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A Program for the Analysis of Long-Period Binaries: The Case of γ Delphini

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Abstract. Binary systems can be analyzed like clusters but with additional constraints from the orbit. The theories of stellar interiors and atmospheres are used to analyze the color-magnitude diagram and spectra to test the consistency of theory and observation and also to provide results on the distance, chemical composition, age, and individual masses, radii, and effective temperatures. Given the total mass resulting from the theoretical analysis, it is possible to determine the complete orbit of even long-period binaries if at least 6 components of the position, velocity and acceleration vectors have been observed.

For this paper we apply such a theoretical and orbital analysis to the γ Delphini binary consisting of a K1 IV primary (γ^2 Del) and an F7 V secondary (γ^1 Del). The primary has been observed with the precise-radial-velocity (PRV) technique and shows a radial acceleration of 2.2 ± 0.7 m s^{-1} yr^{-1} as well as a significant (false alarm probability < 0.01) periodic signature ($P = 1.44$ yr). At least three possible causes of this periodic signature are pulsation, rotation, or a planetary companion. The mass and radius results ($M_2 = 1.72M_{\odot}, M_1 = 1.57M_{\odot}, R_2 = 6.43R_{\odot}$, and $R_1 = 2.21R_{\odot}$) of our theoretical analysis help constrain the possibilities. The pulsational hypothesis requires more investigation of whether it is possible to excite a g-mode or an r-mode period that is much longer than the fundamental period of 0.5 dy derived from the mass-radius results. The rotational hypothesis leads to an inconsistency; the published value of $v \sin i$ is a factor of 4.5 larger than the maximum value allowed by the radius and the PRV period of 1.44 yr. More investigation is required to determine whether increased macroturbulence and decreased $v \sin i$ (by a factor of 4.5) is consistent with observed line profiles. From the PRV period and the primary mass a possible planetary companion would have $m \sin i = 0.7$ Jupiter masses with an orbital semimajor axis of 1.5 AU. The orbital results for the stellar binary (closest approach > 15 AU) shows there is room in the system for such a possible planetary companion of γ^2 Del to survive the gravitational perturbations of γ^1 Del.

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1. Introduction

As part of a larger program to analyze the color-magnitude diagrams, spectra, and orbits of long-period binaries, we concentrate here on the γ Del binary. There are several motivations for analyzing this system. (1) This system tests the theories of stellar interiors and stellar atmospheres over a significant range of stellar conditions. (2) This system has a well-determined Hipparcos parallax which greatly constrains the isochrone fit. (3) γ^2 Del has a significant PRV acceleration of 2.2 ± 0.7 m s^{-1} yr^{-1} (Fig. 1). Most stars do not show such accelerations (Walker et al. 1995). However, in the case of 36 Ophiuchi B, a chromospherically active K dwarf in a long-period binary, the result of the stellar-interior and orbital analysis (Irwin, Yang, & Walker 1996; Paper I) is that a significant part of the observed radial acceleration is likely due to intrinsic spectral changes (e.g., time-variable convective blue shifts). It is of interest to perform a similar analysis for the γ Del system to see whether the observed radial acceleration of γ^2 Del is consistent with center-of-mass orbital motions for this chromospherically *inactive* star. (4) γ^2 Del has a significant (false-alarm probability < 0.01), low-amplitude ($K = 11.6 \pm 2.2$ m s^{-1}), 1.44-yr periodicity in the PRV data (see Fig. 1 and also Larson, Yang, & Walker 1999). Our Ca II $\Delta EW_{866.2}$ index of chromospheric activity (Larson et al. 1993), which is determined at the same epochs as the PRV data, shows no significant periodicities or secular variations. Hypotheses to explain the physical cause of the PRV periodicity (e.g., stellar pulsation, stellar rotation, or a planetary companion) are constrained by the theoretical and orbital analysis of γ Del.

2. The Isochrone Fit

The stellar-interior results were calculated with the University of Victoria code with all semi-empirical parameters calibrated to reproduce observations that are completely independent of the γ Del system (VandenBerg et al. 1999). The helium abundance and mixing-length parameter are calibrated to reproduce the luminosity and effective temperature of the present-day sun. The convective overshooting and color transformations were calibrated to reproduce certain cluster (e.g., Rosvick & VandenBerg 1998) and binary observations.

Table 1. Isochrone Fit

Component	Mass $M_\odot$	Radius $R_\odot$	Grav. Red shift m s^{-1}	T_{eff} K	$\log g$ cm s^{-2}
1	1.57	2.21	451	6303	3.94
2	1.72	6.43	171	4855	3.06

The results of the isochrone fit are given in Table 1 for a γ Del age of 2.2 Gyr. The adopted [Fe/H] value of 0.12 is taken from an abundance analysis of γ^1 Del by Boesgaard & Friel (1990) and is in reasonable accord with an abundance analysis of γ^2 Del by McWilliam (1990) who found [Fe/H] = 0.13 and 0.16. A system parallax determined from the stellar-interior results alone

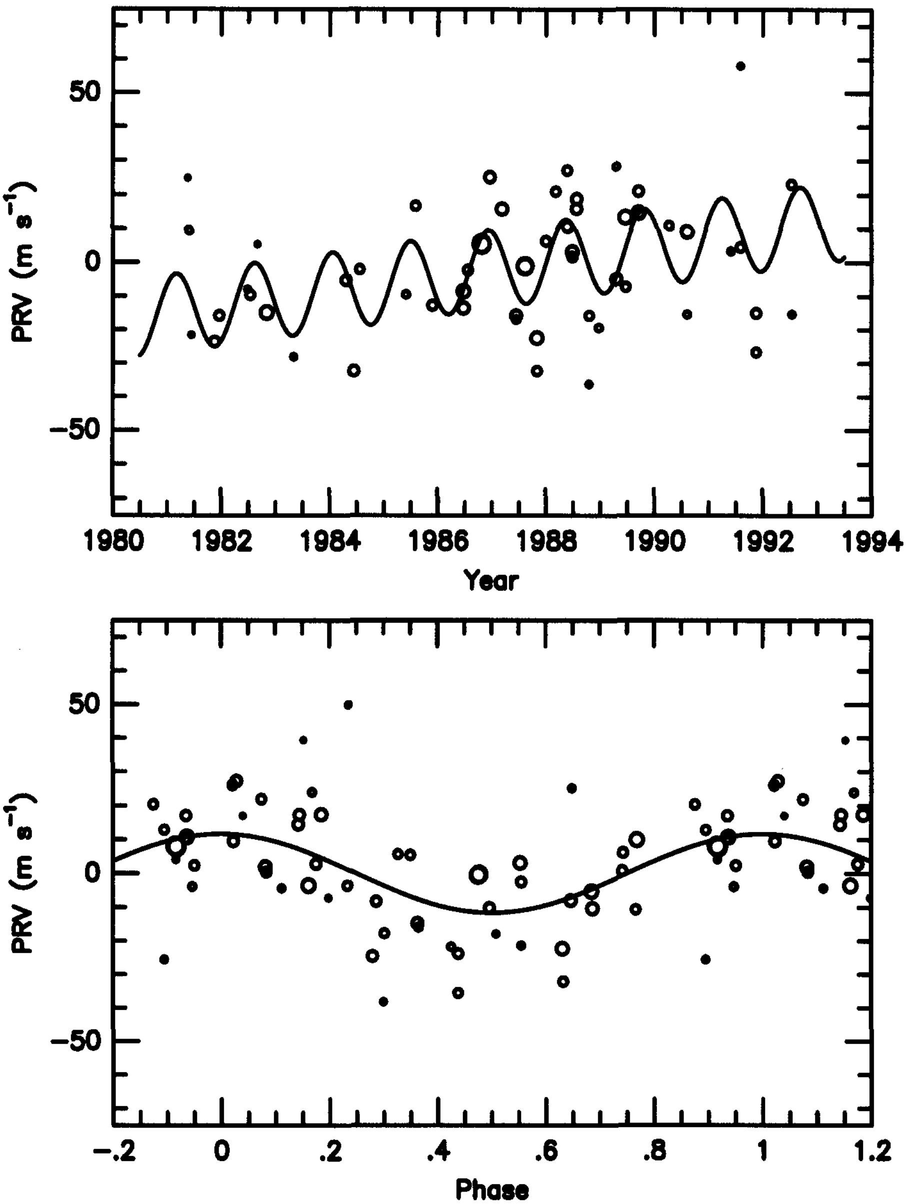

Figure 1. Precise radial-velocity data for γ^2 Del as a function of time and phase (calculated with $P = 1.44$ yr). The data are indicated by circles whose area is proportional to the weight and inversely proportional to the square of the internal error. In the time plot a least-squares model of a constant acceleration + sinusoid is plotted as a solid line. In the phase plot the least-squares acceleration is subtracted from the model (solid line) and data.

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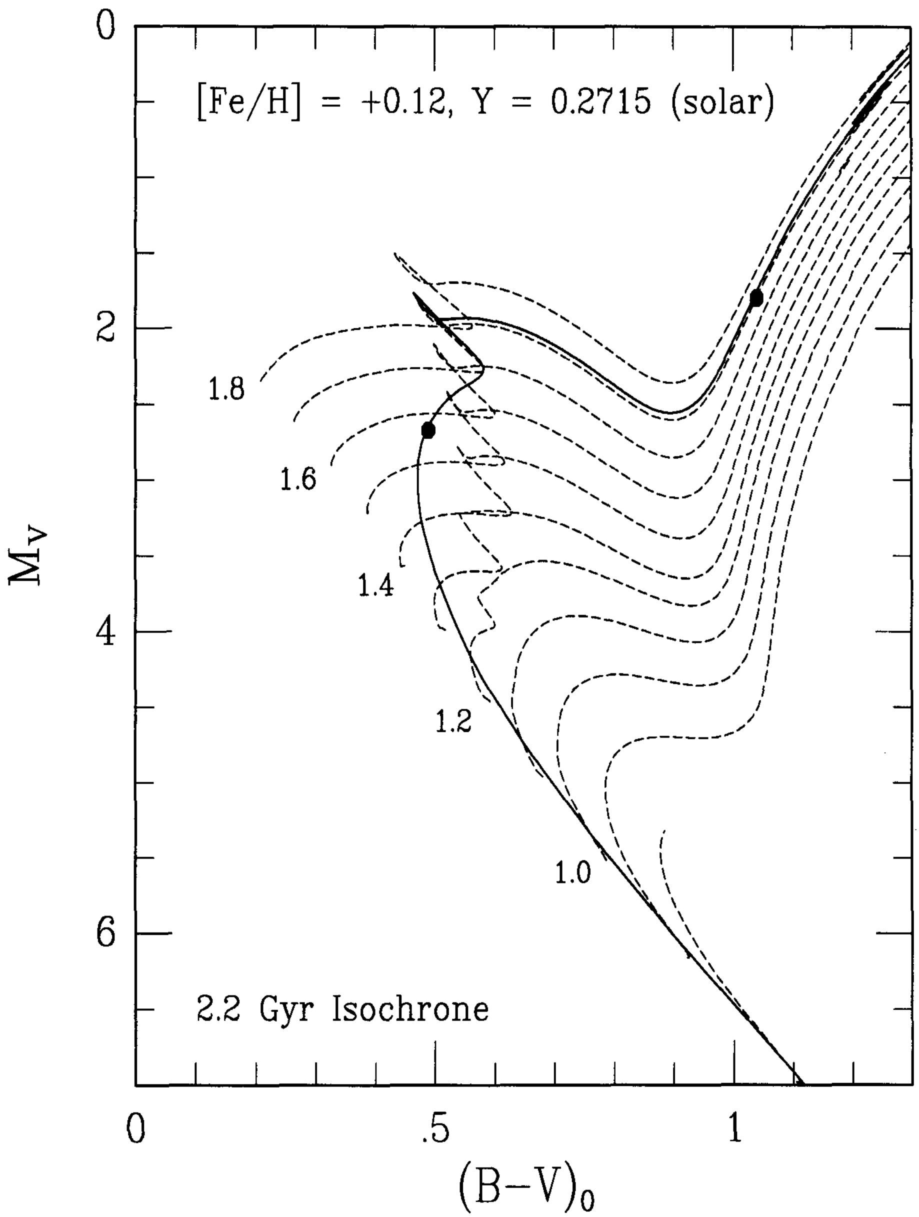

Figure 2. Isochrone fit for the components of γ Del. The semi-empirical calibrations that determine the shape and position of these tracks and isochrone were done independently of this system (see text). We have adopted the weighted mean (0.032 ± 0.001 arcsec) of the Hipparcos parallaxes for the components of this system, [Fe/H] consistent with abundance analyses of γ^1 and γ^2 Del (see text), and zero reddening. The results of the isochrone fit are given in Table 1.

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would be consistent with the weighted-mean Hipparcos parallax but would have much larger uncertainties. Fig. 2 illustrates the superb isochrone fit of this system. We intend to follow up on these results by doing a synthetic spectrum analysis using the derived surface gravities and effective temperatures of the components of γ Del. Our synthetic-spectrum analyses of stars similar to those of the γ Del system show promising preliminary results (Larson 1996, see also Larson & Irwin 1996).

The results of the isochrone fit constrain the pulsational explanation of the observed 1.44-yr periodicity of γ^2 Del. From Table 1 $\overline{\rho}_2/\overline{\rho}_\odot = 0.00647$. If we adopt $Q = 0.04$ for the constant of the period-root mean density relation (see Cox & Giuli 1968, Table 27.2), then the fundamental period of γ^2 Del should be roughly 0.5 dy. Since the observed period is much longer than the fundamental the hypothesized pulsation must be a high-overtone gravity mode (or possibly an r-mode). The question of whether it is possible to excite such modes for γ^2 Del requires theoretical investigation.

The results of the isochrone fit also constrain the rotational explanation of the observed 1.44-yr periodicity of γ^2 Del. From this period and the radius from Table 1 we derive $V_{eq} = 0.62$ km s^{-1} while $V_{eq} \sin i = 2.8 \pm 0.3$ km s^{-1} (Gray & Nagar 1985), where i in this case is the stellar inclination. To avoid $\sin i$ exceeding unity by a factor of 4.5, either our period is too long (there are other shorter-term aliases of our period that are possible although the fit to the data is not as good, see the periodogram plot in Larson et al. 1999); the published $V_{eq} \sin i$ value is too large (the observed spectral-line broadening is due to a combination of macroturbulence and rotation that may be difficult to distinguish); or the rotational hypothesis is invalid.

If the planetary hypothesis to explain the 1.44-yr periodicity of γ^2 Del is correct, then from Table 1 and Kepler's laws we derive a companion mass ($\times \sin i$, where i in this case is the inclination of the planetary orbit) of 0.7 Jupiter masses and a semimajor axis of 1.5 AU. The results of the isochrone fit also affect the orbit derivation (next section) which indirectly constrains the planetary hypothesis through the dynamical effect of the stellar secondary on the possible planetary companion.

3. Determination of the Stellar Binary Orbit

We determine the orbit of γ Del from a simultaneous fit (with parallax and total mass constrained) of radial-velocity and positional data.

3.1. The Radial-Velocity Data

The difference in radial velocity between γ^1 and γ^2 Del, ΔV_{1-2}, and the PRV data for γ^2 Del were determined with the HF technique (Walker et al. 1995) using observations at the CFHT.

Our only spectrum of γ^1 Del was taken within 40 minutes of a spectrum of γ^2 Del, and following Paper I we used these spectra to determine the relative velocity between the two stars. When this velocity result is corrected for the differential gravitational red shift (see Table 1), we find $\Delta V_{1-2} = -1170 \pm 170$ m s^{-1} at epoch $JD = 2446282.96$ dy. The formal internal error of ΔV_{1-2} (estimated from the variance of the mean velocity determined from the individual line velocities) is about 10 times larger than our usual internal errors and this is presumably due to pseudo-random differential line blending effects between the F7 V secondary and K1 IV primary. Some of the scatter may also be due to the variation of differential convective blue shifts from line to line.

The pseudo-random errors can be substantially reduced by using a larger spectral coverage (we observe only 14 nm of the spectrum with the HF technique). However, whatever the spectral coverage a systematic error is always present due to the difference in mean convective blue shifts between the two stars. The mean convective blue shift for the sun is about 300 m s^{-1}, and the effect should generally increase for both earlier spectral types and increasing luminosities (Dravins, 1999). However, in the absence of detailed modeling of the effect for γ^1 and γ^2 Del we can make no correction.. Since the correction is a difference of blue shift values of unknown size for the two stars we don't even know its sign, but we assume the magnitude of the correction is small (say less than $\sim$ 200 m s^{-1}) because the two blue shifts partially cancel in the difference.

We determined 57 precise radial velocities of γ^2 Del (see Fig. 1) with our usual PRV technique, subtracted the least-squares sinusoid corresponding to the 1.44-yr period, and multiplied the residuals by $-(M_1 + M_2)/M_1 = -2.1$ (see Table 1) to transform to the equivalent *relative* values of ΔV_{1-2} that are fitted as part of the orbit determination. These data have an arbitrary common zero point which is determined as part of the fit so they only constrain the orbital acceleration of the fit (which in the mean is -4.6 ± 1.5 m s^{-1} yr^{-1}) not ΔV_{1-2} itself where we must rely on just the single data point discussed above.

3.2. The Positional Data

Our treatment of the positional data follows the procedure given in Paper I, except that we use a correlated fit of the Hipparcos data, and we fit the photographic astrometry in x ($\equiv \rho \cos\theta$) and y ($\equiv \rho \sin\theta$), rather than the separation, ρ, and position angle, θ, of γ^1 Del relative to γ^2 Del.

The Hipparcos catalog gives positions and proper motions of γ^1 and γ^2 Del, and these data and their associated variance-covariance matrix have been transformed (using great-circle formulas) to x, y, dx/dt, and dy/dt, where t is the time. The revised, non-linear least-squares method uses the transformed variance-covariance matrix as part of the procedure to minimize the sum of squares of the *correlated* residuals (see Hamilton 1964, eq. [4-16]).

All positional data other than the Hipparcos results were taken from the Washington Double Star Catalog of Observations (Worley 1993). The raw data from this catalog were in the ρ-θ form and were subsequently corrected for precession and proper motion. The photographic astrometry data were transformed to x and y while the visual-binary data were left in the corrected ρ-θ form for the fit. Following Paper I we use the residuals from fits to determine automatically the weights (defined as the inverse squares of the formal errors) of the photographic astrometry and visual-binary data. As part of this procedure we rejected 7 of 337 complete observations (of x and y or ρ and θ) where at least one component of the orbit residual was greater than 4 times its formal error.

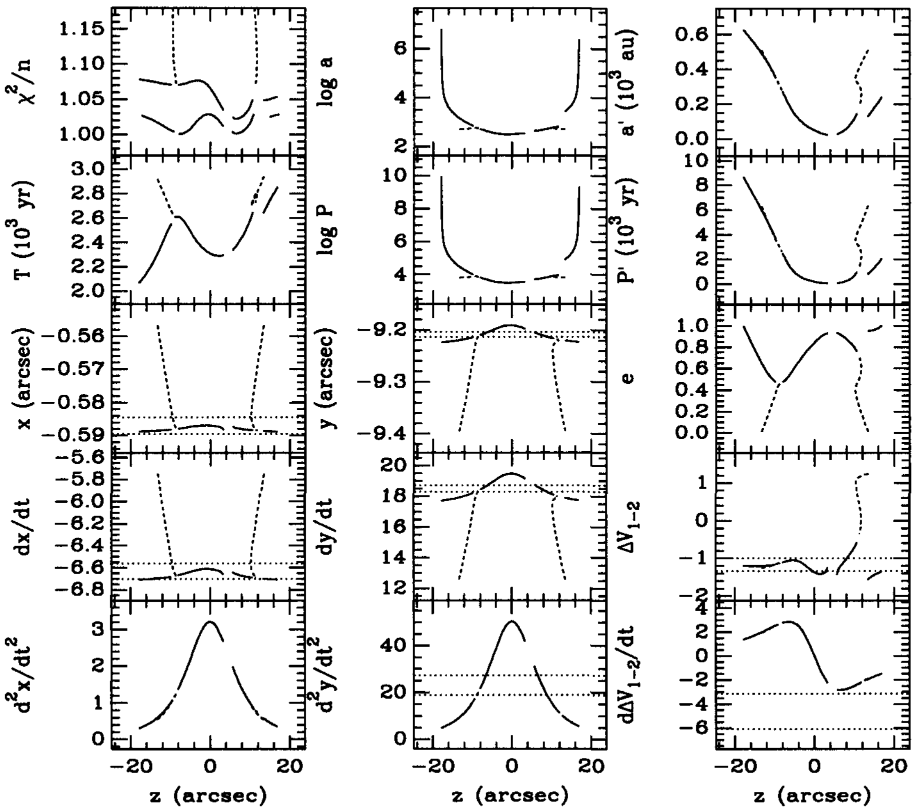

Figure 3. Possible orbital parameters of the γ Del system. We have determined full orbital solutions with frozen eccentricity, e, for a range of e values from 0.01 to 0.9999. We have plotted several parameters determined from these orbits as a function of the z parameter determined from these orbits. x, y, z, and ΔV_{1-2} are defined in the text and these parameters and their derivatives are calculated from the orbits at epoch J2000. The lower χ^2/n curve is the weighted sum of correlated residuals (divided by the number of data points, n) for just the positional data, while the upper χ^2/n curve is the same quantity calculated for both the positional and velocity data. a is the semimajor axis, $a' \equiv a(1-e)$ is the distance of closest approach, T is the Julian epoch of periastron, P is the period, and, $P' \equiv P(1-e)^{3/2}$ is a convenient composite fitting parameter (Paper I). The units of a are AU, the units of P are yr, the units of dx/dt and dy/dt are 10^{-3} arcsec yr^{-1}, the units of d^2x/dt^2 and d^2y/dt^2 are 10^{-6} arcsec yr^{-2}, the units of ΔV_{1-2} are 10^3 m s^{-1}, and the units of $d\Delta V_{1-2}/dt$ are m s^{-1} yr^{-1}. We have connected solutions that are continuous functions of frozen e by solid lines (and also by dashed lines where the solutions correspond to relatively large χ^2/n values). The horizontal dotted lines for the positional and velocity parameters correspond to the $\pm$ 1-σ range about the value determined in simplified fits (see text).

3.3. The Orbit of γ Del

We determine orbital parameters using a simultaneous, non-linear least-squares fits of velocity and positional data. The fits are constrained to be consistent with a total mass (see Table 1) of $3.29 M_\odot$ and the weighted-mean Hipparcos parallax of $\pi_{abs} = 0.032$ arcsec. We have explicitly treated the correlation of errors in the transformed Hipparcos results (see above). Paper I describes the version of our orbit-fitting procedure assuming uncorrelated observational errors.

The results of a number of orbital fits for frozen eccentricities from 0.01 to 0.9999 are given in Figure 3. The interpretation of the acceleration and positional results is straightforward. From Newton's principles the binary orbit can be completely characterized by the total mass and the position and velocity vectors (or equivalently any 6 of the components of position, velocity and acceleration) at any given epoch. From this model the gravitational attraction between γ^1 and γ^2 Del provides a central acceleration with components

$$d^2x/dt^2 = -(365.25\kappa)^2 (M_1 + M_2) x \pi_{abs}^3 / |r|^3, \quad (1)$$

$$d^2y/dt^2 = (d^2x/dt^2) y/x, \quad (2)$$

and

$$d\Delta V_{1-2}/dt = 4740.4\ldots (d^2x/dt^2) z / (x\pi_{abs}); \quad (3)$$

where x and y are defined as before (angular measure in arcsec), z (angular measure in arcsec) is the equivalent component in the line of sight, the units of x and y acceleration are arcsec yr^{-2}, the unit of velocity acceleration is m s^{-1} yr^{-1}, the magnitude of the radius vector (angular measure in arcsec) is given by $|r| = \sqrt{x^2 + y^2 + z^2}$, and κ ($\equiv 0.01720209895$) is the Gaussian gravitational constant. These equations are correct for any epoch, but we choose J2000 to be specific. The relative variations of x and y are quite small for all assumed eccentricities in Figure 3 so from equations (1) to (3) the acceleration curves are virtually a unique function of z with little uncertainty due to errors in the proper motion and radial velocity (and only modest relative uncertainty due to errors in the adopted mass and parallax).

To help interpret our detailed orbital fits we have determined positions, proper motions, and accelerations at epoch J2000 and their formal errors from a simplified fit of the positional data as a quadratic function of time, and we have also determined ΔV_{1-2} and $d\Delta V_{1-2}/dt$ at epoch J2000 from a simplified fit of the velocity data as a linear function of time. The $\pm$ 1-σ range of the parameters derived from the simplified fit are indicated in Figure 3. The y acceleration is the component of the acceleration with the smallest relative formal errors, and it constrains the $|z|$ value to be near 6.6 arcsec as indicated by the double minimum of the lower χ^2/n curve which is calculated for the positional residuals alone. At these z values, the x acceleration from the simplified fit (not illustrated) is 8.0×10^{-6} arcsec yr^{-2} (or 2.9σ) more negative than the model value. This inconsistency is evidence for a systematic error in the x astrometry. If a similar systematic error exists in the y acceleration it only corresponds to 30 % of that acceleration. For $z = 6.6$ the radial acceleration model approaches the observed value (within 1 σ). This near agreement reflects an acceptable consistency of the observed y acceleration and the observed PRV acceleration with the gravitational model (eqs. [1] to [3]) for z near 6.6 arcsec. Thus, within the present fairly substantial radial acceleration errors for γ^2 Del there is no need to invoke spectral changes as an additional source of PRV acceleration.

The upper χ^2/n curve which is calculated for both positional and velocity residuals has only one major minimum, and the solution corresponding to this minimum (with eccentricity fitted) is given in Table 2.

Table 2. The Orbit of γ Del

parameters[a]	π_{abs}	a^3/P^2	i	ω	Ω	e	T	P'
units	arcsec	$M_\odot$	deg	deg	deg	...	yr	yr
values	0.032	3.29	137.	-152.	-44.	0.916	2373.	127.
errors[b]	...	...	10.	11.	12.	0.021	49.	68.
parameters[c]	a	a'	α	z	P	K_{1-2}	χ^2/n	
units	AU	AU	arcsec	arcsec	yr	m s^{-1}	...	
values	450.	38.	14.	6.6	5200.	4400.	1.02	

[a]frozen and fitted parameters where π_{abs} is the parallax, a is the semimajor axis, P is the period, i is the inclination of (in this case) the orbit, ω is the longitude of the periastron for the *secondary*, Ω is the position angle of the ascending node, e is the eccentricity, T is the epoch of periastron, and $P' = P(1-e)^{3/2}$

[b]formal errors, see text

[c]derived parameters where a' ($\equiv a(1-e)$) is the distance of closest approach, α ($\equiv a\pi_{abs}$) is the angular semimajor axis, z is the angular coordinate in the line of sight of the secondary relative to the primary, K_{1-2} is the spectroscopic binary amplitude of the secondary relative to the primary, χ^2 is the correlated, weighted sum of squares of residuals, and n is the number of observations (722 in this case)

The present orbit of γ Del should be more reliable than previously published results. The Hopmann (1973) orbit through some unknown error does not fit the positional data and leads to the ridiculous mass sum of $13M_\odot$. Hale's (1994) correction to this orbit was still forced to rely on Hopmann's period and eccentricity. Thus, the resulting mass sum ($6.5M_\odot$) is still too high although the derived a and i values, the focus of Hale's study, are in reasonable accord with the present work.

From Table 2 the most uncertain parameter in our orbit is P' which implies relatively large uncertainties in the derived parameters a, a', α, P, and K_{1-2}. For such large uncertainties, non-linearities become important and the formal error analysis is only a rough guide to the actual uncertainty due to random observational errors. We performed a 1000-solution numerical simulation (not illustrated) to investigate the effect of random errors, and found, for example, that 99% of the least-squares estimates of a' are less than 2.6 times the actual (model) value. Thus, from our least-squares estimate of 38 AU it is unlikely (at a significance level of 1%) that the actual a' value is less than 15 AU. This minimum value of the closest approach is 10 times larger than the semimajor axis of the possible planetary companion. Since this ratio is larger than 4 (the usual criterion for stars of near-equal mass, see Paper I), the possible planetary companion to γ^2 Del should survive the gravitational perturbations by γ^1 Del.

4. Summary and Future Research

All important detailed results are given in the abstract. We have obtained a superb isochrone fit of the color-magnitude diagram of the γ Del system. This fit provides important constraints on the pulsational and rotational hypotheses to explain the PRV periodicity as well as the orbital analysis. This analysis shows the PRV acceleration of γ^2 Del is consistent (within fairly large errors) with the orbit and also shows the planetary companion hypothesis to explain the PRV periodicity for this star is not ruled out by dynamical considerations.

For the future, the isochrone analysis should be followed up by synthetic spectrum analysis, the differential mean convective blue shift should be estimated, and the $v \sin i$ value of γ^2 Del reevaluated. Most importantly the orbit should be greatly improved by continued monitoring of this system using precise astrometry and the PRV technique. It is especially important to measure an accurate ΔV_{1-2} value, confirm the PRV periodicity and acceleration we have discovered for γ^2 Del, and measure the radial acceleration of γ^1 Del.

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Discussion

Contos: Hale et al. have computed an orbit with $P = 3250$ yr., $e = 0.88$ and $d = 410$ AU. Can you clarify the $d = 25$ AU you calculated? And what are the rms and formal error on your 2 $\mathrm{m\,s^{-1}\,yr^{-1}}$ trend and 1.4 yr. period?

Irwin: The Hale orbit is partially based on a completely wrong orbit published by Hopmann. I don't know what went wrong with the Hopmann orbit, but as Hale remarks it doesn't fit the astrometry. Hale's orbit does fit the astrometry, but you need additional constraints to solve the system. I used the total mass from isochrones, and this is more accurate than adopting arbitrary parameters from a bad orbit. My value of 25 AU is the minimum distance of closest approach, $a(1-e)$, where a is the semi-major axis. The trend is $2.2 \pm 0.7\ \mathrm{m\,s^{-1}\,yr^{-1}}$. The amplitude of the periodicity is $11.6 \pm 2.2\ \mathrm{m\,s^{-1}}$, but the variation was found by a search of period-space, and has a false-alarm probability just under 0.01.

W. Cochran: We have also observed γ^2 Del. We obtained a small acceleration in the early data but this seems to have disappeared in the last five years, for which our data are consistent with zero acceleration. We also find no periodicity at the 0.001 false-alarm probability level, but will check further to see there's anything at the 0.01 false-alarm probability level.

Irwin: We should compare our data in detail.

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