

USE OF METEOROLOGICAL MEASUREMENTS FOR COMPUTING REFRACTIONAL EFFECTS - A REVIEW

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ABSTRACT

Corrections which are dependent on the atmosphere include the first and second velocity corrections and the curvature correction in EDM, and the refraction correction for trigonometric heighting. Generalised correction formulae have been developed to make use of variable values of the refractivity N and the coefficient of refraction k . There are no universally applicable values of these parameters, so atmospheric models of the temperature gradient and, for microwaves, the humidity gradient, are needed to represent the very significant variations. The models should take into account variations due to meteorological factors, surface conditions and the height above the surface. Many models for the vertical gradients have been produced. That of Brocks (1948) is very important and has been widely used and developed by researchers. More recently Monin and Obukhov, basing their work on the physics of the lower atmosphere and dimensional analysis, have included equations for the vertical gradients in their turbulence theory. The Turbulent Transfer Model, which embodies later results in this theory, is currently being refined and developed for geodetic applications.

1. INTRODUCTION

The atmospheric effects on geodetic observations place a limit on their accuracy. The simple approaches - using endpoint meteorological measurements for EDM reductions and the coefficient of refraction in trigonometric heighting - are effective for surveys of low or intermediate accuracy. When higher accuracies are needed, improvements can be found by adopting an atmospheric model which corresponds more closely to the real atmosphere, or using atmospheric dispersion, by measuring with two or more colours. The dispersion method will not be discussed in this paper, beyond noting that there are practical difficulties involved, and that even in this method some atmospheric modelling is essential. In a modified form, the so-called parallel measurement, the practical difficulties are avoided by using existing lightwave and

microwave instruments (KUNTZ and MÖLLER, 1971) but the model adopted for the relationship between temperature and humidity is critical in the reduction.

The atmospheric model is required to give the temperature gradient and, if microwaves are involved, also the humidity gradient. All the required corrections can be derived, using these gradients and meteorological measurements at the geodetic stations. The model must take into account the large number of factors which affect the gradients. No progress will come from trying to find the static, universal model, for example the best average value of the temperature gradient or the best value of the coefficient of refraction. The atmospheric model must be multi-factored to take into account the variations in the atmosphere, which are very large and real. For example, long term temperature measurements (BEST et al., 1952; FLOWER, 1937) show the average error due to observing temperature at 1.5 m instead at the height of the line, say 50 m above the surface. In the month of April (spring) the error, in parts per million (ppm) varies from -3 in the day to +5 at night, at Rye, in Southern England. At Ismailia, Egypt, the average daily variation is -6 to +6 ppm. In mid-summer the errors are similar, but in winter they are quite different, with lower averages, but a far larger variation from day to day (ANGUS-LEPPAN, 1967).

The variations in k , the coefficient of refraction, are even more remarkable. Table I shows values at heights of 1.5, 7.5 and 75 m for the same month, April.

Table I COEFFICIENT OF REFRACTION k

Minimum (day) and maximum (night) values for Spring (April) at Rye, England, and Ismailia, Egypt.

Height above surface	Rye		Ismailia	
	Day	Night	Day	Night
1.5 m	-1.6	2.5	-3.7	3.1
7.5	-0.2	0.8	-0.7	0.9
75	0.14	0.25	0.18	0.35

Ref. ANGUS-LEPPAN, 1967

Even at 7.5 m the values bear little resemblance to the standard value of $k = 0.13$, being -0.2 at Rye and -0.7 at Ismailia. Only at 75 m do the daytime values approach 0.13.

2. EDM REFRACTION CORRECTIONS - CONVENTIONAL FORM

The three corrections to EDM which are related to atmospheric refractive index are:

- the first velocity correction $C_1 = d(n_{ref}-n)$ (1)
 - the second velocity correction $C_2 = (-k+k^2)d^3/12R^2$ (2)
 - the correction for curvature of the wave path $C_3 = -k^2d^3/24R^2$ (3)
- (RUEGER, 1978)

Here, d is the measured distance,
 n_{ref} is the reference refractive index adopted in the particular instrument type,
 n is the refractive index along the wave path,
 k is the coefficient of refraction, defined as the ratio of the radius of curvature of the ellipsoid to the radius of curvature of the wave path,
 R is the radius of curvature of the ellipsoid.

For n and k , values appropriate to the EDM carrier wave are needed. For light waves the group refractive index and the refractive index under atmospheric conditions are calculated according to the well-known formulae of BARREL and SEARS (1939), while the equally familiar formula of ESSEN and FROOME (1951) is used for the atmospheric refractive index of microwaves. The coefficients of refraction normally adopted are 0.13 for lightwaves and 0.25 for microwaves (but see the discussion in BRUNNER, 1977 A).

3. EDM CORRECTIONS BASED ON EIKONAL EQUATION

Correction formulae (1) - (3) are suitable when n and k are assumed to be constant. When variations in n and k are taken into account, more flexible formulae are needed. A suitable general formula has been derived by MORITZ (1967) and developed by BRUNNER and ANGUS-LEPPAN (1976). It has the advantages that it combines all three corrections and that its integration is along the chord between the end points and not along the unknown wave path.

The basis of the derivation is the eikonal equation:

$$(\text{grad } d)^2 = n^2 \quad , \quad (4)$$

a differential equation which corresponds to Fermat's principle, namely that the path followed by electromagnetic waves between two points is that which takes the minimum time. The correction formula is:

$$\Delta s = d-s = 10^{-6} \int_0^s N \, dx - \frac{10^{-12}}{2} \cos^2 \beta \int_0^s \frac{[\int_0^x \frac{dN}{dz} \xi \, d\xi]^2}{x^2} \, dx \quad (5)$$

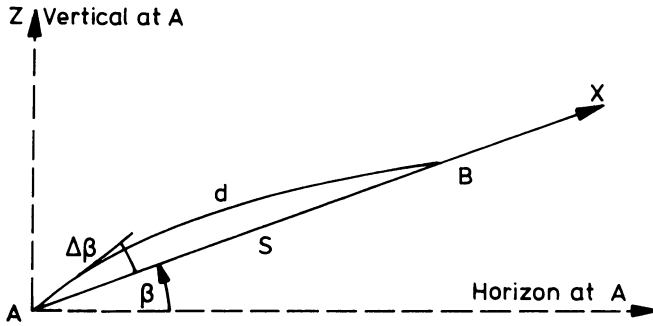


Figure 1 Wave path d and chord s

where s is the chord length AB,
 N is the refractivity = $10^6(n-1)$,
 X is distance along the chord,
 $\frac{dN}{dz}$ is the vertical gradient of refractivity,
 β is the vertical angle,
 ξ is an integration variable along the chord.

See Figure 1. The integrations are along the chord AB.

The determination of N and the refractivity gradients dN/dz will be discussed later. These will be values at discrete points. Rather than expressing them as functions of X for (5), numerical integration is used. Simpson's Rule is appropriate for the main integrals. However, for the minor integration with respect to ξ the Trapezoidal Rule is more suitable, as the number of ordinates is sometimes odd and sometimes even, and Simpson's Rule requires an odd number of ordinates; also, the correction is small.

If for example it is decided to divide the line into 6 sections, there will be 7 values of N : $N_1, N_2 \dots N_7$, where N_1 is the value at A and N_7 the value at B. The first integral is a straightforward application of Simpson's Rule. Before applying Simpson's Rule in the second term, each ordinate must be formed by integration and squaring. For example at the second ordinate $X = s/6$ and $\{(dN/dz)_\xi\}$ must be integrated from 0 to $s/6$, applying the Trapezoidal Rule.

4. TRIGONOMETRICAL HEIGHTS WITH VARYING k

It has been shown that if a line, length S , is divided into sections $s_1, s_2, s_3 \dots s_n$ with refraction coefficients $k_1, k_2, k_3 \dots k_n$, then with some minor restrictions, the trigonometrical refraction correction is given, to sufficient accuracy, by:

$$\Delta z = \frac{1}{2R} \{ k_1 s_1 (2S - s_1) + k_2 s_2 (2S - 2s_1 - s_2) + k_3 s_3 (2S - 2s_1 - 2s_2 - s_3) \dots \dots + k_n s_n^2 \} \quad (6)$$

The sections $s_1, s_2 \dots s_n$ need not necessarily be equal.

More rigorously, the vertical angle of refraction $\Delta\beta$, the angle between the tangent to the light ray and the chord AB (Figure 1), can also be derived from the eikonal equation:

$$\Delta\beta = \frac{10^{-6}}{S} \cos \beta \int_0^S \frac{dN}{dz} (S-X) dX \quad (7)$$

5. ATMOSPHERIC MODELS-GENERAL

Meteorological measurements at the end points of an EDM line can be expected to give representative values if observations are from towers and the line traverses over flat terrain with even surface conditions. However tower observations do not always give a higher accuracy, as noted by JONES (1971) from observations by the Geodetic Survey of Canada. The need for an atmospheric model can also be avoided by taking direct measurements along the line. Meteorological observations from planes set to fly along the EDM wave path are reported by MEADE (1969) and PRESCOTT and SAVAGE (1974). Such measurements have not been adopted more generally, probably because of the expense and the difficulties of organisation. Other possible problems are the low flight altitude required and uncertainties in the friction heating effect and time lag of the temperature sensor. The use of tethered balloons or kites for intermediate measurements has also been advocated, for example by MITTER (1962). In practice, the balloons and kites are troublesome and not compatible with sensitive electrical apparatus.

Many different atmospheric models have been suggested for refraction corrections. In order to provide a good representation, the model needs to take into account, directly or indirectly, variations arising from:

- daily and annual cycles of sun and temperature,
- amount of cloud cover,
- height above the surface,
- wind velocity,
- roughness and heat properties of the surface.

The first two factors might be covered by measurements of radiation.

Generally the models are based on measurements at special meteorological stations. The stations are carefully chosen in the midst of large areas of flat terrain with even surface conditions. In contrast, geodetic stations cannot be chosen for their meteorological suitability. The models could be systematically different from conditions above the typical geodetic station, which is on a hilltop. The models leave out

the important factors of topography, uneven surface conditions and the effects of movements of masses of air (advection). While it is true that schematic representations of temperature structure under typical geodetic lines are implied or shown by many writers, for example SCHÄDLICH (1975), ANGUS-LEPPAN (1967) and MAIER (1977), they are somewhat conjectural and may not be generally applicable.

It can be concluded that, at present, the horizontal variations of meteorological conditions are inadequately represented. It will be worthwhile in the future to undertake research towards the development of suitable models.

6. REFRACTION IN TRIGONOMETRIC HEIGHTS

The coefficient of refraction k has a long history. In the celebrated geodetic survey in Lapland, 1736-7, Maupertius made calculations which implied the use of the coefficient. In the 19th century Jordan compiled values from the surveys of various countries and derived a mean value which is still valid today. There is an extensive literature on investigations to find the value of k . In using it, we are assuming that the atmospheric gradient of refractivity is constant, and, in practice, that the temperature gradient is also constant.

Refraction posed an early challenge in determining the heights of the high Himalayan peaks. From the foothills they appeared to rise and fall 150 m per day. J. de Graaff-Hunter, at the time Mathematical adviser to the Survey of India, made investigations which enabled the refraction correction to be calculated with some precision (de GRAAFF-HUNTER, 1913).

In another field, precise levelling, the investigations of KUKKAMÄKI (1938) were important. He adopted a function for the temperature gradient,

$$dT/dz = a z^b \quad (9)$$

in which a is the temperature gradient at height 1 m and the exponent b was determined from a series of temperature observations by BEST (1935).

A fundamental contribution was that of BROCKS (1948). He adopted the same function as Kukkamäki, equation (9), for the gradient in a layer up to about 30 m, above which the gradient is adiabatic, $-1^\circ\text{C}/100\text{ m}$, for a few hundred meters. For daytime conditions, the index b in (9) is generally equal to -1 . Based on long series of temperature measurements in France, Germany, England and Egypt, Brocks deduced values of the parameters a and b for various heights, times of day and seasons of the year. These parameters also define the values of k for corresponding conditions.

The work of Brocks has been very influential in refraction research. Numerous investigators have adopted his functions as the basis of their atmospheric models. However, this influence did not extend into general practice. Brocks' work should have put an end to the concept of a single fixed value for k . For each observation it was possible to look up an appropriate value of k . Since 1948 many more sets of temperature observations have become available, enabling modified values to be determined for different localities and conditions. Unfortunately this did not happen, at least not in the English-speaking parts of the world, and the search for the elusive k continued.

At about the same time a group from the Institut Géographique National was investigating refraction in the French Alps. Their report, in 1953, introduced a model for atmospheric temperature as a function of height and time of day (LEVALLOIS and d'AUTUME, 1953). Fixed parameters in the function were determined from earlier radiosonde observations at a nearby weather station, and variable parameters from the temperatures at the geodetic station throughout the day of observation.

ANGUS-LEPPAN (1967) tried using the same model in flat terrain but found it needed modifications. His investigations resulted in a comprehensive function to model temperature gradients, but it required the determination of 10 constants.

Recent developments in technology have made it easier to measure temperature gradients, but the use of direct temperature gradient measurements in calculating refraction has not been successful. This is because of the large fluctuations in the temperature ($1 - 2^\circ$), and because local measurements are not representative of the line. Reciprocal vertical angles have been used for a long time, but it has only recently been realised that simultaneous reciprocal observations over short lines ($1 - 2$ km) have an accuracy almost up to that of third order levelling (BRUNNER, 1974).

Another model proposed is the Heat Balance-Turbulent Transfer Model (TTM). Unlike the earlier, basically empirical, models, the TTM has a basis in atmospheric physics and turbulence theory. It will be described in the following section on EDM. It is also applicable in vertical refraction. BRUNNER (1977) reports results in which the coefficient of refraction, which varied from -1.0 to $+0.6$ during a day, is calculated with a standard deviation of ± 0.13 by the TTM method. The method also represents rapid changes in k which occur when shadow moves over or away from the line.

7. ATMOSPHERIC MODELS IN EDM

In the 1960's the emphasis in refraction research shifted to EDM, where the basic problem is how to determine the representative temperature and humidity for a line, given the values at the end points. As early as 1961 it was realised that better meteorological data was the

key to improved accuracy in EDM (RINNER, 1961). An effective model for temperature and humidity in the lowest atmospheric layers should provide the solution, but it has not been easy to find.

Many functions have been tried. Brock's function, equation (9) has been applied, for example, by ADLUNG (1963), FELLETSCHIN (1978), LANG (1969), MAIER (1977), SCHÄDLICH (1975) and others. BRETTTERBAUER (1966) used a non-linear representation of the refractivity-height relationship to deduce the mean refractivity. KUNTZ (1970) developed a parabolic profile, termed the "spherical-parabolic-model". Generally the results have been inconclusive. SCHÄDLICH (1975), for example, claims that his "topographic-atmospheric reduction" gives good agreement with test data, but provides no evidence, while PARM (1967), representing his meteorological parameters as logarithmic functions of height, finds after comparison with normal reductions, that the technique does not give the expected improvement.

A useful review of research on microwave EDM is given by LANG (1969). He notes particularly that there is disagreement on three matters:

- the most favourable time of day for EDM observations,
- the determination of the representative refractivity,
- the minimum height of the line above the surface.

A number of writers have noted systematic differences between geodimeter (lightwave) and tellurometer (microwave) measurements, and between day and night measurements. Responsibility for these has been traced to systematic errors in the representative refractivity, by Bretterbauer and others. Extensive test measurements in Canada are described by JONES (1971). These were compared, and the systematic differences deduced. Applying the temperature and humidity observations of BEST et al. (1952), the expected systematic errors were calculated, and were found to be in good agreement with those from measured distances. Perhaps surprisingly, there has been no general agreement to apply corrections for such systematic errors, nor to adopt a model based on observations such as those of Best.

In 1962 Saastamoinen proposed the use of zenith distances, measured with the EDM, to improve the refraction correction. The concept of using the values of k to provide integrated information on the line, is appealing. The theory was fully developed by KUNTZ (1970) in conjunction with his spherical-parabolic model. Practical tests were conducted by DIECHL and REINHART (1969), who concluded that the use of zenith distances at one end only was not favourable, whereas reciprocal zenith distances gave the same result as the normal meteorological reduction, that is, no improvement.

The Heat Balance-Turbulent Transfer Model, put forward by Angus-Leppan and Webb (ANGUS-LEPPAN, 1971; ANGUS-LEPPAN and WEBB, 1971) differs from most other models in being based on atmospheric physics and also in being highly sensitive to a wide range of meteorological conditions at the time of observations. It is based on theories of turbulent transfer in the boundary layer to which contributions were made by Monin, Obukhov, Priestley, Panovsky, Webb, Deacon and many others.

Although the TTM has a different origin, it has some parallels with Brocks' model. A distinction has to be drawn between stable and unstable thermal stratification in the atmosphere. For unstable (daytime) conditions there are three strata, defined in terms of a stability parameter L , the Obukhov length. L fluctuates rapidly with time, varying as the cube of the wind speed and inversely as the upward heat flow due to turbulence. On the typical breezy summer day the lowest stratum might average up to 1 m, and the middle stratum, 1 to 30 m. In the upper stratum the temperature gradient is $-1^{\circ}/100$ m as in Brocks' model. The parameter b is -1 in all cases except the middle stratum where it takes the value -1.33 . The parameter a is different in each stratum, being a function of meteorological parameters, including those which determine turbulent heat flow (sun and sky radiation, cloudiness, heat properties of the surface, surface moisture etc.), wind velocity, surface roughness and air temperature. BRUNNER and FRASER (1977 A) have compared values from the model against test data, with very favourable results.

8. CONCLUSIONS

Effective atmospheric models for temperature and humidity are badly needed. Perhaps geodesists have not been collaborating sufficiently closely with meteorologists, and have not made sufficient use of existing meteorological data. Even if changes in measurement techniques do away with the need to determine refractivity along an EDM line, studies of the atmosphere will remain important in geodesy, since the atmosphere is the medium in which the geodesists make their observations. There is another aspect of geodesy-meteorology collaboration which should be borne in mind. Geodetic observations can be valuable in meteorology. They are very efficient in determining spatially integrated values of meteorological parameters.

9. ACKNOWLEDGEMENTS

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DISCUSSION

L. Hradilek: As far as the trigonometric leveling is concerned, is it satisfactory to measure the temperature gradient at the observation station only, or is it necessary to have some information about the gradients over the whole line of sight?

P.V. Angus-Leppan: Meteorologists can tell us quite a lot about the vertical gradients but not so much about variations in the horizontal direction. The typical site for meteorological research is specially selected with very flat terrain and very even surface conditions. In special circumstances, such as a mountain top, there may be information on the variations in the horizontal. In general not enough is known about the changes which will occur in vertical gradients as one moves from point to point in the horizontal.

L. Hradilek: When starting our refraction research, we measured at the station all basic meteorological data, especially the temperature grad-

ient by two independent methods. When introducing these values into the formulas of Jordan, we obtained a result which was 30 times larger than the actual variation in refraction determined by precise theodolite measurements.

P.V. Angus-Leppan: In general the direct measurements of temperature gradients have been very disappointing, because they are representative only of a single point. It is very difficult to get representative values over the length of a line whose height above the surface varies.

T.J. Kukkamäki: In Finland we observed daily periodic variation in the lateral direction of a 5 km long sight running rather low along a steep sideward slope. The difference between day and night reached 6 seconds of arc. We made efforts to derive this lateral refraction from vertical temperature gradient observations carried out simultaneously on the spot. Assuming that the isothermic layers ran parallel to the sloping ground surface, we calculated the horizontal component of the temperature gradient and on that basis its effect on the lateral direction of the sight. The correlation between the observed and the calculated refractions was good. (T.J. Kukkamäki: On lateral refraction in triangulation, Bull. Géod. No 11, 1949).

P.V. Angus-Leppan: There is a line in the Australian Geodetic Survey, where the angle varied by 13 seconds, in a daily cycle. It lies along a low bluff some distance from the sea. It is easy to measure the variation in lateral refraction, but very difficult to measure the absolute value.

T.J. Kukkamäki: Also in Finland we have carried out observations on lateral refraction along the seashore. The variation between day and night directions of a 30 km long sight was 2 seconds of arc. The sight ran parallel to the shoreline. The horizontal temperature gradient was determined from the temperature recordings at two stations, one on a small island and the other on land, situated symmetrically on either side of the sight. Correlation between the observed and the calculated refraction was as good here as on the slope.

B. Garfinkel: In making the calculation of terrestrial refraction, it is crucial that the choose of a good physical model. You suggested Brock's model with the parameters a and b . Now these parameters are presumably subject to fluctuations with time, depending on the time of the day and perhaps the season of the year. Instead of using formulas with constant parameters for such a dependence it would be practical to measure at the sight three values of temperatures along some vertical direction, and from these values calculate those parameters a and b for the particular time of the observation.

P.V. Angus-Leppan: I do not recommend Brock's model any more. It was good at the time, but the turbulent transfer model is much better. It is based upon theoretical and empirical research in the boundary layer

by meteorologists, and it comes close to a true representation of the physical processes that take place: the heat-balance at the surface and the transfer of heat upwards, taking into account the various factors such as the radiation intensity, wind and surface conditions.

B. Garfinkel: I have a question regarding that other model. Is that the one that involves the potential temperature, the temperature which you denote with letter θ ? You suggest there are some difficulties in using such an equation, because the potential temperature is not directly measurable in the field. Is that correct?

P.V. Angus-Leppan: No, it is just more convenient to use θ rather than T in the theory.

B. Garfinkel: What about the practise?

P.V. Angus-Leppan: One can convert from one temperature parameter to the other without difficulty.

B. Garfinkel: Do you have any reference to this other model, which involves the potential temperature?

P.V. Angus-Leppan: There are papers by Webb listed in the references to my paper.