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Letter to Editor

DYNAMICS OF PSEUDOINVERSES' NULL SPACE VECTORS IUT Dépt. GE, 22 Allée, J. Rostand 91025 Evry (France)

Sir,

In a recent paper, Ma and Nenchev[1] have investigated some interesting aspects of the pseudoinverse transformation which is required in robot control schemes. These authors present an exact equation of pseudoinverse's null space vectors, but apparently they only consider the mechanical dynamics. As shown in reference 2, this short Letter points out that a stable behaviour of null space vectors results of the implementation of convenient null space feedback laws in programming pseudoinverse expressions. The usual control laws are derived from on a slightly different version of the formulation provided in reference 1.

The kinematic equation which links workspace and joint space represents the basic relationship of robot motion. But as the Cartesian acceleration is generally needed, such a mapping relationship can be directly derived with respect to time. Alternatively, it is also possible to transform its aspect via the use of a pseudoinverse $\mathbf{J}^+$, before to consider its time derivative. This implies both the projection into the null space of a vector $\mathbf{\Phi}_e$, as well as the handling of its time derivative

$$\dot{\mathbf{q}} = \mathbf{J}^+\dot{\mathbf{x}} + \mathbf{\Phi}_e \tag{1}$$

where $\mathbf{J}^+ = \mathbf{J}^T(\mathbf{J}\mathbf{J}^T)^{-1}$, and

$$\mathbf{\Phi}_e = (\mathbf{I} - \mathbf{J}^+\mathbf{J})\mathbf{v} \tag{2}$$

represents the projection in null space of a vector related to the mechanism, i.e., $\mathbf{\Phi}_e = \mathbf{\Phi}_m$, or the one used in feedback law $\mathbf{\Phi}_e = \mathbf{\Phi}_c$, when equation (1) is implied in the construction of the control algorithm. So, energy conservation law of a workspace controlled mechanism is given by

$$\dot{\mathbf{q}}^T\mathbf{M}(\mathbf{J}^+\ddot{\mathbf{x}} + \dot{\mathbf{\Phi}}_m - \mathbf{J}^+\{\ddot{\mathbf{x}}_d + \mathbf{K}_v\dot{\varepsilon} + \mathbf{K}_p\varepsilon\} - \dot{\mathbf{\Phi}}_c) = 0 \tag{3}$$

where $\mathbf{M}$ is the inertial matrix of the redundant mechanism. The two first expressions of (3) concern the mechanism itself, while the last two terms are due to the control laws. From the derivation of (3), observe that the time derivative of $\mathbf{\Phi}_e$, (i.e., $\mathbf{\Phi}_m$ or $\mathbf{\Phi}_c$) implies the time derivative of both $\mathbf{J}$ and $\mathbf{J}^+$. As given in [1], its general form is given by

$$\dot{\mathbf{\Phi}}_e = (\mathbf{I} - \mathbf{J}^+\mathbf{J})\dot{\mathbf{v}} - (\mathbf{J}^+\dot{\mathbf{J}}\mathbf{J}^+ + \dot{\mathbf{J}}^+)\mathbf{J}\mathbf{v}. \tag{4}$$

where $\mathbf{v}$ is an usual vector. Note that due to (1), this vector describes some speed in null space, as $\mathbf{v} = \mathbf{g} - \dot{\mathbf{q}}$, for example, and $\mathbf{g}$ is a vector function. In this particular case, the time derivative of the mechanical $\mathbf{\Phi}_m$ vector becomes

$$\dot{\mathbf{\Phi}}_m = (\mathbf{I} - \mathbf{J}^+\mathbf{J})(\dot{\mathbf{g}} - \ddot{\mathbf{q}}) - (\mathbf{J}^+\dot{\mathbf{J}}\mathbf{J}^+ + \dot{\mathbf{J}}^+)\mathbf{J}\mathbf{v}. \tag{5}$$

When implementing a replica of previous relationship into the control algorithm, it is possible to modify its expression since such a design allows a large freedom. Taking advantage of this fact to control the null space dynamics, as in [2], it is possible to implement in the control algorithm the following term

$$\dot{\mathbf{\Phi}}_c = (\mathbf{I} - \mathbf{J}^+\mathbf{J})(\dot{\mathbf{g}} + \mathbf{K}_\phi\mathbf{\Phi}_m) - (\mathbf{J}^+\dot{\mathbf{J}}\mathbf{J}^+ + \dot{\mathbf{J}}^+)\mathbf{J}\mathbf{v} \tag{6}$$

where $\mathbf{K}_\phi$ is a positive feedback gain matrix, the elements of which specify the dynamics of vectors in the null space. Reworking (3) to get $\ddot{\mathbf{q}}$, combining with (5, 6), and taking into account the fact that $(\mathbf{I} - \mathbf{J}^+\mathbf{J})\mathbf{J}^+ = 0$, the introduction into (5) of the mechanism dynamics yields

$$\dot{\mathbf{\Phi}}_m = -(\mathbf{I} - \mathbf{J}^+\mathbf{J})\mathbf{K}_\phi\mathbf{\Phi}_m. \tag{7}$$

In particular, this shows us that the following mechanical energy

$$I = \frac{1}{2}\mathbf{\Phi}_m^T\mathbf{\Phi}_m \tag{8}$$

becomes a decreasing function, since its time derivative reduces to

$$\frac{dI}{dt} = \mathbf{\Phi}_m^T\dot{\mathbf{\Phi}}_m = -\mathbf{\Phi}_m^T\mathbf{K}_\phi\mathbf{\Phi}_m. \tag{9}$$

As the right-hand side of (9) is always a negative number, this implies that $\|\mathbf{\Phi}_m\| \to 0$, $\|\mathbf{v}\| \to 0$, and in the previous example, $\|\mathbf{g} - \dot{\mathbf{q}}\| \to 0$. Then, the null space energy goes down to zero in a stable manner, and this can be done in performing a stable minimization subtask, with a user prescribed dynamics. So, note that there is no kinetic energy stored in null space when (6) is implemented in the governing control laws. Finally, this Letter notes that no instability should appear in workspace redundant controllers, if in addition to the usual control laws, the null space dynamics is also controlled.

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