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Representations of quivers over finite fields

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Let $\mathbb{F}_q$ be the finite field of q elements. Let Γ be a connected graph with vertices $\{1, 2, \cdots, n\}$, and let a_{ij} be the number of edges connecting i and j. Let $\Delta \subset \mathbb{Z}^n$ be the root system associated with Γ. For any $\alpha \in \mathbb{N}^n$, let $A_\Gamma(\alpha, q)$ be the number of classes of absolutely indecomposable representations of Γ (with respect to a fixed orientation) over $\mathbb{F}_q$ with dimension α. A theorem of Kac asserts that $A_\Gamma(\alpha, q) \neq 0$ if and only if $\alpha \in \Delta^+$. It is known that $A_\Gamma(\alpha, q)$ is a polynomial in q with integer coefficients; these have been conjectured to be non-negative by Kac. Thus, we may assume that $A_\Gamma(\alpha, q) = \sum_{i=0}^{u_\alpha} t_i^\alpha q^i$ with $t_i^\alpha \in \mathbb{Z}$.

Let $\mathcal{P}$ denote the set of all partitions. For $\lambda \in \mathcal{P}$, we let $\lambda' = (\lambda'_1, \lambda'_2, \cdots)$ denote the partition conjugate to λ. For any $\lambda, \mu \in \mathcal{P}$, we define $\langle \lambda, \mu \rangle = \sum_{i \geqslant 1} \lambda'_i \mu'_i$. For any $\lambda = (1^{n_1} 2^{n_2} \cdots) \in \mathcal{P}$, we define $b_\lambda(q) = \prod_{i \geqslant 1} (1-q)(1-q^2) \cdots (1-q^{n_i})$. Let $X_1, \cdots, X_n$ be n independent commuting variables, and for $\alpha = (\alpha_1, \cdots, \alpha_n) \in \mathbb{Z}^n$, we set $X^\alpha = X_1^{\alpha_1} \cdots X_n^{\alpha_n}$.

The main result in this thesis can be stated as the following formal identity:

$$\sum_{\lambda_1, \cdots, \lambda_n \in \mathcal{P}} \frac{\prod_{1 \leqslant i \leqslant j \leqslant n} q^{a_{ij} \langle \lambda_i, \lambda_j \rangle}}{\prod_{1 \leqslant i \leqslant n} q^{\langle \lambda_i, \lambda_i \rangle} b_{\lambda_i}(q^{-1})} X_1^{|\lambda_1|} \cdots X_n^{|\lambda_n|} = \prod_{\alpha \in \Delta^+} \prod_{i=0}^{\infty} \prod_{j=0}^{u_\alpha} \left(1 - q^{i+j} X^\alpha\right)^{t_j^\alpha}.$$

Kac has conjectured that if Γ does not contain edge-loops then the constant term of $A_\Gamma(\alpha, q)$ equals the multiplicity of α, which is defined to be the dimension of the root space corresponding to α of the Kac-Moody algebra determined by Γ. By assuming this conjecture, the above identity turns out to be a q-analogue of the Weyl-Macdonald-Kac denominator identity.

Another result in this thesis is as follows. Let A be a finite dimensional algebra over a perfect field $\mathbb{K}$, and let M be a finitely generated indecomposable module over

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$A \otimes_{\mathbb{K}} \overline{\mathbb{K}}$, where $\overline{\mathbb{K}}$ denotes the algebraic closure of $\mathbb{K}$. If there exists a module N over $A \otimes_{\mathbb{K}} \mathbb{E}$, where $\mathbb{E}$ is a finite extension of $\mathbb{K}$, such that $M \cong N \otimes_{\mathbb{E}} \overline{\mathbb{K}}$, then $\mathbb{E}$ is called a *field of definition* of M. It is proved that for each M there exists a unique indecomposable module $M^\dagger$ over A such that M is a direct summand of $M^\dagger \otimes_{\mathbb{K}} \overline{\mathbb{K}}$, and there exists a positive integer s such that $M^s = M \oplus \cdots \oplus M$ (s copies) has a unique minimal field of definition which is isomorphic to the centre of the division algebra $(\mathrm{End}_\Gamma(M^\dagger)) \big/ \big(\mathrm{rad}(\mathrm{End}_\Gamma(M^\dagger))\big)$. If $\mathbb{K}$ is a finite field, then s can be taken to be 1.

Part of this thesis has already been published in [**1**].

References

[1] J. Hua, 'Generalizing the recursion relationship for the partition function', *J. Combin. Theory Ser. A* **79** (1997), 105–117.

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