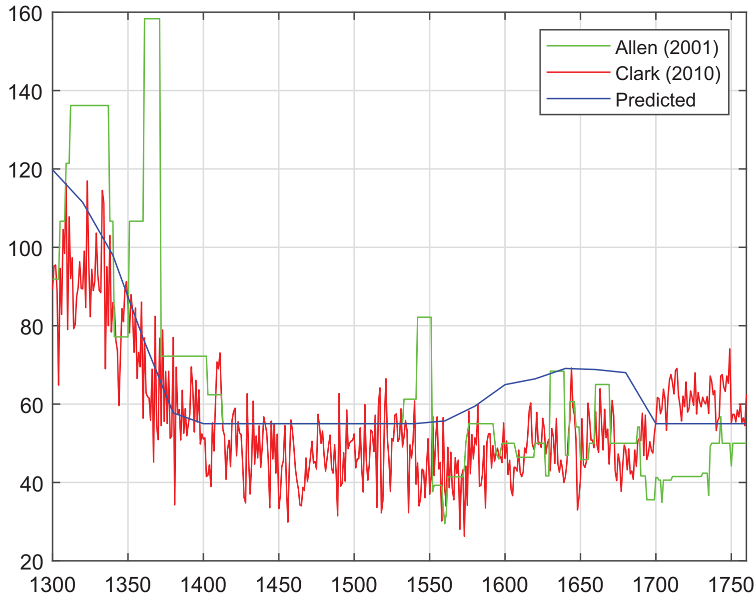

Figure 9. Simulated vs. actual skill premium.
Notes: Actual skill-premium data is the yearly skill-premium in construction from Allen (2001).

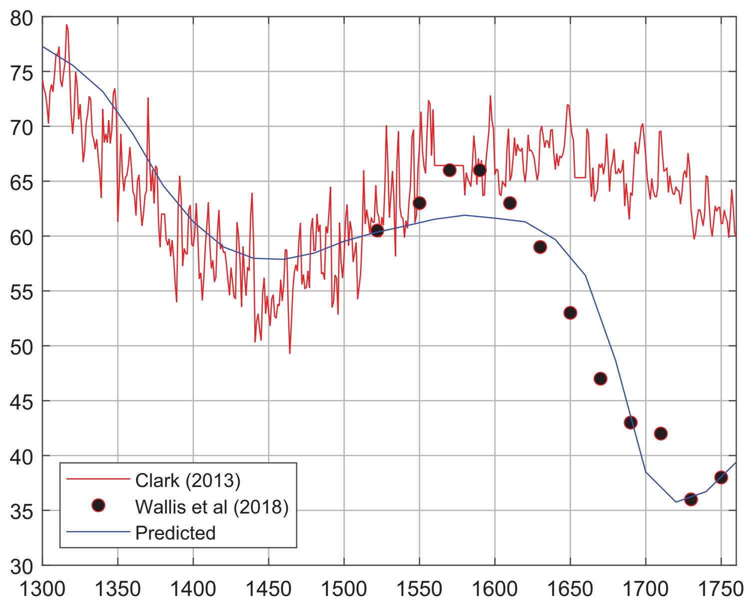

Figure 10. Simulated vs. actual agricultural labor shares.

enhanced by the non-agricultural sector's ability to substitute toward skilled workers (which, as we have seen, it did). This change in the economy's cost structure is why the plague-induced increase in non-agricultural demand is not in equilibrium met by a more strongly off-setting decline in the relative price of food (this point is subtle but critical and we elaborate on it in a footnote).[32]

The result is a very substantial shift toward non-agricultural production and a corresponding shift of workers away from agriculture. Note that an important reason why this shift is so significant is that the human capital investments are stuck at the skilled workforce entry requirement prior to 1400—increased human capital shifts workers out of the unskilled workforce (as opposed to individual human capital levels increasing).

Figure 10 depicts our simulated agricultural labor shares along with two sets of estimates by economic historians. Here the focus is the years prior to 1454, where the (only) comparison is with Clark (2013).[33] In our simulation, the agricultural labor share declines during this Late Medieval transformation from around 70% just before the Plague to 57% at its pre-17th century trough around 1450. As seen, this prediction (we do not match this data in our calibration) is highly consistent with Clark (2013). In particular, the minimum is found at precisely the same time as the minimum in Clark (2013).

4.3. 1454–1660: population pressure, narrow escape, and the Early Modern sectoral transformation

The year 1454 marks a turning point because this is when Britain's population began to recover after the Great Mortality and subsequent epidemic flare-ups. At first, recovery was slow due to repeated, if less virulent outbreaks [*e.g.* in 1471 and 1479; see Gottfried (1983)]. Around 1524, however, a growth spurt set in, and by 1625, the British population had surpassed its pre-plague high.[34]

Considering the whole 1454–1660 period, Britain's population increased by 172%, which corresponds to an average per-annum growth rate of 0.5%. According to our calibration, non-agricultural productivity increased by approximately 5% over that same period (0.02% per annum), and the comparable (Hicks neutral) agricultural productivity increase was 17% (0.08% p.a.).

It is evident from these numbers that population growth far outstripped any underlying productivity gains between 1454 and 1660, especially during the Early Modern population boom (1524–1654). The consequences are evident in all of the figures: wages, income per capita, and the share of skilled workers in the population all decline, and the skill-premium and the agricultural sector's labor share rise. As we emphasized in the Introduction, these reversals are no mystery in light of the Malthusian mechanisms that caused the opposite adjustments in the first place. What is puzzling in light of the imbalance between the pace of population and productivity growth is that the reversals are so muted. What explains this?

The first reason is "blunting" of decreasing returns: Because of the Late Medieval sectoral transformation, the non-agricultural sector's labor share sits in our simulation at 43% at the start of the population recovery. The non-agricultural sector exhibits constant returns; hence the larger the share of the non-agricultural sector in the economy, the smaller the impact of population pressure on economy-wide diseconomies of scale. In particular, the Malthusian channel's direct impact on wages and per-capita incomes is correspondingly reduced. However, this blunting of the Malthusian channel is conditional on a limited reversal of the sectoral transformation which, as we have seen, depends on the physical-to-human capital ratio k_t not falling too much.

Now, as long as k_t is above the threshold $\bar{k}$ in equation (21), the skill-premium remains stable and there is therefore no reversal of the relative cost changes discussed in the previous section. The blunting therefore remains potent because the sectoral composition changes very little. In the figures, this description is seen to apply until around 1560 when k_t drops below the threshold and the skill-premium slowly begins to increase. But looking at Figure 10, this year does not mark any meaningful trend-shift in the agricultural labor share.

The reason is that our model contains a second "blunting mechanism": It limits the skill-premium's elasticity with respect to k_t when k_t is close to the threshold. We discussed the economics of this mechanism after Proposition 1. Its quantitative strength can be gauged by

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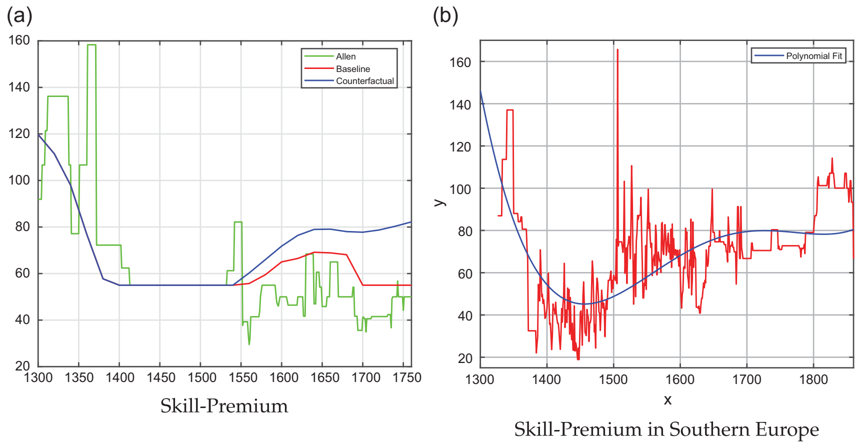

Figure 11. Skill-premium counterfactual and Southern European skillpremium.

comparing Figures 7b and 9: If we look at the end point in 1660, it is seen that at the end of the Early Modern population boom, the physical-to-human capital ratio has reversed 65% of its plague-induced increase and returned to a level last seen in 1375 (a quarter of a century after the Great Mortality). But this only reverses 30% of the skill-premium decline induced by the Plague (specifically, the skill-premium increases by 13 percentage points from its stable level). As can be seen by going further back in time than 1375 and recalling the one-to-one relationship between k_t and the skill-premium in Proposition 1, a fairly modest additional physical-to-human capital ratio decline of, say, 20 percentage points, would have led to an additional 26 percentage points increase in the skill-premium, that is, to a near-complete reversal of the post-plague decline.[35]

The third and final blunting mechanism in our model ties in with the first one through the demand channel: As long as per-capita incomes remain (relatively) high, so does the relative demand for non-agricultural goods.

In combination, the previous mechanisms suggest that the productivity gains in the 1454–1660 period, although very small in comparison with the pace of population growth, could be of outsized significance for the outcomes. To shed light on this, we did a numerical counterfactual holding the productivities ($A_{X,t}$ and A_t) fixed after 1454 while keeping everything else unchanged. The main time series are reported in the next two figures.

As seen, the loss of a cumulative 17% increase in agricultural productivity and 5% increase in non-agricultural productivity over this roughly 200 years period reverses by 1660 the key post-Plague changes back to levels corresponding to the end of the Great Mortality. This is notable, considering how small the counterfactual productivity reductions are. To draw a modern comparison, it takes just 5 years at a 2% p.a. technological growth rate to obtain a cumulative gain of the same (sector-weighted) magnitude. This is especially interesting in light of the comparison we wish to bring out by placing the Southern European historical data in Figure 11b side-by-side with the counterfactual skill-premium.

As seen, Southern Europe experienced precisely the type of skill-premium reversal our counterfactual suggests Britain would have experienced, given a very modest reduction in productivity growth. As shown by van Zanden (2009b), divergence in skill-premia correlates with subsequent divergence in income per capita [van Zanden (2009b), Section 8; see also Freeman and Oostendorp (2001) and Clark (2008)]. This comparison—keeping Britain and Southern Europe's subsequent economic performances in mind—paints a tantalizing picture of historical developments where very small factors may account for very large differences. What ultimately

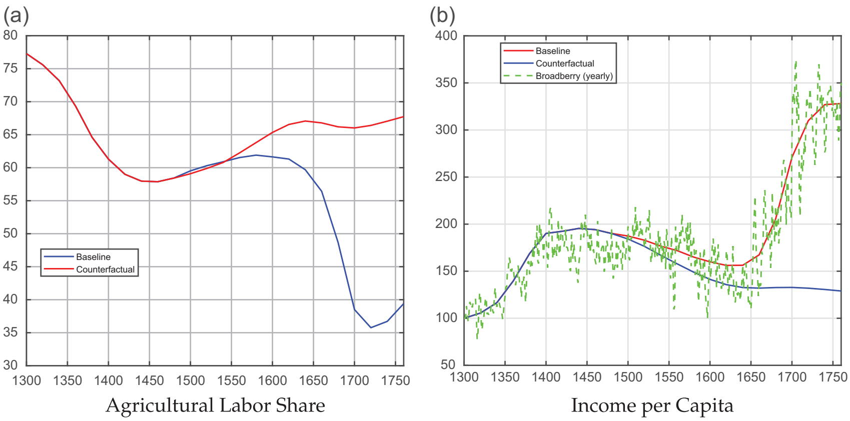

Figure 12. Labor share and income per capita counterfactuals.

drove the productivity differences is beyond the scope of our model because productivities are exogenous/calibrated [on this, see *e.g.* Mokyr (2005), van Zanden (2009a), Clark (2013), and van Zanden and de Pleijt (2016)]. But it is interesting that, whatever the drivers and their relative significance, their impact on productivities need not have been large and they could still account for the subsequent divergence in economic performance (see Figures 12a and 12b).

Finally, let us compare the simulated labor shares in Figure 10 with the historical data in the 1454–1660 period. As seen, the estimates of Clark (2013) and Wallis et al. (2018) are roughly the same until the end of the 16th Century. Before 1550, our simulated labor shares fit this data well. For the next 50 years, our simulation displays less reversal than the data. Specifically, at the point of maximum deviation around 1575, the difference is 4%-points. After approximately 1620 our predictions are very close to Wallis et al. (2018).

We hesitate to draw any firm conclusions from this in light of the disagreement between the historical sources. Certainly, the point of maximum deviation mentioned a moment ago can be accounted for by pointing to our relatively simplistic description of consumer demand. Indeed, because of this, our model is incapable of reproducing some of the relatively substantial price swings seen in the data in the relevant period—see Figure 13a which shows the historical data together with our simulated relative price. In our model, the agricultural demand curve is downward sloping but the expenditure is increasing in the price; hence, a less pronounced increase in the relative price implies a less pronounced relative increase in agricultural demand—hence, a less pronounced shift of workers toward agriculture.[36] After approximately 1620, this price effect is insignificant in comparison with the changes in relative demand caused by the rise in income per capita which is consistent with our model performing better after that point. But, as mentioned, it is difficult to assess in light of the historical sources.

4.4. 1660: take-off and miscellaneous observations

This paper's primary focus is the years before 1660. We therefore limit this section to a few miscellaneous observations.

First, our simulation predicts that the non-agricultural sector's labor share overtakes the agricultural sector's at exactly the same time as the recent literature on the sectoral transformation in Britain [see Keibek (2017) and the references therein]. As Keibek (2017) discusses, this has important consequences for our understanding of the Industrial Revolution—obviously so, because it dates the end of the sectoral transformation to before the start of the Industrial Revolution.

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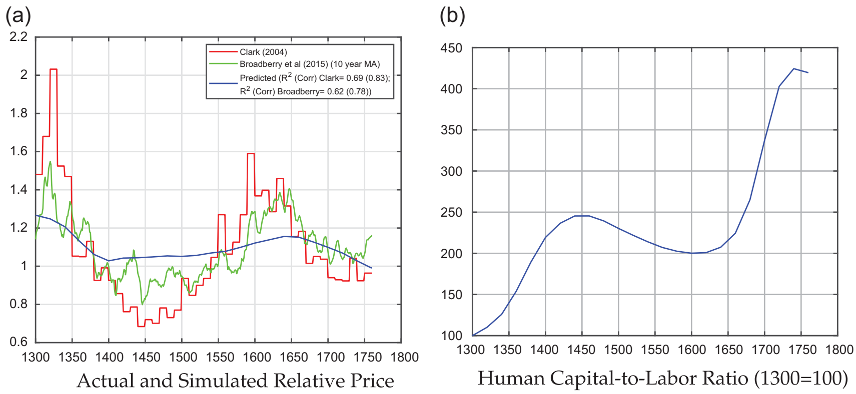

Figure 13. Relative prices and human capital-to-labor ratio.
Notes: The actual price index of the non-agricultural sector is calculated as a real output weighted average of industry and services from the data in Broadberry et al. (2015) [BoE (2017), Tables A6 and A7]. Both actual and simulated relative prices are indexed to 1 in 1341.

See also Clark (2005b), and Voigtländer and Voth (2006) for a discussion of the Industrial Revolution that is methodologically quite similar to ours.

Second, our simulation suggests that there was (meaningful) growth in human capital investments from the late 17th century (Figure 8a) and an increasing human capital-labor ratio in the run-up to the Industrial Revolution (Figure 13b), consistent with Clark (2005b). If correct, 1620 was a kind of turning point: Before 1620, increasing human capital was driven primarily by an increase in the number of skilled workers but after 1620, it was driven by increasing individual human capital investments.

Now, it is arguably right to not make too much of this.[37] The reason we highlight it is because of the relationship with Clark (2005b). In his conclusion, Clark writes: "*We also see in the premium paid for skills that while increased investment in human capital may lie at the heart of the Industrial Revolution, the causes of this increased investment, evident in England as early as 1600, are mysterious*" [Clark (2005b), p. 1320]. What our model does then is solve this mystery. It does so by not insisting that human capital accumulation must be explained as a (partial equilibrium) phenomenon solely from the supply side.

5. Concluding remarks

This work attempts to take the historical sources seriously and at the same time impose detailed model structure on the data. We are under no illusions about the difficulties involved. If one compares a less model-based approach to economic history with our approach, the former's clear advantage is its ability to give a coherent account that takes any number of disparate factors of influence into consideration. By comparison, our analysis is crude. The comparative advantage of a structural approach is its ability to discriminate between multiple factors of influence—several of which may individually explain the data, and all of which generally influence each other causally. Our main conclusions are precisely informed by this strength and, we believe, provide valuable new insights:

The skill-premium—although clearly influenced by endogenous factors in our model such as income per capita and the sectoral composition—was prior to the year 1660 or so one of the key fundamental determinants of developments (all conditioned, of course, on demographic change). In particular, its steep decline in the wake of the Plague—which we explain in a novel way that receives strong empirical support (Section 4.2)—was crucial for the Late Medieval sectoral transformation, which our analysis explains also.

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Later—when in the Early Modern period the skill-premium was no longer a key driver of events—the prior sectoral transformation played a key role in shielding the British economy from the pressures of the Early Modern population boom. But again, the skill-premium played a role because its determination "buffered" any negative impact on sector shares of a decrease in the physical-to-human capital ratio (Section 4.3). This mattered greatly since the British economy was very much balancing on a knife edge as we demonstrated through a counterfactual experiment in Section 4.3.

Yet the outcome of the interplay between productivity growth, prior sectoral transformation, and the skill-premium was ultimately to avoid a significant reversal of the post-Plague adjustments. The British economy was thus able to sustain its momentum into the 17th century and on to the Industrial Revolution, unlike, for example, Southern Europe whose skill-premium evolution looks very much like our British counterfactual.

These findings contribute to our understanding of pre-industrial historical developments and, by extension, to our understanding of the possible routes to economic take-off and development today. It follows, for example, that a "Kaldor-inspired" development strategy—that is, a strategy that seeks to simultaneously transform sectors and set off capital accumulation, innovation, and technological adoption—is misplaced if its design is thought to emulate British pre-industrial development. Instead, our analysis suggests, development strategies must take a much more complex interplay into account involving both sectoral transformation and the skill-premium.

Methodologically, this paper is as far as we are aware the first to attempt to fit all main macroeconomic time series with historical data in a manner similar to other new economic history works such as for example Bar and Leukhina (2010), Voigtländer and Voth (2013, 2006), or Leukhina and Turnovsky (2016). What enables us to do this, is the recent effort of economic historians to develop long-run economic data [*e.g.* Clark (2010), Broadberry et al. (2015)]. A similar approach, using similar data, could potentially address many other interesting historical questions.

Notes

1 These decline and stability patterns are well-established facts among economic historians, and there is broad agreement that they are indicative of much more general patterns in Western Europe [see *e.g.* Figure 2, p. 128 in van Zanden (2009b) and the discussion pp. 129–130 of why the skill-premium in construction is indicative of a pattern across different occupations—as one would also expect absent *very* substantial rigidities in the labor market]. As for the interest rate, Clark (1988) puts the English interest rate prior to the Plague at 10% and the post-plague interest rate at 5%. Again see van Zanden (2009b) for a general discussion of the decline and stability pattern, as well as for data on other European countries.

2 The "transition" is normally dated to the turn of the 20th Century [Galor and Moav (2004)]. Note also that any significant increase in individual human capital levels does not appear to have taken place until, at the earliest, in the 17th Century [Clark (2005b); see also Section 4.4 below].

3 To be clear, economic historians have long ago moved away from such a view of economic development [e.g. consider the Little Divergence literature, or more broadly, works such as van Zanden (2009a)].

4 Note that because we treat the population size as exogenous, we remain agnostic about why Malthusian checks did not more fully assert themselves and depress income per capita or wages to subsistence levels after the initial onset of bubonic plague (our approach in this respect, is explained in more detail in Section 2.4.3). Since both war and disease patterns played critical roles throughout the period we study, endogenizing the labor supply will as a minimum require an extension of our model that takes such factors into account, similar to, for example, Voigtländer and Voth (2013) but with physical and human capital accumulation and endogenous occupational choice decisions. While very challenging, such an analysis (fitting everything together), would obviously be interesting in light of the issues raised in this paper.

5 Note that for this notation to be accurate, skilled workers must appear first in the indexation of workers (a convention we adopt throughout).

6 We are thankful to an anonymous referee for suggesting this paragraph's framing of what follows.

7 Since these payments would necessarily come out of bequests, this can also be thought of as young individuals' parents directly paying masters. Note that since we assume that young individuals do not work, we are prevented from capturing the opportunity cost of training. However, if apprentices are paid the unskilled wage less the aforementioned payment to the master, then the wage payment when young will cancel out in equilibrium because skilled and unskilled workers must earn the same lifetime income.

8 If we normalize the fixed factor to unity, the human capital production function can be written $h^i_{t+1} = B \cdot (e^i_t)^{\gamma} 1^{1-\gamma}$. $1-\gamma$ is then the rent-earning fixed factor's share, and under perfect competition, the fixed factor will consequently earn income/accrue rents $(1-\gamma)w^S_{t+1}$ where w^S_{t+1} is the total revenue to employing the human capital production technology (=becoming skilled).

9 Note that here we are excepting the irrelevant case where there is a simultaneous discontinuous shift in demand. As explained in the figure's caption, the demand response is generally not significant for the skill-premium response (as returned to in Section 4.2, demand is on the other hand very significant for the resulting change in the equilibrium level of human capital).

10 As we mentioned in the Introduction, this is consistent with the historical evidence, which does not support any substantial increase in individual human capital levels at this point in time [*e.g.* Clark (2005b)]. Note also that in our model, just as in van Zanden (2009b) (returned to in the next paragraph), the reduction in the physical-to-human capital price ratio is the key driving force.

11 Note that by the results in the previous subsection, $e^i_t \in \{0, e_t\}$, where e_t is given by equation (14).

12 In our numerical simulation, we approximate σ by a linear function, $\sigma(p) = \eta_1 p + \eta_2$, where $\eta_1 < 0$ and $\eta_2 > 0$ (this is justified by the fact that there is very little variation in $\sigma(p_{t+1})$ in our simulation, so this just corresponds to a local approximation).

13 Strictly speaking, Wallis considers only the period from the 16th century onwards but there is no obvious reason to think quits were less common before then.

14 We are grateful to Patrick Wallace who alerted us to the significance of the Statute of Artificers for our account.

15 According to Clark (2005b) "the skill-premium was, if anything, higher in rural areas and small towns than in the largest cities. And the decline in the premium over time was just as profound in the countryside" (p. 1316). This suggests that outside of the largest cities, employers did hire masters (or, more likely, their journeymen) even if guilds did not actively prevent them from hiring semi-professionals. Indeed (leaving aside a possible allowance for travel and related costs), it seems most unlikely that they would have paid a higher premium to hire a semi-professional than what a master/journeyman could command in a city.

16 Note that the same is true for our treatment of the supply of land.

17 Of course, in the immediate aftermath of the Plague, this is less important as the explanation is fairly obvious. But as we mentioned in the Introduction, this is no longer the case with, for example, the magnitudes of the Early Modern population boom.

18 Specifically, from equation (22), the elasticity is $-(\gamma \bar{k})/(k_t - \gamma \bar{k})$.

19 Since there is very little absolute variation in $\sigma(p)$ in our simulation, the functional form for σ can be viewed as a local approximation. Constant price elasticity of demand is motivated mostly by goodness-of-fit and familiarity.

20 We include this "alternative" simulation in the MATLAB implementation accompanying this paper.

21 During the Great Mortality, people who survived the Plague moved to more fertile land, or to tenure with land owners who offered better terms at more fertile land, see, for example, Gottfried (1983). Furthermore, incentives improved markedly because peasants were increasingly able to farm their own land instead of working in serfdom for a lord (again, see Gottfried). Combined, this must have markedly increased labor-embodied productivity, which is what our calibration finds. As for the (small) increases before 1650 mentioned next, these are easily accounted for by Smithian progress due to agricultural specialization [Mokyr (2005)] and increased work hours [De Vries (1994); Broadberry et al. (2015), Section 6.3; Humphries and Weisdorf (2019)] countering the diminishing returns caused by rising populations.

22 The relationship between wages and income per capita, that is, the effect on other sources of income such as capital income, is returned to below. Note also that the Great Mortality reduced land use which, all-else equal, reduces the relevant agricultural economies of scale index, $(A_{X,t}X_t)/L_t$ (compare with Lemma 4). According to our calibration, however, this effect of the decline in land use was fully offset by increased agricultural productivity.

23 The previous numbers slightly overestimate the improvement in purchasing power because the agricultural good is the numeraire. On a CPI basis, the unskilled wage increases corresponding to 50 and 100 years after the Great Mortality are 49% and 53%, respectively.

24 The relationship between the agricultural scale index and the relative price is clear in light of equation (25). The relative price is discussed more in Section 4.4.

25 In reality, the capital-labor ratio would for obvious reasons have exploded higher during the Great Mortality, and the interest rate must have plummeted. Our model and simulation captures not such short-run adjustments but long-run general equilibrium adjustments.

26 The relationship between the physical-to-human capital ratio and its corresponding price ratio is governed by equation (8) in our model. In particular, the two are one-to-one in percentage terms (so a 1% increase in one corresponds to a 1% decrease in the other). Note that we focus mostly on the quantity ratio from now on because it is the model's state variable.

27 Note that while individual human capital investments do begin to increase around 1400, as seen, the increase is moderate—reflecting the moderate decline in the physical-to-human capital price ratio between 1400 and 1450 mentioned a moment ago—and as soon as the population starts to recover 50 years later, it is reversed. For these reasons, it has no meaningful impact in our simulation and can therefore be ignored in the following.

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28 See also the discussion after Proposition 1.
29 Note that the previous statement is empirical: A supply-side "take-off" in individual human capital levels is consistent with the model (in particular, it happens later as seen in the figure).
30 For other skilled professions, data availability is poor. However, conditional on the underlying cause being changes in the demand for and supply of skills (as argued by both Clark and van Zanden), it is reasonable to extrapolate this pattern to other skilled professions as well.
31 See the discussion beginning on page 15, especially regarding the skill-premium prior to 1300.
32 It is in order to match this detail that a two-sector model with a flexible relative price is essential. As seen from the relative price data of Broadberry et al. (2015) as well as our simulation (Figure 13a below), the decline in the relative price of food was in fact *not* especially pronounced after the Plague. This is a mystery (given the enormous improvement in the agricultural scale) *until* we remember that the relative cost improvement of the non-agricultural sector—by way of the skill-premium as just described—exerted a very strong downward pressure on the non-agricultural sector's price as well. It is thus in the intricacies of the price data that we find, perhaps, some of the strongest support for our skill-premium-based account of the first phase of the sectoral transformation.
33 It should be pointed out that there is a lack of agreement among economic historians about the drop in agricultural labor share in the wake of the plague (the data is relatively scarce prior to the 1522 muster rolls).
34 Which was likely also the historical high at that point [see Jordan (1996)].
35 In fact, turning to the sectoral transformation, the reversal in the physical-to-human capital ratio and the skill-premium by 1660 is seen to only lead to a quite modest 2 percentage points decline in the share of skilled workers (Figure 8b), and a 4 percentage points increase in the agricultural sector's labor share (Figure 10). Note that the fact that agricultural productivity grows faster than the non-agricultural productivity might also play a role, but the relative importance is difficult to identify quantitatively due to general equilibrium effects in our model. Note also the previous explanation is lacking in comparison with historians' insights in at least one dimension (again, in our model this would simply be captured by the productivities): In reality, much of the productivity increases were likely driven by increased hours worked as people were unwilling to reduce their level of consumption back to pre-plague levels.
36 In equilibrium, this relationship is especially clear from equation (26).
37 The reason is that (non-agricultural) producers soon after this uptick in human capital investments replaced the skills thus acquired with machines.
38 Note that this is actually income per worker but the two are identical in indexed terms, assuming a constant participation rate (see "Labor Supply" below).
39 The agricultural land share θ is a "deep" parameter.
40 This normalization is necessary for determinacy because (L_t), $(A_{X,t})$, and (X_t) only enter equation (28) through the terms L_t/L_{t+1} and $(A_{X,t}X_t)/L_t$.
41 This corresponds to solving $p_0(\bar{c}(p_0))^{\frac{1}{1-\theta^{-1}}} = \left(\frac{A_{X,0}X_0}{L_0}\right)^{\frac{1}{1-\theta^{-1}}} (1-\theta)^{-1}\beta A_0 k_0^{\alpha}(\varsigma(k_0))^{1-\beta}$ and $k_0 = \sigma(p_0)A_0k_0^{\alpha}(\varsigma(k_0))^{-\beta} - B^{-1}(e(k_0))^{1-\gamma}$.
42 This is described in more detail in the console of the MATLAB implementation.

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Appendix A: Data sources. mapping between model and data

- (Agricultural labor shares) Model: U_t/L_t. Data: Clark (2013) and Wallis et al. (2018).
- (Agricultural output) Model: Y_t^P. Data: Output-based measure from Broadberry et al. (2015) ["Real agricultural GDP" in BoE (2017), Table A6].
- (Income per capita, agricultural numeraire) Model: $Y_t/(p_t L_t)$.[38] *"Broadberry"*: Nominal GDP per capita divided by the agricultural price index from Broadberry et al. (2015) [BoE (2017), Table A6].
- (GDP/output, agricultural numeraire) Model: Y_t/p_t. Data: Nominal GDP divided by the agricultural price index from Broadberry et al. (2015) [BoE (2017), Table A6].

- (Land) Model: X_t $(= X_t^S)$. Data: Total arable acreage and total sown acreage in millions of acres from Broadberry et al. (2015) [BoE (2017), Table A3]. Indexed. Note: This implicitly assumes that arable acreage is proportional to total agricultural acreage.
- (Land rent, agricultural numeraire) Model: ρ_t^{U}/p_t. Data: "*Clark*": Nominal non-wage farm income from Clark (2013) divided by acres of arable land [BoE (2017), Table A3], deflated with the agricultural price index from Clark (2004). Index measure (see the note under "Land"). "Humphries and Weisdorf": $(\theta/(1-\theta)) \cdot (w_t U_t)/X_t$ where w_t is the "Humphries and Weisdorf" farm wage with agricultural numeraire, X_t is land, U_t agricultural workers [calculated from Clark (2013) prior to 1522 and Wallis et al. (2018) after 1522], and $\theta = 0.4$ the land share. Indexed. Note that in either case, the implicit assumption is that arable land use is proportional to total land use, and with the second measure, it is implicitly assumed that population size times agricultural labor share is proportional to number of agricultural workers (constant participation rate).
- (Labor supply) Model: L_t $(= L_t^S)$. Data: Indexed population. Note: This implicitly assumes a constant participation rate.
- (Population) Data: Broadberry et al. (2015) [BoE (2017), Table A2].
- (Relative price) Model p_t. Data: "Broadberry et al": p_t^a/p^{nag} where $p_t^{nag} = (\omega_t^I p_t^I y^I + \omega_t^S p_t^S y^S)/\,(\omega_t^I y^I + \omega_t^S y^S)$ is the composite deflator of Industry and Services with the sectoral share weights ω_t^I and ω_t^S from Broadberry et al. (2015), p. 194 and sectoral price indices and sectoral real outputs from BoE (2017), Table A6. p_t^a is the agricultural price index of Broadberry et al. (2015) [BoE (2017), Table A47]. "Clark": Clark (2001), Table 5.
- (Skill-premium) Model: w_t^S/w_t^U. Data: *"Allen," "Southern Europe"* Calculated from the wage of skilled craftsmen and unskilled laborers in Allen (2001). "Clark": Calculated from the wages of building craftsmen and building laborers in Clark (2010) [also available in Table A47 in BoE (2017)].
- (Unskilled/farm wage, agricultural numeraire) Model: w_t^U/p_t Data: *"Humphries and Weisdorf"*: Real wage from Humphries and Weisdorf (2019) multiplied with the GDP deflator and divided by the agricultural price index, both from Broadberry et al. (2015) [BoE (2017), Table A47]. "Clark": Nominal farm wage from Clark (2013) divided by the agricultural price index from Clark (2004).

Appendix B: Calibration and model validation

The period length is set at 20 years, and throughout this appendix, "actual data" refers to 20-yearly actual data. The agricultural sector's production function (2) can be rewritten as follows:

$$A_{X,t} = U_t^{(1-\theta^{-1})} X_t^{-1} (Y_t^{P})^{\theta^{-1}}. \tag{B1}$$

For fixed exogenous variables (deep and non-deep parameters and non-agricultural productivities (A_t)), and given actual data for land (X_t) and agricultural output (Y_t^{P}), there is therefore a one-to-one relationship between agricultural employment U_t and agricultural productivity $A_{X,t}$.[39] Because of this, both $(A_{X,t})$ and the agricultural labor shares (u_t), $u_t = U_t/L_t$, can be endogenously determined by imposing the actual data in the loop of following pseudo-code. The loop initiates with an initial guess for the agricultural labor shares (u_t).

1. Given actual data for (X_t) and (Y_t^P), and the input agricultural labor shares (u_t), use equation (B1) to compute $(A_{X,t})$.
2. Given $(A_{X,t})$ and the exogenous variables, simulate the model.
3. If the simulated agricultural labor shares coincide with (u_t), then stop. Otherwise, update (u_t) by setting it equal to the simulated agricultural labor shares and return to step 1.

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In the MATLAB implementation, this loop typically finds a fixed point in three or four iterations. We always use $(u_t) = (0.5, 0.5, \ldots, 0.5)$ as the initial guess to emphasize the endogeneity of the outcome. Note that while it is necessary to "anchor" the labor share in the year 1300 for identification purposes (Target (T1) below), this is the only target which at any point involves agricultural labor shares.

Because of what was just said, the variables that must be calibrated are the non-deep parameters and the non-agricultural productivities. The assumption that allows us to identify the non-deep parameters is that there is no productivity growth in the non-agricultural sector prior to 1420. This assumption is strongly supported by historians [*e.g.* Mokyr (2005) and Clark (2005b)], and it is easily reconfirmed via sensitivity analysis (simply put, any productivity increases in A_t prior to 1420 leads to extremely poor matches with the data). Also, since our model is extremely over-identified, we are able to indirectly test this assumption as returned below.

B.1. Calibrating the Non-Deep Parameters

There are seven (non-deep) parameters as specified in Table 1. To calibrate these, we focus on the period prior to 1420 (1300–1420) and assume no non-agricultural productivity growth. From now on, "simulation" refers to the termination point of the iterative fixed point loop described previously. Given any choice of parameters, the 1300–1420 subperiod simulation returns endogenously determined agricultural labor shares and agricultural productivities, as well of course as all of the other data points. We match the following targets:

- (T1): The agricultural labor share equals 77% in 1300. [The exact center point of the "channel" around Clark's data seen in Figure 10].
- (T1): The skill-premium equals 120% in 1300 [The average over the previous years based on Allen's data].
- (T2): Indexed income per capita in agricultural units matches the data. Specifically, the target is to match each period and when that is impossible to rely on minimizing the R-squared between the simulated income per capita and the two-sided 20-year moving average of the data.
- (T3): The relative price of food declines by 16% between 1340 and 1380 [This is the relative decline of the 20 years two-sided moving average of the relative price between the primary/agricultural and secondary (industry and services) computed from Table A6 in BoE (2017) using the methodology of Broadberry et al. (2015) (p. 200; with weights taken from Table 5.01, p. 194). Note that these years are chosen to mimic the price change caused by the Plague].

We normalize L_0 to 1[40] which pins $A_{X,0}X_0$ down (because of (T1) and clearing of the agricultural market). k_0 and p_0 are then solved for their steady state values.[41] We allow an "epsilon derivation" from k_0. The rationale is that allowing for such an epsilon derivation corresponds to imposing that the economy is initially (year 1300) in a neighborhood of a steady state. This restriction is as much technical as economical: Without it, the dynamic equilibrium path can begin at an arbitrary point in the phase space and the calibration becomes indeterminate. Together with A_0, this adds an additional two free parameters.

Because we impose the previous targets exactly (except for the income per-capita changes which we can obviously not hit exactly at every date), the parameters we calibrate in this way are very firmly identified. Note that while the exact targets may be debated, minor changes in them lead to no meaningful differences in any results. We also did extensive sensitivity analysis of the deep parameters but as returned to below, the primary robustness concern is not actually related to the calibration procedure but to the historical data sources used (hence that is the focus in what follows).

B.2. Calibrating the Non-Agricultural Productivities

In this step, we fix the parameters calibrated previously and calibrate (A_t) from the year 1420 and onwards targeting *only* income per capita. We smooth all exogenous data in order to avoid arbitrary swings resulting from our 20-year time period (the data is yearly).[42] This calibration is straightforward to implement because all initial values are fixed from the previous step and (A_t) roughly "controls" income per capita although, of course, we cannot perfectly match the data because the fixed point loop determination of the agricultural productivities and the fixed parameters do impose substantial cross-restrictions.

Significantly, when we implemented this second part of the calibration, it immediately became clear that the endogenously determined agricultural labor shares must deviate from Clark (2013)'s series after approximately 1600. The significance of this is discussed in more detail at the end of Section 4.3.

In light of the output-based methodology of our primary data source [Broadberry et al. (2015)], the advantage of the previous approach to estimating agricultural productivities is that it makes no use of price data with the single exception of the relative price decline match in (T3) above. There is an alternative approach that instead relies entirely on price data. This is the approach adopted by for example Clark (2001), Bar and Leukhina (2010) and Leukhina and Turnovsky (2016). The starting point is the following formula, which follows directly from the inverse demand functions in Section 2.1:

$$A_{X,t}^{\theta} = \left(\frac{\rho_t/p_t}{\theta}\right)^{\theta} \left(\frac{w_t^{\mathrm{U}}/p_t}{1-\theta}\right)^{1-\theta}. \tag{B2}$$

Note that $A_{X,t}^{\theta}$ is the Hicks neutral agricultural productivity, and ρ_t/p_t and w_t^{U}/p_t are the real land rent and unskilled wage with the agricultural output as numeraire.

It is clear that if we use this formula to calculate $(A_{X,t})$ from the price data predicted by our simulation, we get precisely the same agricultural productivities as before. Comparing our estimates with "price-based" agricultural productivity estimates thus amounts to a reduced form external validity test. Figure 14 displays this comparison for three different combinations of wage and land rent data. The first compares with productivities calculated via equation (B2) using the farm wage and land rent of Clark, and, respectively, the Humphries and Weisdorf (H-W) wage and land rent (again the reader is referred to Appendix A for the details). The second subfigure displays the same comparison except that we use Clark's land rent throughout.

Digging deeper, Figure 15 adds to the relative price match in Figure 13a by directly comparing the relevant price predictions with the data. Both visual and formal inspection show that our model predicts the land rent extremely well whatever the data source. Our unskilled/farm wage predicts Humphries and Weisdorf (2019) very well too, but does relatively poorly when held up against Clark's wage data.

Taking also Figure 13a into consideration, it is clear that the primary cause of the difference between our agricultural productivity estimates and the estimates based on Clark's data is driven by the difference between our predicted unskilled/farm wage and Clark's. To test our analysis' sensitivity to this, we redid the whole analysis as follows: First, we imposed the agricultural productivities corresponding to Clark's wage and land rent depicted in Figure 14 instead of determining these in a loop. We then replaced the Broadberry et al. (2015) income per-capita data with Clark's and recalibrated non-agricultural productivities (we kept the parameters unchanged) (see Figures 16a and 16b).

The figures below summarize the outcomes (we omit the wage, land rent, and the relative price, but these fit extremely well too; they can be seen by running the associated MATLAB code). The takeaway is surprisingly unambiguous: The paper's main conclusions are robust to this change in data source. Upon reflection, this is not too surprising: As can be seen in Figure 14, the main difference between our (output-based) estimates and the (price-based) Clark ones is a more pronounced "swing" between the Great Mortality and roughly 1500. Comparing with our main conclusions,

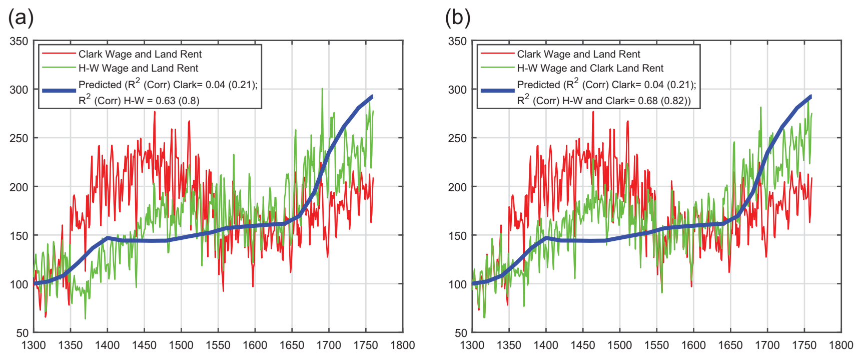

Figure 14. Structurally estimated agricultural productivities. R^2 is the R-squared of the linear regression of the actual data on the model predictions (blue curves).

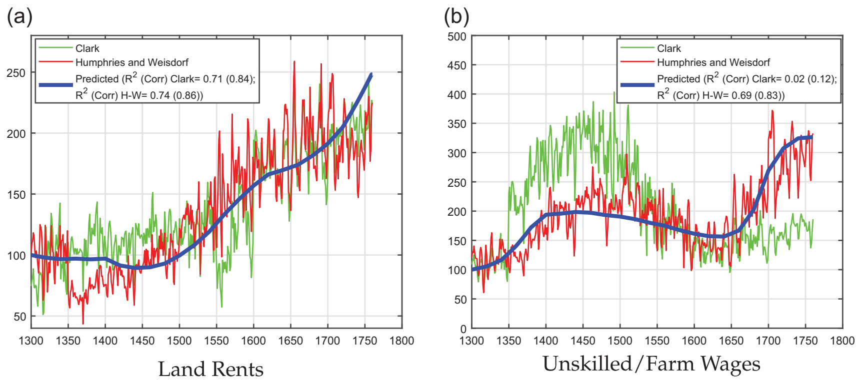

Figure 15. Land rents and unskilled wages (agricultural numeraire). R^2 is the R-squared of the linear regression of the actual data on model predictions (blue curves).

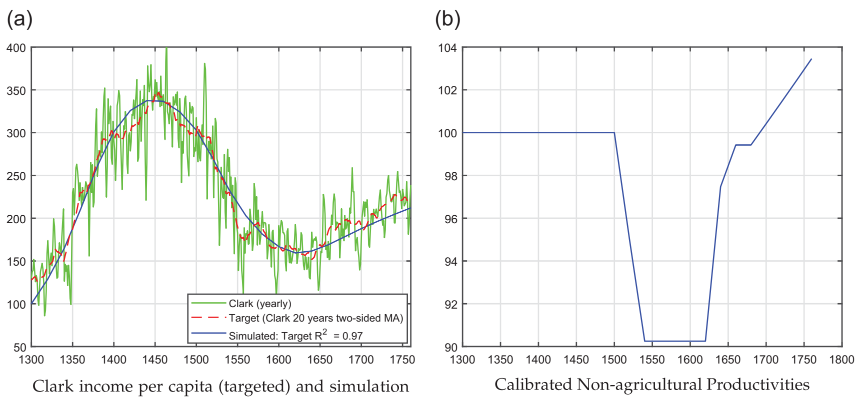

Figure 16. Income per-capita match via non-agricultural productivities. R^2 is the R-squared of the linear regression of the actual data on model prediction (blue curves).

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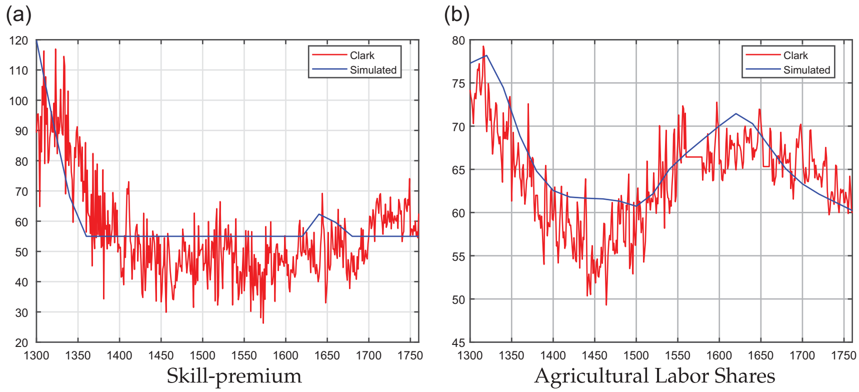

Figure 17. Skill-premium and agricultural labor shares.

this is of little consequence because the reversal after 1450 follows a higher initial level in 1450. After 1660, there are differences also, which precisely accounts for the "re-alignment" of the agricultural labor shares with Clark's data (Figure 17b). But since our main conclusions all relate to the years prior to 1660, this does not impact anything in Sections 4.2 and 4.3. On the other hand, that our model performs so well with this alternative data source does lend additional support to the model and the main economic mechanisms it identifies.

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