

Abstracts of Australasian PhD theses

On the optimal control of delay differential systems

Sahala M. Nababan

In this thesis, we study the following three problems:

1. existence of optimal controls for systems governed by ordinary differential equations with time delayed arguments appearing in both state and control variables;
2. necessary conditions for optimality for systems governed by parabolic partial delay-differential equations in "divergence form" with first boundary conditions; and
3. necessary conditions for optimality for systems governed by parabolic partial delay-differential equations in "divergence form" with Cauchy conditions.

In §2.3 of Chapter II, we establish the existence theorems for optimal controls for a class of systems in which the functions on the right hand side of the differential equation are linear in the control variables, using a direct approach. Further, the existence of optimal controls for the control problem with an additional integral inequality constraint condition is also proved. In §2.4, a Filippov-type lemma for functions involving delays is established using the technique based on Guinn's scheme [3] and Filippov's lemma [2]. This lemma is then applied to establish the existence of optimal controls for a class of system in which the functions on the right hand side are nonlinear in the control variables. Further, it is also shown that the Filippov lemma can be used directly to establish the

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existence of optimal controls for a certain class of systems.

In Chapter III, we consider the systems governed by second order linear parabolic partial delay-differential equations in "divergence form" with first boundary conditions. All the coefficients and the forcing terms contain controls and delays in their arguments. Further, all the coefficients are assumed bounded measurable and the delays are allowed to appear in both state and control variables. The existence and uniqueness of weak solutions in the sense of Ladyženskaja, Solonnikov, Ural'ceva [4] for the system is proved. An integral maximum principle and its pointwise version are derived. Further, the difficulties in proving the existence of optimal controls for the control problem are also discussed.

In Chapter IV, we consider the system governed by second order linear parabolic partial delay-differential equations in "divergence form" with Cauchy conditions. As above, all the coefficients and the forcing terms contain controls and delays in their arguments. Further, all the coefficients are assumed bounded measurable and the delays are allowed to appear in both state and control variables. The existence and uniqueness of weak solutions in the sense of Aronson [1] for the system is proved. An integral maximum principle and its pointwise version are derived. Further, the difficulties in proving the existence of optimal controls are also discussed.

The constructions of an optimal control using necessary conditions are also outlined (see The Additional Remark).

References

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