7. OB STELLAR WINDS

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THE WIND-COMPRESSED DISK MODEL

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Abstract. We discuss the effects of rotation on the structure of radiatively-driven winds. When the centrifugal support is large, there is a region, at low latitudes near the surface of the star, where the acceleration of gravity is larger than the radiative acceleration. Within this region, the fluid streamlines "fall" toward the equator. If the rotation rate is large, this region is big enough that the fluid from the northern hemisphere collides with that from the southern hemisphere. This produces standing shocks above and below the equator. Between the shocks, there is a dense equatorial disk that is confined by the ram pressure of the wind. A portion of the flow that enters the disk proceeds outward along the equator, but the inner portion accretes onto the stellar surface. Thus there is *simultaneous* outflow and infall in the equatorial disk. The wind-compressed disk forms only if the star is rotating faster than a threshold value, which depends on the ratio of wind terminal speed to stellar escape speed. The spectral type dependence of the disk formation threshold may explain the frequency distribution of Be stars. Observational tests of the wind-compressed disk model indicate that, although the geometry of the disk agrees with observations of Be stars, the density is a factor of 100 too small to produce the IR excess, Hα emission, and optical polarization, if current estimates of the mass-loss rates are used. However, recent calculations of the ionization balance in the wind indicate that the mass-loss rates of Be stars may be significantly underestimated.

1. Introduction

Rapid rotation has an important effect on the structure and lives of many stars. There is gravity darkening at the equator according to the von Zeipel theorem. Rotation affects the internal structure, which changes the non-radial pulsation modes, as well as the subsequent evolution of the star. If there is a magnetic field, angular momentum is transported by the stellar wind, causing the star to "spin down". The most extreme effects of rotation are seen in the Be stars. These stars, which are among the most rapidly rotating stars, are thought to have a dense equatorial disk in their circumstellar envelope (see the review by Waters elsewhere in these Proceedings). However, the connection between rotation and the formation of this equatorial disk is poorly understood. Since rotation and mass-loss affect the structure and evolution of massive stars, and since it leads to the formation of equatorial disks, it is important to understand the effects of rotation on the structure of the stellar wind.

2. Structure of Rotating Winds

In this section we review recent results concerning the two-dimensional structure of the winds from rotating stars. In particular, we discuss the Wind-Compressed Disk (WCD) model of Bjorkman & Cassinelli (1992, 1993) as

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well as the time-dependent hydrodynamics simulations of Owocki, Cranmer & Blondin (1993).

In a rotating 2-D axisymmetric model, the streamlines are not radial, but instead bend toward the equator due to the centrifugal and Coriolis forces. To determine the structure of the wind, we must solve the fluid equations and find the location of the resulting streamlines.

For simplicity we assume a steady-state isothermal wind. With these assumptions, the continuity and momentum equations are

$$\nabla_j\left(\rho v^j\right) = 0\,, \tag{1}$$

$$\nabla_j\left(\rho v^j v_i\right) = -a^2\partial_i\rho + \rho F_i\,, \tag{2}$$

where ∇ is the covariant derivative, ρ is the fluid density, v is the fluid velocity, a is the isothermal sound speed, and F is the external force per unit mass. These equations are quite difficult to solve; however, an enormous simplification occurs in the supersonic portion of the flow.

2.1. Fluid Equations in the Supersonic Limit

Consider the forces acting on the fluid. For an axisymmetric geometry, the pressure gradient only has r- and θ-components. Although the θ-component is large at the stellar surface (to enforce hydrostatic equilibrium), it drops rapidly beyond the sonic radius, r_s. The other forces are gravity and radiation, which are central forces. Thus, beyond the sonic point, there are no external torques, so both the θ- and ϕ-components of the velocity are determined by angular momentum conservation. This implies that v_θ and v_ϕ are $O(V_{\rm rot}R/r)$, where R is the stellar radius. Typically for an early-type star, the rotation speed, $V_{\rm rot}$, is highly supersonic. So, as long as $r \gg r_s$ and $r \not\gg R$, all three velocity components are highly supersonic.

Note that the left hand side of the momentum equation (2) is $O(v^2)$, but the pressure gradient on the right hand side is $O(a^2)$. As long as all three velocity components $v_r, v_\theta, v_\phi \gg a$, then we may completely ignore the pressure gradient. If there are no pressure forces, there are no interactions between the individual fluid particles. This implies that the streamlines are free particle trajectories corresponding to the external forces. To find the location of the streamlines, we simply integrate Newton's equations of motion using gravity and radiation for the forces.

2.2. Motion in the Orbital Plane

Much can be learned about the location of the streamline by recalling that gravity and radiation are central forces. Therefore, the *total* angular momentum is constant along a streamline. Consequently, the streamline lies in a plane (shown in Fig. 1) that contains the initial radius and velocity, V_0.

Fig. 1 shows two trajectories labeled (a) and (b), that correspond to different initial conditions. Trajectory (a) has a slow initial acceleration and

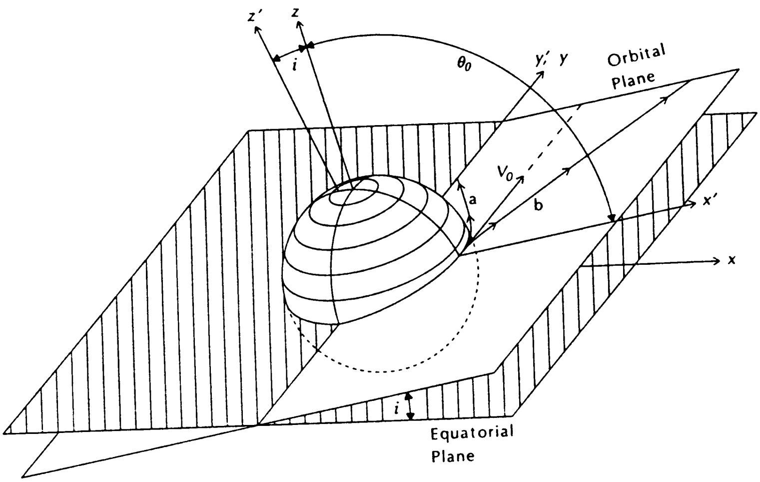

Fig. 1. Orientation of the orbital plane for a streamline originating at a polar angle θ_0. The streamline labeled (a) is a case with a high rotation rate and the streamline labeled (b) denotes a low rotation rate. (Figure from Bjorkman & Cassinelli 1993).

occurs when there is a large rotation rate. Trajectory (b) has a fast initial acceleration and occurs when there is a slow rotation rate. Note that as trajectory (a) wraps around the star, it has a decreasing altitude, z, and eventually crosses the equator. Conversely, trajectory (b) deflects outward and has an increasing altitude.

The curvature of the streamline depends on the forces and is most easily understood in the non-rotating reference frame. In this frame, there are only two forces, and each is in the radial direction; gravity points inward, and the radiation force points outward. To produce a net force with a negative z-component, we must have $F_{\rm grav} > F_{\rm rad}$. Thus, the equator-crossing trajectory (a) corresponds to initial conditions where the force of gravity exceeds the radiation force, and trajectory (b) occurs when the radiation force is larger than gravity.

2.3. Forces in Rotating Winds

The location where the radiation force exceeds gravity depends on the subtle interaction of the radiation force with the velocity gradient, dv_r/dr. In the orbital plane, the r-component of the momentum equation is

$$v_r \frac{\partial v_r}{\partial r} = -\frac{a^2}{\rho}\frac{\partial \rho}{\partial r} + F_{\rm grav} + F_{\rm rad} + \frac{{v_\phi}^2}{r} \ . \tag{3}$$

The last term on the right hand side is the centrifugal force, so the velocity gradient, dv_r/dr, is determined in the *rotating* reference frame. To maintain an outward flow, a line-driven wind must constantly accelerate to higher velocities, so that there is always a supply of unattenuated stellar photons.

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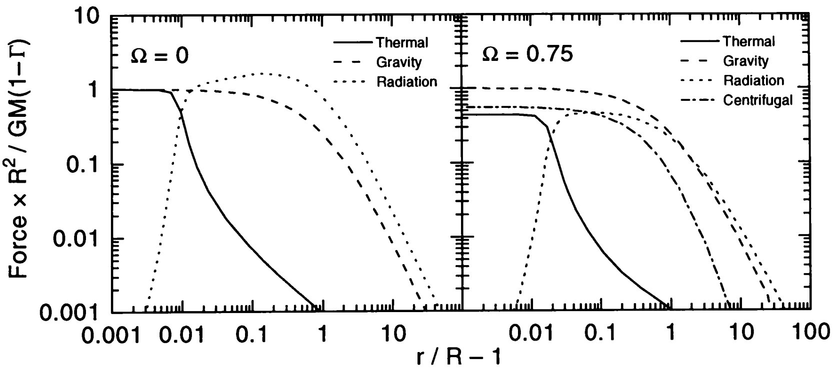

Fig. 2. **Forces vs. radius in a Friend & Abbott (1986) 1-D equatorial rotating wind model that has $v_\infty/v_{esc} = 1.4$. Shown are a non-rotating case, $\Omega \equiv V_{rot}/V_{crit} = 0$, and a rapidly rotating case with $\Omega = 0.75$. (Figure from Bjorkman & Cassinelli 1993).**

Since the radiation force depends on dv_r/dr, the velocity adjusts until the radiation force maintains a positive dv_r/dr, and the velocity increases monotonically.

Fig. 2 compares the forces for rotating and non-rotating winds. In the non-rotating case, $\Omega \equiv V_{rot}/V_{crit} = 0$, thermal pressure supports the flow out to the sonic radius, $r_s \approx 1.01R$. Beyond the sonic point, the thermal pressure support is negligible; therefore, the radiation force must increase until it is larger than gravity (so that dv_r/dr is positive). Note that the radiation force exceeds gravity at all locations beyond the sonic point. In the rapidly rotating case, $\Omega = 0.75$, most of the support is instead from the centrifugal force. When the thermal pressure support is lost at the sonic point, the radiation force again increases to supply the missing force, but because of the large centrifugal support, the required amount is smaller than gravity. The centrifugal support falls as $1/r^3$ (much slower than the thermal pressure) and it is not until the centrifugal support is lost that the radiation force finally exceeds gravity at about $3R$. Thus there is a region between the sonic point and about $3R$ where gravity is larger than the radiation force. Within this region, the streamlines fall toward the equator, and if the outer radius of this region is large enough, the streamlines attempt to cross the equator.

2.4. Streamlines

To build a model for the entire 2-D structure of the wind, we calculate the shape of the streamlines as a function of initial latitude on the surface of the star. If the star is not rotating, then the streamlines are entirely in the radial direction. On the other hand, if the star is rotating, then the streamlines fall toward the equator between the sonic point and the location where the radiation force exceeds gravity (see Fig. 3). Near the pole, the

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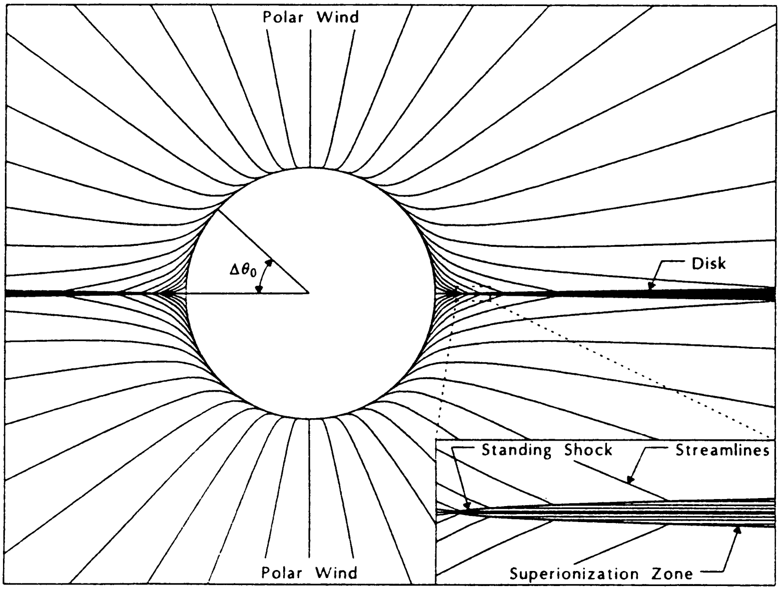

Fig. 3. Diagram of the stellar wind and wind-compressed disk. Shown are the wind streamlines, which fall toward the equator. The expanded view shows the standing shocks that form above and below the disk. (Figure from Bjorkman & Cassinelli 1993.)

rotation velocity is small, so the streamlines are radial. But for streamlines closer to the equator, the rotation velocity is higher, the region where gravity exceeds the radiation force is larger, and the streamlines fall farther before turning in the radial direction. If the equatorial rotation rate of the star, V_{rot}, is above a threshold value, V_{th}, then for latitudes less than $\Delta\theta_0$, the streamlines attempt to cross the equator (see Fig. 3). This equator-crossing latitude, $\Delta\theta_0$, is approximately given by $V_{rot} \sin \Delta\theta_0 \sim V_{th}$.

When the streamlines cross the equator, they collide with the streamlines from the opposite hemisphere of the star. Streamlines cannot cross because the density diverges. Instead, the increase in density causes a large pressure gradient, and since the flow velocity (perpendicular to the equator) is supersonic, a pair of shocks form above and below the equator. The pressure at the equator must balance the ram pressure of the wind, so between the shocks there is a dense equatorial disk. Additionally, the shock temperature is large enough (a few 10^5 K) that the disk is bounded by a thin superionization zone (shown in Fig. 3). Note that this disk forms only when the star is rotating faster than the equator-crossing threshold.

The disk formation (equator-crossing) threshold depends on the ratio of the terminal speed of the wind to escape speed of the star, v_∞ / v_{esc}. This is because a faster wind implies a larger radiative acceleration, which decreases the size of the region where the streamlines fall toward the equator. To compensate, the stellar rotation rate must be increased. Thus the disk formation threshold increases with increasing v_∞/v_{esc}.

2.5. Time-Dependent Hydrodynamics

The Wind-Compressed Disk (WCD) model developed by Bjorkman & Cassinelli (1993, hereafter BC) is only valid in the supersonic region of the flow, and thus requires initial conditions at the sonic point. To obtain these initial conditions, BC assume that the subsonic expansion is in the radial direction, i.e., θ is constant and $v_\theta = 0$ for $r < r_s$. Another approximation BC employ is to assume a shape for the WCD shock surface, because the actual shape depends on the detailed dynamics of the disk.

To assess the validity of the WCD approximations and to examine in detail the dynamics of the disk, Owocki, Cranmer & Blondin (1993, hereafter OCB) developed a 2-D time-dependent numerical simulation of the wind from a rotating star. Aside from properly including shocks and gas pressure, OCB also included an oblate lower boundary condition that accounts for the rotational distortion of the star. Starting with a wind that is initially spherically symmetric, OCB find that, after about 50000 s, the time-dependent solution relaxes to a steady-state solution with a thin equatorial disk (see Fig. 4).

The qualitative appearance of the disk agrees quite well with that predicted by BC. The thickness of the disk is about 3° in latitude (BC predicted 0°.5), and the disk density is about two orders of magnitude higher than the density at the pole, which is somewhat lower than predicted by BC. Interestingly, a weak disk persists even at rotation rates below the rotation threshold predicted by BC.

At first, quantitative comparison of the velocity of the wind (before entering the disk) did not agree with the analytic results of BC. The discrepancy arose because BC integrated their streamline trajectories starting at the stellar surface, $r = R$, instead of the sonic radius, $r = r_s \approx 1.01R$. Surprisingly, this small correction lowers the θ-component of the velocity by about a factor of two. After correcting this, BC's analytic approximations are in good agreement with OCB's numerical results (see Fig. 12 of Owocki *et al.* 1993); however, many details (mostly concerning the properties of the disk) are somewhat different in the numerical simulations.

There are two fundamental differences between OCB's results and the predictions by BC. Firstly, the disk is not detached from the stellar surface (compare Figs. 3 and 4). Secondly, there is a stagnation point in the disk. Exterior to the stagnation point, the disk material flows outward. Interior to the stagnation point, the material falls back onto the stellar surface. Thus there is simultaneous outflow and infall in the disk.

In the previous sections, we examined the structure of a radiatively-driven rotating wind and discussed the conditions whereby an equatorial disk forms.

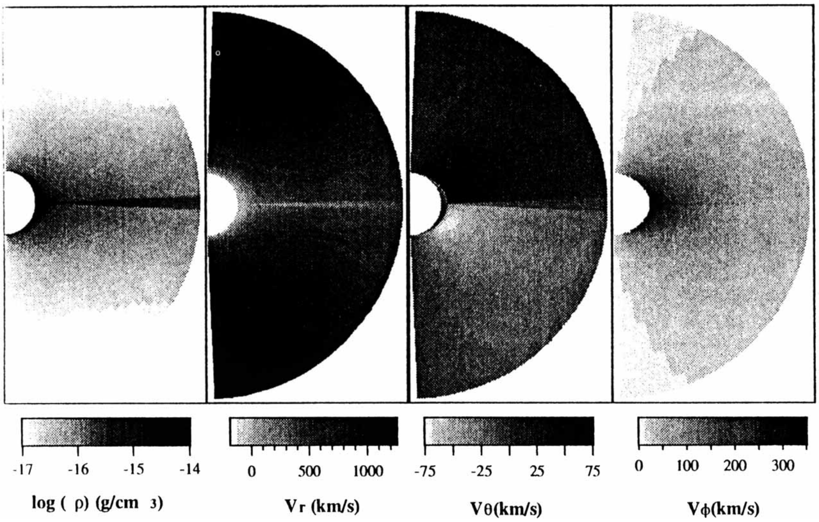

Fig. 4. Time-dependent numerical simulation of the wind-compressed disk for a B2.5 V star with $V_{rot} = 350$ km s^{-1}. Shown (from left to right) are the density and the r-, θ-, and ϕ-velocity components. (Figure from Owocki *et al.* 1993).

We have seen that the formation of a thin disk is inevitable, *if* the terminal speed of the wind is slow enough, *and if* the star is rotating fast enough. We suspect that disk formation in rotating winds is a general feature that occurs whenever there is the combination of rapid rotation and slow initial acceleration. In the next section, we explore the possibility that wind-compressed disks may be responsible for the Be star phenomena.

3. Application to Be Stars

To establish that Be stars might have wind-compressed disks, we must first determine if the stars are rotating faster than the disk formation threshold.

3.1. Rotation Threshold

The disk formation threshold depends on the ratio of the wind terminal speed to stellar escape speed. Fig. 5 shows both the observed and theoretical values of the wind terminal speed ratio as a function of main-sequence spectral type. The observed and theoretical values mostly agree for O stars; however, for late B stars, the observed terminal speeds are significantly lower than the theoretical terminal speeds. The observed terminal speeds of B stars are estimated from the edge-velocities of the C IV and Si IV line profiles (K.S. Bjorkman 1989). Typically these profiles are quite weak, so it is quite likely that the observed B star terminal speeds are systematically underestimated. On the other hand, there are many uncertainties in the the-

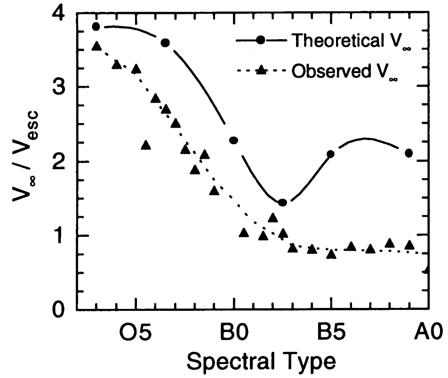

Fig. 5. Ratio of wind terminal speed to stellar escape speed vs. spectral type. The solid curve is obtained from theoretical calculations of the terminal speed, and the dashed curve is from observations. (Figure from Bjorkman & Cassinelli 1993.)

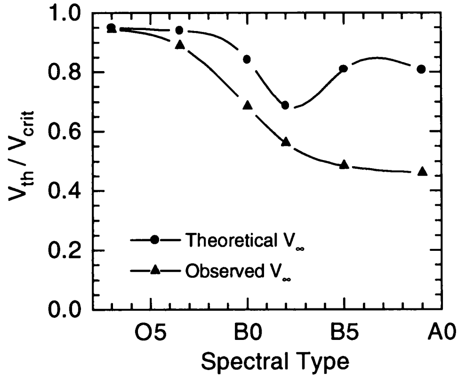

Fig. 6. Disk formation threshold vs. spectral type. The curve with filled circles is the threshold obtained using the theoretical values of the terminal speed, and the curve with triangles employs the observed terminal speeds.

oretical calculations. At this time, it is unclear which terminal speeds are more reliable for B stars.

Using the terminal speeds shown in Fig. 5, we find the disk formation threshold, shown in Fig. 6, as a function of spectral type. Note that these (and all subsequent) results contain the correction to the streamline locations found by OCB that is discussed in section 2.5. Assuming that the winds of B stars are radiatively-driven, Fig. 6 indicates that any B star that rotates faster than the rotation threshold will have a thin equatorial disk. Since observations of Be stars seem to indicate that they possess a dense equatorial disk (see the review by Waters in these Proceedings), one is tempted to conclude that wind-compressed disks may be responsible for the Be phenomena. Note that the rotation threshold has a minimum at B2, if one uses the theoretical terminal speeds. This minimum may qualitatively explain the frequency distribution of Be stars, i.e., why Be stars are most common at a spectral type of B2.

To investigate more quantitatively whether or not the WCD model can explain the Be phenomena, we must determine the properties of the disk. For a concrete example, we present results for a B2 V star with a mass-loss rate of $10^{-9}\ M_\odot\ \mathrm{yr}^{-1}$ and a terminal speed ratio $v_\infty / v_{esc} = 1$.

3.2. Shock Temperature and Disk Density

The WCD shock temperature is determined by the shock velocity, which is approximately the θ-component of the velocity of the wind when it enters the disk. The shock temperature, shown in Fig. 7, is typically a few 10^5 K, which is large enough to produce C IV and Si IV by collisional ionization.

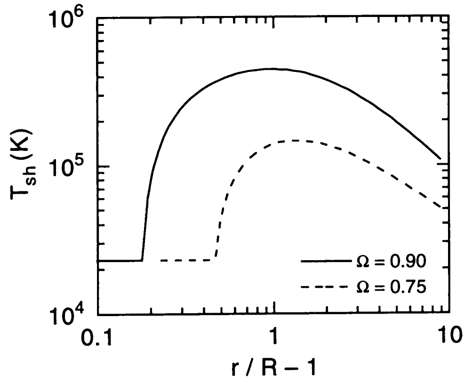

Fig. 7. Shock temperature as a function of radius for two values of the rotation rate, Ω. Note the increase in the maximum shock temperature as the rotation rate increases.

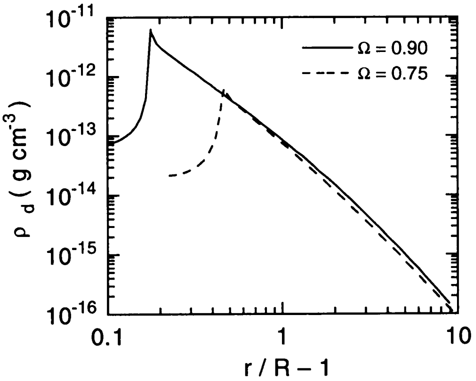

Fig. 8. Disk density as a function of radius for two values of the rotation rate, Ω. Note that the slope of the curve indicates that the density falls as $(r/R-1)^{-3}$.

Thus the disk is bounded above and below by a thin superionization layer. Note that the maximum shock temperature depends on the stellar rotation rate, so more rapidly rotating stars will produce higher ionization states in the WCD shocks. In addition to the shocks that produce the disk, there is also an accretion shock (due to the disk inflow) in the equator at the stellar surface. The disk infall velocities are typically a few hundred km s^{-1}, so the accretion shock can have temperatures of order 10^6 K and may produce soft X-rays. Note that the temperature due to the accretion shock is not shown in Fig. 7. Since all of these shocks can produce superionization, they may be responsible for the excess superionization of Be stars compared to normal B stars (Grady, Bjorkman & and Snow 1987; Grady *et al.* 1989).

The disk density may be estimated by equating the gas pressure in the disk with the ram pressure of the wind entering the disk. Assuming that the shocked material that enters the disk cools to the same radiative equilibrium temperature as the stellar wind (OCB's numerical simulations indicate that the shock cooling length is thinner than the disk), we find $\rho_{\rm disk} \approx \rho_{\rm wind}[1 + (v_\theta/a)^2]$, where the right hand side is evaluated in the wind just prior to the shock that forms the disk.

The disk density is shown in Fig. 8. Note that the disk density is $\rho = \rho_0(r/R-1)^{-n}$, where $\rho_0 \approx 10^{-13}$ g cm^{-3} and $n \approx 3$. Using the slope of the IR free-free continuum excess, Waters, Coté & Lamers (1987) estimated values of n in the range of 2–3.5; however, the required disk density is $\rho_0 \approx 10^{-11}$ g cm^{-3}. Thus the WCD model predicts the correct radial distribution of material, but if a polar mass-loss rate of 10^{-9} $M_\odot$ yr^{-1} is assumed, it produces a density that is about two orders of magnitude too small.

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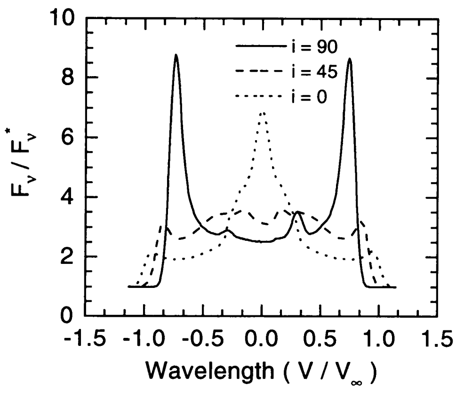

Fig. 9. Hα line profile produced by a wind-compressed disk for three different inclination angles, i. The polar mass-loss rate of 2×10^{-7} $M_\odot\,\mathrm{yr}^{-1}$ was chosen to produce a peak intensity (relative to the continuum) that is typical of a strong Hα emission line produced by a Be star.

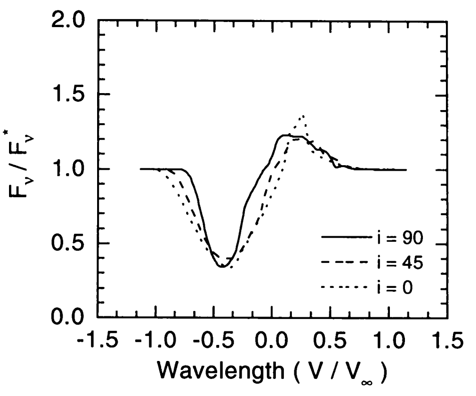

Fig. 10. C IV line profile produced by a wind-compressed disk for three different inclination angles, i. The polar mass-loss rate of $\dot{M}q = 5\times10^{-10}$ $M_\odot\,\mathrm{yr}^{-1}$ was chosen to produce an absorption depth (relative to the continuum) that is typical for a Be star.

3.3. Observational Predictions

Similarly, the optical linear polarization and Hα emission produced by the disk are too small. The optical polarization of a typical Be star is a few tenths of a per cent, but the wind-compressed disk for our example star produces only about 0.004 per cent. Since the polarization is linearly proportional to the disk mass, the disk must contain about 100 times more material. Preliminary calculations of the Hα emission are shown in Fig. 9 (the small bumps are numerical artifacts). The edge-on ($i = 90°$) profile shows more or less symmetric, double-peaked structure (characteristics that are often observed in Be stars); however, to produce as much emission as is shown requires a polar mass-loss rate of 10^{-7} $M_\odot\,\mathrm{yr}^{-1}$, which is again a factor of 100 times larger than our example star.

The IR excess, Hα, and optical polarization observations all indicate that the disk density must be larger. To find out whether there is too little shock compression or instead too little mass-loss from the star itself, we have examined synthetic UV line profiles for the WCD model. The UV lines are sensitive to the material in the polar portion of the flow, so they provide a measure of the polar mass-loss rate. Fig. 10 shows a WCD model C IV line profile. The polar mass-loss rate required is only $\dot{M}q = 5 \times 10^{-10}$ $M_\odot\,\mathrm{yr}^{-1}$, where q is the ionization fraction. This agrees with Snow's (1981) mass-loss rates as well as the mass-loss rate used for our example star, *if* C IV and Si IV are the dominant ionization states in the wind (so that $q \sim 1$).

Recently, MacFarlane (elsewhere in these Proceedings) has calculated NLTE ionization fractions for B stars. He includes an X-ray source that

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matches the emission measure inferred from *ROSAT* observations. Assuming an X-ray luminosity, $L_X = 10^{29}$ erg s^{-1}, and temperature, $T_X = 10^6$ K, for our example star, he finds that Si V and C III are the dominant ionization states in the polar regions and that the ionization fractions of Si IV and C IV are of order 0.1. Additionally, the ionization fractions are sensitive enough to the density that it is quite likely that there are ionization gradients from the pole to equator. With these considerations, it appears that the mass-loss rates of Be stars could be underestimated by an order of magnitude or more, depending on the X-ray luminosity and temperature.

This is still one order of magnitude less than required for producing the IR excess, Hα emission, and optical polarization observations with a wind-compressed disk. However, increasing the mass-loss rates would reduce the radiative line-driving force, and might decrease the initial acceleration of the wind. If this is the case, then the streamlines would fall closer to the stellar surface, the θ-component of the velocity would be larger, and there would be more compression of the disk. Furthermore, the higher disk density would decrease the expansion velocity of the disk, which in turn increases the total mass of material in the disk.

4. Concluding Remarks

We have seen that rapid rotation of a B star leads naturally to the formation of a wind-compressed disk. Whether or not this disk causes many of the Be phenomena remains to be seen. It can only explain Be stars if there is a mechanism that increases the mass of the disk. Some possibilities are: 1) the mass-loss rate of the underlying star might be larger than previously thought, 2) the velocity in the disk might be smaller if the radiation-driving forces have been over estimated, and 3) a weak magnetic field might transfer angular momentum to the disk, stop the infall of material, and instead store the material in a Keplerian disk.

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Discussion

Hummel: How can you get symmetric Hα line profiles from an expanding disk?

Bjorkman: There is infall as well as outflow in the disk, and the maximum infall velocity is about the same as the maximum outflow velocity. This produces more or less symmetric absorption.

Hubert: Can you explain the presence of shell lines in Be stars with moderate $v \sin i$?

Bjorkman: I agree that it is difficult to explain shell stars with a thin disk for two reasons. Firstly, unless the inclination angle of the star places the observer within the opening angle of the disk, a thin disk can cover at most half of the solid angle of the star. So, even if the disk has infinite optical depth, it cannot produce profiles that are more than 50 per cent black. If the observer is instead within the opening angle of the disk, then only the disk material at large radii completely covers the stellar disk. Secondly, the shell lines are narrow and at low velocity. This implies that they would have to be formed only near the stagnation point in the disk—not at large radii. Perhaps shell stars, which are only a small fraction of Be stars, are formed by a completely different mechanism, or, since some Be stars change between B, Be, and Be-shell phases, perhaps the mechanism that creates the variability forces the disk into different configurations at different times.

Percy: Given that there is now a mechanism for producing a disk, what is the role of non-radial pulsations (NRP's), which are found in most Be stars? Do the NRP's enhance the Be phenomena, could the NRP's be caused by the disk infall, or are the two unrelated?

Bjorkman: If NRP's increase the mass-loss rate from the star, they will increase the disk density and enhance the Be phenomena. Whether or not they could supply the extra mass-loss needed by the WCD model is unclear. Looking at the disk connection to NRP's, I suppose the disk infall could act as a small noise source which might excite NRP's, but the mass involved in infall is tiny compared to the mass involved in the pulsations and it is doubtful that the disk infall would be "tuned" to any particular frequency.

Smith: Of course the devil lies in the details. There are a few problems with the actual numbers. Firstly, Ω for Be stars is less than about 0.5; very few Be stars show $\Omega > 0.7$. Secondly, as you point out, there is a problem with the densities of the disk. Third, the episodic appearance of Be outbursts and disks is even more important. I would like to suggest that all these problems together do not negate the WCD. They merely argue strongly that a time-dependent "something else" is needed to make the WCD model work.

Bjorkman: First of all, Owocki's simulations show a weak disk even when $\Omega = 0.5$, which is less than the equator-crossing threshold of the WCD model. This indicates that the disk formation thresholds are somewhat too large. If you believe that Owocki's results support the basic WCD scenario, then many (if not all) Be stars will have a wind-compressed disk, whether we like it or not. The real question is whether the disk densities are large enough to explain the Be phenomena. I would suggest that your "something else" is the mechanism that increases the mass of the disk. Finally, as far as time variability is concerned, there are many possibilities including: time-dependent mass-loss from the star, changes in the ionization balance that affect the driving forces in the wind, and stability of the disk itself.

Harmanec: Firstly, I found your model quite convincing and acceptable; however, since one can find B absorption stars (usually classified as Bn) with $v \sin i > 300$ km s^{-1}, why are all rapidly rotating stars not Be stars? Secondly, since you find infall in the inner part of the disk, it becomes even more difficult to observationally distinguish accretion disks from disks created by outflow.

Bjorkman: Remember that the disk formation threshold depends on the spectral type of the star, but more fundamentally on the ratio v_∞ / v_{esc}. Before concluding that any given star is rotating faster than the disk formation threshold, one must also determine the terminal speed of its wind. If it turns out that some of these Bn stars are rotating faster than their threshold, then a disk will be present. This would indicate that something beyond the mere presence of the disk is required for making a Be star. As far as distinguishing between accretion disks and wind-compressed disks, I agree that it would be useful to find an observational indicator that differentiates the two. Certainly there is more than one way to make a disk, and some Be binaries must have accretion disks.

Lafon: One should be very cautious when trying to observationally distinguish between disks with or without infall, because the shock is probably subject to instabilities—both sides of the strong shock have opposing flow at large velocities, which probably produces two-stream instabilities.

Bjorkman: The question of the stability of the disk is very interesting. Firstly, if the disk is unstable, then the instability itself could cause the notorious variability of Be stars. Secondly, the instability would likely impose a very complicated multiple shock structure that propagates outward through the disk and thus alters the geometry and density of the disk. This in turn affects the radiation driving forces in the disk, which changes the disk velocity and indirectly changes the disk thickness. Thus the disk could be "clumpy" and geometrically thicker, which would help with the shell star problem.

Waters: The WCD model predicts infall near the star with high velocities.

If this is the case, then the optical and IR emission lines should have very different shapes, but this is not observed. So probably infall does not occur.

Bjorkman: We need to calculate the IR hydrogen line profiles for the WCD model. If the difference between the optical and IR hydrogen lines can prove that there is no disk infall, it would be a very important constraint, because one suggestion for increasing the disk density is to stop the inner disk inflow.

Dougherty: Waters mentioned in his talk that the far IR observations may indicate a flaring (increase in opening angle) of the disk at about 5–$10R_*$. Likewise the radio spectrum can be explained by a flaring of the disk at somewhat larger radii. At what radius does the WCD begin to "flare"?

Bjorkman: The wind streamlines enter the disk at very oblique angles when r becomes large, so the disk will begin to "flare" when the ram pressure of the wind becomes small. Flaring can be seen as soon as few stellar radii ($\sim 3R$) in the highest rotation rate cases in Owocki's simulations.

Lafon: The expression for the CAK radiation force has been determined assuming spherical symmetry and includes many lines. It is probably highly perturbed close to the shock and stagnation point.

Bjorkman: There are many simplifications that have been employed in the radiation-driving force. An example arising from the lack of spherical symmetry is that the azimuthal velocity gradient produces an asymmetry in the shape of Sobolev surface. The resulting asymmetry in photon escape probabilities produces a radiative torque acting on the fluid. As you point out, the ionization balance in the wind could play an even more important role. Our calculations so far have assumed that the CAK force multiplier parameters (k, α, δ) are constants independent of position. I think one of the potentially most important effects that has been neglected is how changes in the ionization balance change the CAK parameters as a function of position.

Moss: Can you make any qualitative comments (e.g., disk thickening) about the possible effects of a modest (unobservable) magnetic field in your model?

Bjorkman: A weak magnetic field might be the missing ingredient needed to increase the disk mass. One problem with the disk is that it leaks—material falls back onto the star. To plug the leak requires adding angular momentum to the disk material so that it is rotationally supported. A magnetic field is one of the few ways to exert a torque on the fluid. There are many advantages to producing a Keplerian disk, namely the long timescales associated with the variability. One interesting scenario involves wrapping up the magnetic field lines in the rotating disk. Eventually the magnetic stresses might disrupt the disk causing large scale changes.