Hence, postulating axi-symmetry, and eliminating the a's, we find

$$c_{qr}\, c_{(q+1)(r+1)}\, c_{r(q+1)}\, c_{(r+1)q} = c_{rq}\, c_{(r+1)(q+1)}\, c_{q(r+1)}\, c_{(q+1)r}.$$

The $(n-1)(n-2)/2$ equations of this type are conditions under which the roots shall be real.

If we use the term "image minors" with reference to any minor $c_{qr}\, c_{(q+1)(r+1)} - c_{q(r+1)}\, c_{(q+1)r}$ and the corresponding one in which q and r are interchanged, and if we use the term "cross products" with reference to the two quadruple products, we can say that

The roots of an n-ic *are real if the cross products of any pair of image minors, in the determinantal form of the equation, are equal.*

When $q+1=r$, the coefficient c_{rr} is common to both cross products, so that the above condition includes Tait's condition for the case $n=3$. When $r=q$, the condition becomes a mere identity.

From these conditions it is easy to prove Muir's conditions, such as $c_{12}c_{24}c_{41} = c_{21}c_{14}c_{42}$; as also more complicated relations, such as

$$c_{n(n-1)}\, c_{(n-2)(n-2)} \ldots c_{21}c_{1n} = c_{n1}c_{12} \ldots c_{(n-2)(n-1)}c_{(n-1)n}.$$

Since we have $c_{pq}a_p^2 = c_{qp}a_q^2$, we see that, so far as this condition goes, c_{pq} and c_{qp} might be of opposite signs if we regard a_p *or* a_q as an imaginary quantity. But, since $c_{pq}a_p/a_q$ and $c_{qp}a_q/a_p$ must be equal and real, all the a's must be imaginary if one a is so. Therefore c_{pq} and c_{qp} must have like signs when we consider real coefficients only. In the case in which the roots represent, in proper units, the squared frequencies of the fundamental vibrations of a system of n masses, under the action of forces which are homogeneous and linear in the coordinates, c_{pq} and c_{qp} are necessarily of like sign since the masses are positive and the third law of motion holds.

On a simple theodolite suitable for use in schools.

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