THE AXISYMMETRIC STABILITY OF THE MILKY WAY

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1. Motivation

The problem of axisymmetric Jeans stability of a stellar disk was essentially completely solved in the classic paper by Toomre (1964). While his analysis was strictly local, the stability criterion it yielded has been found to be remarkably reliable in global studies; I review earlier results and give a further example in Sellwood (1995, hereafter S95). Toomre (1974) concluded that the MW is safely stable to axisymmetric Jeans modes.

Here, however, I am principally concerned with bending instabilities which were again first considered by Toomre (1966) also using a local approximation. He showed that random motion can cause an infinitesimal corrugation to grow exponentially, but that the instability can be shut off by gravitational restoring forces on large scales and by finite thickess on small scales. His analysis, and one more detailed by Araki (1985), indicated that the vertical random velocities need be no larger than some 30% of the in-plane random velocities for a uniform slab to be stable.

Large-scale bending instabilities have been reported in a number of recent N-body simulations of thin stellar systems, which ultimately caused them to thicken. At first this behavior was seen only in bars (e.g., Combes & Sanders 1981; Combes *et al.* 1990; Raha *et al.* 1991; Merritt & Hernquist 1991) but Sellwood & Merritt (1994, hereafter SM) and Kalnajs (unpublished) found that large-scale bends could also occur in axisymmetric disks. The instability had been missed in many other simulations either because the particles were confined to a plane, or random motions were too small to drive any but small-scale instabilities, which are easily suppressed by a modest disk thickness or poor spatial resolution. The bending instability can, however, even outpace the more familiar bar instability in a fully self-

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gravitating disk with a single directly rotating population, when random motions are large enough.

Merritt & Sellwood (1994, hereafter MS) showed that the Toomre-Araki stability criterion breaks down when the dominant bending mode has a radial wavelength comparable to, or greater than, the scale on which the surface density varies, and argued that it fails because the gravitational restoring force at the disk center arising from the displaced outer disk is weaker than the Toomre-Araki analysis assumes. From much higher quality simulations, but restricted to axial symmetry, I (S95) was able to show that the local stability criterion fails significantly only where pressure support is comparable to, or exceeds, that from rotation.

MS concluded that the bending instability precludes the existence of elliptical galaxies more flattened than about E7, as Fridman & Polyachenko (1984) had speculated. On the other hand, only the very centers of most disk galaxies would have enough in-plane random motion to thicken significantly; the instability therefore offers a potentially important process for bulge formation, as noted in the above cited papers.

While the local stability criterion is not strongly violated in rotationally supported parts of the disk, it seemed to MS at least possible that some overshoot may occur on all scales. They therefore wondered whether the Milky Way could after all be locally close to the bending stability boundary and that the current thickness might have been set by bending instabilities, rather than through some other mechanism. This consideration prompted the present study in which, however, I find that these suspicions were unfounded and that Toomre (1966) concluded correctly that "the Galaxy is well clear of this stability boundary" also.

2. A Milky Way Model

The Kuz'min-Toomre disk I used in S95 is a poor model of the Milky Way; I have therefore tried here to simulate a more realistic model, both to verify the stability of the MW directly and also to determine the scale height of the local disk if bending instabilities were the sole thickening agent.

The model for the Milky Way I have used in this study is based on a Rybicki (unpublished) disk model which has the surface density distribution

$$\Sigma(r) = \frac{M}{2\pi a^2}\left(1 + \frac{r^2}{a^2}\right)^{-1/2}, \tag{1}$$

and squared circular velocity

$$v_c^2 = \frac{GM}{a}\left(1 - \frac{a}{\sqrt{a^2 + r^2}}\right). \tag{2}$$

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Here, a is the core radius and M is a mass unit, but the total disk has infinite mass and extent, as it must have if it is to give rise to an asymptotically flat rotation curve without a halo component. Evans & Collett (1993) have given a family of DFs for this model characterised by an index n which determines the degree of pressure support; $n = 1$ yields a hot disk (but still with significant rotation) while all orbits become circular as $n \to \infty$.

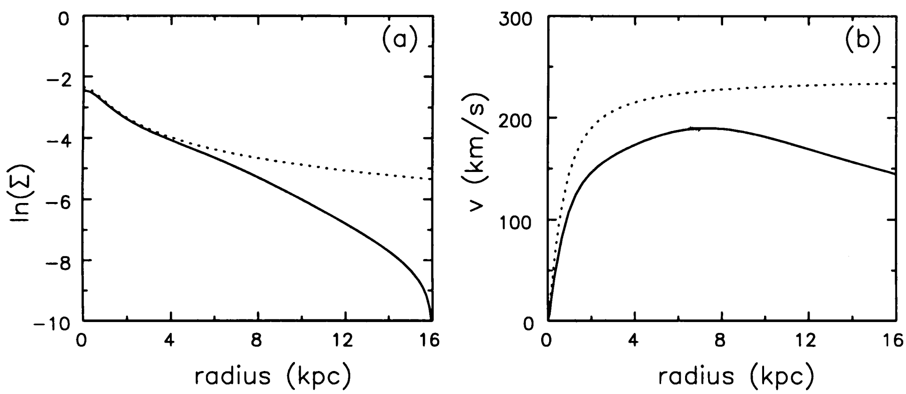

Figure 1. Properties of the adopted Milky Way model. (a) The dotted curve shows the surface density profile of a 60%, untruncated Rybicki disk, the full-drawn curve that obtained after applying the truncation rule described in the text. (The surface density unit is M/a^2.) (b) The circular velocity (km s^{-1}) arising from the truncated, disk only (full drawn) and total rotation curve of the model (dotted).

Since the surface density (1) declines too slowly for a realistic galaxy, and the disk anyway has to be truncated for use in a simulation, I applied the following rules to limit its radial extent: I multiplied the DF by the taper function

$$T(h) = \left[1 + \left(\frac{h}{h_0}\right)^4\right]^{-1}, \tag{3}$$

where h is the specific angular momentum and $h_0 = 10\sqrt{GMa}$. I also applied the further restriction that no particle has sufficient energy that its unperturbed orbit would take it beyond an outer cut-off radius of $r_{\mathrm{max}} = 20a$. With these rules, and choosing $n = 15$, I obtained the surface density profile of active matter shown in Figure 1(a), which is approximately exponential with a mean scale-length of about $4a$. The rotation curve of this considerably altered disk is, of course, no longer given by (1) and it is necessary to supplement the self-consistent central attraction with an additional function in order to maintain the equilibrium of this truncted DF.

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A reasonable scaling of this model to the Milky Way would be to choose $a = 0.8$ kpc and $\sqrt{GM/a} = 240$ km s^{-1}, which implies that $M \simeq 1.07 \times 10^{10}$ M$_\odot$ and one dynamical time ($= \sqrt{a^3/GM}$) is 3.26 Myr. With this scaling, the radial velocity dispersion at the position of the Sun ($r = 8$ kpc) is approximately 47 km s^{-1} and the asymmetric drift is 18.5 km s^{-1}.

There are two undesirable features of the full disk mass distribution arising from this DF. First, the central attraction of the truncated disk *exceeds* that of the untruncated disk at radii between $4a$ and $13a$ (because the missing exterior mass would pull outwards locally); the correcting forces to restore the radial balance would therefore have to be repulsive over this range. Second, the radial velocity dispersion is too low to prevent axisymmetric Jeans instabilities in the truncated disk; the locally defined Q has a minimum of about 0.75 around $r = 6a$. I therefore reduced the active surface density of the disk to 60% of that given by the above description; the extra forces required to restore the radial balance of the disk now arise from a mass distribution (assumed spherical) having a positive density everywhere, and the disk is locally Jeans stable. The rotation curves of the active disk mass and of the total disk-plus-halo distributions are shown in Figure 1(b). The total mass of the disk in this model is $\sim 6.1 \times 10^{10}$ M$_\odot$ and the surface density at the position of the Sun is ~ 83 M$_\odot$ pc^{-2}, which is deliberately on the high side.

Thus far, the vertical structure of the disk has not been specified. The purpose of this study is to determine whether the Milky Way is locally stable to bending modes and whether bending instabilities could in fact have thickened the disk to something close to its current value. These questions can both be answered by a single simulation which is initially thin enough that bending instabilities are certain to be provoked; the non-linear evolution will reveal the extent to which instabilities can thicken the disk and indicate the thickness at which instabilities are shut off. I have therefore adopted the very low value of $z_0 = 0.04a = 32$ pc for the Gaussian scale height of the disk. The vertically integrated, vertical velocity dispersion resulting from this choice is no more than one quarter of the radial dispersion anywhere.

3. Results

I computed the evolution of this model using the grid-based axisymmetric N-body code described in S95. The grid had 200 points in radius and 135 vertically, the spacing in each direction was separately constant but the vertical spacing was 0.16 that in the radial direction in order to ensure five mesh spaces per initial Gaussian scale height of the disk. I employed 50 K particles and a time step of 0.02 dynamical times. Tests indicated that the results were insentive to changes in grid resolution and other numerical parameters.

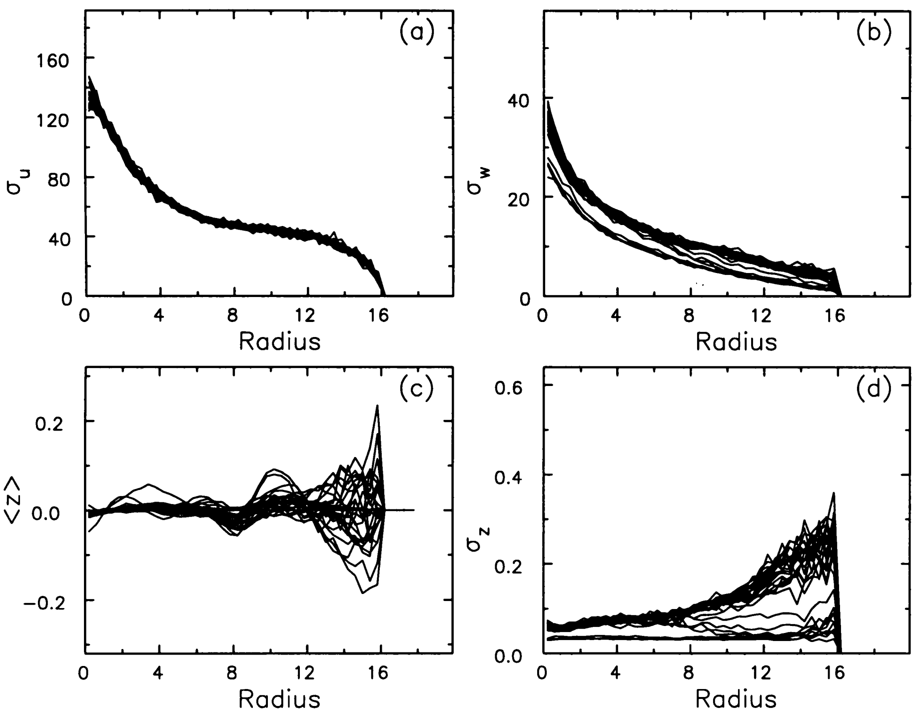

Figure 2. Properties of the Milky Way model, as functions of radius (in kpc), measured at intervals of 20 dynamical times. (a) The radial velocity dispersion (km sec^{-1}) does not change significantly throughout the evolution. (b) The vertical velocity dispersion rises by some 60% at the solar radius ($r = 8$); the vertical scale is set to have a full extent precisely 30% of that in (a). (c) The mean z displacement of the particles (kpc), showing a long lived bend. (d) The rms vertical thickness which started out at a small constant value and increased most at large radii.

The evolution of the model reavealed a number of bending modes which caused a peak displacement of the mid-plane of almost 200 pc, but bending activity appeared to cease after about 1 Gyr. Figure 2 shows profiles of the radial and vertical velocity dispersions and of the mean displacement and rms thickness of the plane at a number of equally spaced moments during the run. The vertical velocity dispersion rises during the first Gyr of evolution only, but does not reach even 30% of the radial dispersion, because the additional restoring force to the plane from the spherical rigid material alters the Toomre-Araki stability criterion. An additional experiment with a less massive disk led to a still flatter final ellipsoid.

Figure 2(d) shows that the disk flares outwards, while Figure 2(c) indicates that it does not flatten completely immediately after the final instability saturates, but maintains a long-lived bend which shows little sign of decay for at least a further Gyr. The persistence of this feature suggests that the observed departures from a perfect plane of the Milky Way may possibly have been created by bending instabilities in the distant past when the plane might have been thinner. I would caution, however, that rings which form in Jeans-unstable models are very persistent when non-axisymmetric forces are inhibited but quickly dissolve when the evolution is unrestricted; if bending modes behave analogously, the bend in the present model might not persist in an unrestricted simulation.

This result confirms Toomre's (1966) conclusion, based on his local criterion alone, that the disk of the Milky Way is sufficiently thick to be well clear of the bending stability boundary. The observed vertical velocity dispersion of old disk stars in the solar neighborhood, some 18–20 km s^{-1} (e.g. Mihalas & Binney 1981), is almost twice that which the simulation indicates could have been created through bending instabilities, even with a high disk surface density. The current thickness must therefore have developed through some other mechanism, probably scattering by gas clouds.

Admittedly this study has been restricted to axial symmetry; non-axisymmetric instabilities are possible, though the prospects of a violation of the Toomre-Araki criterion in the azimuthal direction seems less likely because the azimuthal dispersion is lower and surface density varies only in the radial direction.

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DISCUSSION

K. Chamcham: Observations of Kennicutt (89) show that there is a close correlation between the stability properties of discs at the onset of star formation i.e. star formation starts when the disc becomes unstable and therefore you should include the effect of stars in your criterion. Moreover, once star formation starts, you should also include the effect of cosmic rays and magnetic fields. Don't you think that these effects will change your results?

Sellwood: You are obviously thinking of Jeans-type instabilities, whereas my principal interest in these stellar dynamical simulations was the stability into bending modes. It is well known that Jeans instabilities in discs are significantly affected by a cool dissipative gas component, but I would not expect it to have much effect on the bending stability boundary.