GRAVITATIONAL LENS EFFECTS OF DARK MATTER OF MILKY WAY

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ABSTRACT. In this paper, dark matter of Milky Way is discussed as a transparent gravitational lens. The mass distribution of Milky Way is composed of spheroid, disk and dark matter (corona). The trajectory of light ray influnced by Milky Way has been calculated numerically. The deflecting angle to a extragalactic souce is changed with different observational direction and is about several seconds in maximum. The brightness amplification ratio K_V of extragalactic background radiation is also derived from the redistribution of intensity due to the gravitational lens effects of Milky Way, $\Delta K_V/K_V$ is about 10^{-5} in maximum. K_V has been expanding to multipole with spherical harmonics by means of K_V distribution in celestial sphere. Dipole and qudrapole are about 3×10^{-6} and 7×10^{-6} respectively, which might be added to the observational data of microwave background radiation. A measurement method to pick out the gravitational lens effects from real cosmological anisotropy has been proposed. (ref. 8)

1. INTRODUCTION

Dark matter (DM) is one of the most mystery problem in astrophysics and cosmology. Althought we do not know what is the dark matter (Sancisi 1986), it is convinced that DM exist from rotation curve. Any way ,every kind of DM influences the trajectory of light rays. Since DM is dissipationless, it could be taken as a transparent gravitational lens. The gravitational lens effects, in the case of light source enclosed by a spherical symmetric transparent halo, have been discussed by Xu et al. (1983) The solar system is also surrounded by a spheroid and corona (DM) and located at the galactic disk. The mass and potential distributions of Milky Way (spheroid, corona and disk) have been advanced in many forms (Caldwell and Ostriker 1981; Ostriker and Caldwell 1983) to fit the rotation curve (Knapp 1983). A preferable model is D150 (Ostriker and Caldwell 1983). The null geodesic equations are solved numerically for model D150.

Some new effects could be caused by the mass distribution of Milky Way itself. The new effects predict that the observational angular

A. Hewitt et al. (eds.), Observational Cosmology, 771–773.

positions of extragalactic sources are deflected, the observational brightness of the extragalactic background radiation is redistributed. The brightness amplification ratio K_{ν} and its multipolar expansion in spherical harmonics have been calculated numerically. Since the microwave background radiation is measured by Dicke Radiometer (Wilkinson 1982), in which to measure the temperature is equivalent to measure the radiation power from a standard resister at the same local oscillating frequency. Therefore, if some deviation of energy density power is observed in different point of celestial sphere, we could not distinguish weather it is caused by gravitational lens effects or by real cosmological anisotropy. Perhaps a part of anisotropy of microwave background radiation is caused by those new effect in normal measurement. But we could measure the every deviation of energy density in two different frequency at the same direction, which are located in the both sides of extremum value of Planck spectrum. Then, for the lens effect, since at the same direction the two frequency points have the same K_{ν}, so the energy density power of two points to compare with the standard Planck spectrum change identically, but for the real cosmological anisotropy, the variation of pure temperature may cause the Planck spectrum move along the frequency axis, the energy density power of two frequency points would change in opposite direction. The formula of the amplification ratio and the variation of temperature between two space points in terms of two different frequency can be easily obtained.

2. THE LIGHT RAY IN MILKY WAY

In Milky Way, the potential is weak (10^{-5}) and the motion of the axial symmetric disk could be omitted, then the metrics $g_{0j}=0$ (Misner et al. 1973). Since the corona, spheroid and disk are concentric, then $g_{ij}=\delta_{ij}A(\rho,\theta)$, $g_{00}=-B(\rho,\theta)$, here ρ, θ are spherical coordinate from center. Therefore

$$dS^2 = A(\rho,\theta)\cdot(d\rho^2+\rho^2 d\theta^2+\rho^2 \sin^2\theta \, d\varphi^2) - B(\rho,\theta)dt^2 \tag{1}$$

In the weak field approximation,

$$A(\rho,\theta) = 1+2u, \text{ and } B = 1-2u \tag{2}$$

where u is the gravitational potential, which is composed of three parts: disk u_1, spheroid u_2 and corona u_3. u_1, u_2 and u_3 have the analogous formulas to appendix A in Caldwell and Ostriker (1981), and the parameters are taken from D150. Now the metrics can be calculated numerically. The Christoffel symbol $\left\{ {\mu \atop \nu\lambda} \right\}$ are obtained also. Then the four geodesic equations are

$$\frac{d^2x^{\mu}}{dp^2} + \left\{ {\mu \atop \nu\lambda} \right\} \frac{dx^{\nu}}{dp}\frac{dx^{\lambda}}{dp} = 0 \tag{3}$$

The three constants of motion from the component equations of (3) are obtained. Substituties constants of motion to the above equations, there are

$$\begin{aligned} &\frac{d^2\rho}{dp^2} + \left(\frac{1}{\rho} - \frac{2}{A}\frac{\partial u}{\partial \rho}\right)\left(\frac{\partial \rho}{\partial p}\right)^2 - \frac{2}{A}\frac{\partial u}{\partial \theta}\left(\frac{d\rho}{dp}\right)\left(\frac{d\theta}{dp}\right) - \frac{1}{\rho} + \frac{2}{A}\frac{\partial u}{\partial \rho} = 0 \\ &\frac{d^2\theta}{dp^2} + \left(\frac{2}{\rho} - \frac{2}{A}\frac{\partial u}{\partial \rho}\right)\left(\frac{d\rho}{dp}\right)\left(\frac{d\theta}{dp}\right) - \frac{2}{A}\frac{\partial u}{\partial \theta}\left(\frac{d\theta}{dp}\right)^2 - \frac{1}{A^2\rho^4} J^2 \frac{\cos\theta}{\sin^3\theta} + \frac{2}{A^2\rho^2}\frac{\partial u}{\partial \theta} = 0 \end{aligned} \tag{4}$$

The distance and mass are normalized with radius of corona R_{TOT} and R_{TOT}/G respectively. The position S of the solar system (8.5 KPC far from the center) is taken as the initial condition. At first we put the light source in the position of solar system. The trajectory of light ray is integrated from $\rho = 0$ to infinity(take $\rho=100$ as infinity) with initial direction of spherical angular coordinate η_0, χ_0 from S with the method of Merson variable step in double precision. Because of the static metrics, the optical reciprocity therem is valid. The trajectory of light ray is the same, when source exchange for observer each other.

3. THE RESULTR OF NUMERICAL CALCULATION

3.1. The Variation of Angular Position

From above calculation the total deviation of angle $d\Omega \approx \sqrt{d\eta^2 + \sin^2\eta \, d\chi^2}$ where $d\eta$ and dx are the angular difference between existence and non-existence of the gravitational lens. When the light ray is closer to the disk, the deflecting angle is larger ($d\Omega \approx 2.2$ Sec). But for convenient to observe, the angle to disk at least 10°, in this case the deflecting angle is about 1.05Sec.($\eta_0=80°$; $x_0=152°$). The distribution of deflecting angle with celestial sphere has been obtained.

3.2. Brightness amplification ratio K_V and multipole of K_V

The larger K_V-1 is about -2.49×10^{-5} at $\eta_0=74°$ and $x_0=149°$. The distribution of deflecting angle is also obtained.

K_V could be expanding with spherical harmonics. The coefficients of multipole are obtained as follow: The dipole $a_{1,\pm1}=\mp2.8\times10^{-6}$; The quadrupole $a_{2,\pm2}=3.6\times10^{-6}$ and $a_{2,0}=7.2\times10^{-6}$. Others are all equal to zero. $a_{1,\pm1}$ and $a_{2,\pm2}$ are caused by the deviation of solar system from concenter and $a_{2,0}$ by the anisotropy of the disk.

4. DISCUSSION

Some of above mentioned effects are expected to detect in future, if we could find these effects, the dark matter of Milky Way might be understood further more and the distribution of DM can be calculated.

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